LS/LSIT VIDEO EXAM PREPARATION COURSE

WORKBOOK

EDITED BY

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Department of Transportation
Division of Engineering Management
Office of Engineering Technology
Geometronics Branch

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INTRODUCTION

This course is designed to help you prepare for the Land Surveyor in Training Examination (LSIT) and the California Land Surveyor Exam (LS). It should be used as a part of your preparation for the exams, not your only preparation. The course introduces typical topics covered on the two exams and can be used as a bench mark against which to measure your readiness for exam questions.

The course includes a set of video tapes and a workbook and is composed of 16 units: 15 on different surveying topics and one on general exam preparation. Each unit was authored by a practicing licensed land surveyor. The course is designed for both the LSIT and LS exams, so you will find some units more useful than others depending on which exam you are taking.

The units designed for the LSIT exam are Basic Survey Math, Basic Measurements, Traversing, Leveling, and Areas and Volumes. Units written with LS candidates in mind are The Global Positioning System, California Coordinate System, U. S. Public Land Survey System, Principles of Boundary Determination, Water Boundary Location, Legal Descriptions, Photogrammetry and California Law for Surveyors. The units on Exam Preparation, Route Surveying and Azimuth Determination by Celestial Observation are not as easily categorized.

This is not to say that an LS candidate need not review Basic Measurements or an LSIT candidate need not review Water Boundary Location. Those taking the LS exam should be thoroughly familiar with all of the material covered for the LSIT. LSIT candidates must be familiar with the LS topics, with the understanding that LSIT problems will not be as complex or require the same expertise as problems on the LS Exam.

The course can be used to best advantage by treating the table of contents like a menu. Units can be reviewed in any particular order you choose. Prioritize the units based on your personal needs. Work through each unit you select by reviewing the workbook, watching the video tape and solving the problems. Completing work on a unit will help you develop an accurate assessment of your exam readiness for the topic.

Each workbook unit begins with an introductory statement and a list of the types of problems you are most likely to encounter on an exam. Units also include a list of key terms, a video outline, a set of sample problems and their solutions and a reference section.

The “Introduction” and “Performance Expected on the Exams” sections of the workbook list types of questions posed on exams as well as the breadth of the subject matter in the unit. These sections are designed to give you a taste of what will appear on the video.

The “Key Terms” and “Sample Test Questions” sections of the workbook are designed to test your knowledge of the topics. You will have difficulty on the exam if you do not acquire the basic vocabulary of the profession. Sample test questions are designed by presenters to both test general knowledge of surveying and to give you practice with the type of questions you are most likely to encounter on the tests. Keep in mind that the LSIT is a closed book test with a multiple choice format.

Some of the sample test problems are from past LS exams. These problems are identified by stating the problem number and year the problem appeared on the exam as in the following example: Problem D-5, 1984 LS. You should work as many old exam problems as possible. LS exams from previous years can be purchased from the California State Board of Registration for Professional Engineers and Land Surveyors for $3.23 per test.

The “Answer Key” not only gives you solutions to the problems, but in most cases shows the solution methodology. Knowing the problem solving methodology is the key to success on the exams.

Finally, the “Reference” section at the end of each unit is provided as a guide to further study. References given are not meant to be all inclusive, but reflect the preference of each individual instructor.

Each workbook unit contains a “Video Presentation Outline,” which serves as a preview of the information presented in the video. More importantly it provides a framework for note taking while viewing the video. You can personalize the video outline, which includes formulas, sketches and problem solutions from the video, with your own comments and notes. The workbook Video Presentation Outlines, annotated with your own notes, can be used as a reference when you take the LS exam.

Video tapes for each unit vary in their format, length and method of presentation. In presenting the material, the instructors have used their experience and personal teaching styles to emphasize certain important facts. The tapes, being only one to two hours long, are not meant to be a comprehensive teaching lecture. They are designed to review topics likely to appear on the exams, offer exam-taking tips, and present useful information such as mathematical formulas and problem-solving methodologies.

This course can help organize your test preparation. After using the course to identify topics for further study, you should develop a plan to increase your knowledge in these areas. Your program of study might include seminars, college courses or simply studying one of the texts recommended in the reference sections in the workbook. Certainly you should find and solve as many surveying problems as possible. There are several books of problems available and referenced in the workbook chapters.

When all is said and done, the key to success on the exam is based on thorough knowledge of the subject matter and lots of practice solving problems. As Robert McComb says, the most important principle of exam preparation is “study, study, study.”

ACKNOWLEDGMENTS

We are all indebted to the creative efforts of the instructors, who worked so hard in an unfamiliar medium. Thank you Richard Burns, Lester E. Carter, Jr., Don D'Onofrio, Mitchell Duryea, Jeremy Evans, R. W. "Russ" Forsberg, Don Hunter, Bill Jackson, Carrol Leong, Jim McCavitt, Robert McComb, Roy Minnick, Charles Safford, John Sands, Vincent J. Sincek, C. J. Vandegrift, and Edward Zimmerman.

The instructors are highly visible, but the project was equally dependent on many other good people and their organizations. Thanks to Larry Fenske and Dave Goodman of Caltrans Geometronics for their inspiration; Edward Zimmerman, chief of Caltrans Geometronics Publications and Training Section; who was both instructor and administrator; and Jim Comins and Elisa Holmes of Los Rios Community College District Training Source, who guided us through the publishing process.

Thanks also to Kirk Wiecking, Director of the Sacramento City College Media Resources Center, whose sure-footedness and insistence on proper planning kept us on track; Terry Hajec, Graphic Artist at Sacramento City College, who managed production of graphics and the workbook; Jennifer Martin, video producer, who eased everyone's anxieties; Dave Sauer, Engineer of the Sacramento City College TV Studio, and his studio crew, Mike Colozzi and Allen Elston; Ric Hornor of Electric Canvas, who created the video and print graphics; Sarah Toll of Electric Page, who typeset the workbook and Kakwasi Somadhi of Sacramento City College, who proofread the workbook.

L.E. Luke Wilson

Editor

UNIT 1

EXAM PREPARATION

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Introduction

This video course is designed to help you prepare for what will probably prove to be the most important tests in your career as a professional land surveyor, the Land Surveyor in Training (LSIT) and the Land Surveyor (LS) exams. This unit will cover application for admittance to the exams, content of the exams, preparation for the exams, test taking strategy, and appeals.

Natural abilities and past experience both have an impact on how individuals perform in exam situations. Some people test well because they handle the stress of the testing situation well, or have had the benefit of testing experience from college work. The single most important attribute necessary for success is the self-confidence that comes from thoroughly understanding the subject.

Performance Expected on the Exams

Apply successfully to take the exam.

List the topics covered and the type of questions used on the exam.

Develop a study plan. Elicit support for your plan from family, friends, and employer.

Develop a test taking strategy for the exam.

Outline the appeals process for the exam.

Key Terms

Responsible field training

Multiple choice

Responsible office training

Factual questions

Application questions

References (on exam)

References (on application)

Reciprocity

Comity

Grandfather clause

Certified transcript

Video Presentation Outline

Applying to Take the Exams

- **For Applications and Information**

  Board of Registration for Professional
  Engineers and Land Surveyors
  P. O. Box 659005
  Sacramento, CA 95865-9005
  (916) 920-7466

- **Board Publications**

  Information for Licensing as a Land Surveyor Plain Language Pamphlet of the Land Surveyor's Act and Board Rules

- **Eligibility Requirements**
  - **Land Surveyor in Training Exam**
    For information and sample test, write:
    National Council of Engineering Examiners
    P. O. Box 1686
    Clemsen, SC 29633-1686

  - **Land Surveyor Exam**
    Experience
    - Responsible field training
    - Responsible office training
    - Employment verification forms

    Education
    References

Exam Preparation

- A place to study
- A schedule for study
- A strategy for study
  - Review
  - Problem solving
- Study aids

Taking the Exam

- What to bring to the exams
- Exam strategy

Overview of California Land Surveyor Exams

- **Land Surveyor in Training Exam**
  - Closed book
  - Multiple choice questions on the fundamentals of land surveying
  - More emphasis recently on boundary and public land principles

- **Land Surveyor Exam (eight hour open book)**
  - Emphasis on:
    - Application of principles
    - Making professional judgments

  - Topics covered on exams 1987–1992 *(by % of total points for each topic)*

61%	Boundary determination and legal descriptions
8%	California statute law
9%	State plane coordinates
8%	Photogrammetry
7%	Construction, route surveying
5%	Astronomy
2%	Other (error analysis, GPS)

  - Other topics
    - From past exams
    - Probable future topics

- **Review of 1990 Exam**

- **Review of 1991 Exam**

The Appeals Process

- 60 day appeal period
- Materials provided by the Board
- Strategy

References

Brinker, Russell C., *4567 Review Questions for Surveyors*, 1978.

Chelapati, C. V., ed., *Land Surveyor License Examination Review Manual*, Professional Engineering Development Publications, Long Beach, CA, 1987.

Harbin, Andrew L., *Land Surveyor Reference Manual*, Professional Publications, San Carlos, CA, 1985.

Minnick, Roy, *Land Survey Test Training Manual*, Landmark Enterprises, Rancho Cordova, CA, 1972.

Safford, Charles, *Syllabus for Land Surveyor Examinations*, Landmark Enterprises, Rancho Cordova, CA, 1988.

UNIT 2

BASIC SURVEY MATH

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California Department of Transportation

Introduction

The purpose of this video unit is to present basic math concepts and principles useful to survey computations. It has been assumed that most viewers are already familiar with some or most of the topics presented in the beginning of the unit. It is important to have a developed understanding of the basic operations of arithmetic, algebra, geometry, and trigonometry. This unit is not designed as a complete math course, but rather as an overview and guide to computation processes unique to surveying and mapping. Surveyors who need to work on math operations and fundamental skills addressed in the video will find sources for further study in the reference section at the end of this unit.

Survey mathematics generally consists of applications of formulas and equations that have been adapted to work toward the specific needs of the surveyor such as:

- Units of measurement and conversions
- Check and adjustment of raw field data
- Closure and adjustment of survey figures
- Calculations for missing elements of a figure
- Working with coordinates (COGO)
- Intersections of straight lines and of circles

It is hoped this video unit will help viewers to recognize solution formats for problems and then make correct and effective use of appropriate methods to solve these particular survey problems.

Performance Expected on the Exams

Recognize solution formats, and make correct and effective use of appropriate mathematical solutions to particular survey applications.

Key Terms

Absolute value
Adjacent side
Algebra
Arc
Arithmetic
Azimuth
Bearing
Central angle
Chord
Circular curve
Circumference
Complementary angle
Coordinate conversion
Cosecant
Cosine
Cotangent
Cubes
Decimal system
delta x, delta x
Departure
External distance
Geodetic north
Grads
Grid north

Hexagon

Horizontal curve

Hypotenuse

Intersections

Intersection of straight line and arc

Intersections of straight lines

Inverse processes

Latitude

Law of cosines

Law of sines

Length of arc

Magnetic north

Meter

Mid-ordinate distance

Most probable value

Oblique triangle

Opposite side

Order of operations

Parabola

Parallelogram

Pentagon

Percent of slope

Percentage

pi

Plane geometry

Polar coordinates

Polygon

Pythagorean theorem

Quadrants

Quadratic equation

Quadrilateral

Radian

Radius

Radius point

Random error

Rate of change

Rectangular coordinates

Residual

Rhomboid

Right triangle

Roots

Rounding off

Sag curve

Secant

Sector of a circle

Segment of a circle

Sexagesimal system

Signed numbers

Significant figures

Simultaneous equation

Sine

Square root

Squares

Standard error

Supplementary angles

US survey foot

Tangent

Trigonometry

Video Presentation Outline

Arithmetic

- Decimal system
- Rounding off and significant figures
- Percentage
- Squares, cubes and roots

Conversion of Units of Measure

- Converting lineal units
- Converting angular units
- Converting units of area

Random Error Analysis

- Error definitions
- Error residuals
- Statistical error matrix
- Propagation of error
- Error in summation
- Error in product
- Error in series

Algebra

- Signed numbers
- Equations
- Order of operations
- Parentheses
- Evaluating equations and combining terms
- Solving equations
- The quadratic equation formula

Plane Geometry

- Angles
- Geometrical theorems
- Geometrical figures
- Polygons
- Triangles

Trigonometry

- Right triangles
- Pythagorean theorem
- Trigonometric functions
- Oblique triangles
- Directions: bearings and azimuths
- Latitudes and departures
- Plane coordinates

Coordinate Geometry

- Intersection of straight lines
- Intersection of straight line and arc
- Intersection of two arcs

Sample Test Questions

1. The product of 416.78 multiplied by 210.98 is?

   A. 879.32
   B. 8,793.32
   C. 87,932.24
   D. 879,322.44

2. The quotient of 36.11 divided by 191.67 is?

   A. 188.40
   B. 18.84
   C. 1.88
   D. 0.19

3. Square the number 0.713729, showing the results to the nearest five decimal places.

   A. 0.50941
   B. 0.50940
   C. 0.50942
   D. 0.50943

4. The percentage of slope for a proposed ramp is +3.55%. What is the change in elevation of this ramp for a horizontal length of 356 ft?

   A. -126.38 ft
   B. +12.60 ft
   C. +12.64 ft
   D. +126.38 ft

5. Where the centerline slope of a highway has a vertical drop of 14.75 ft in 265 ft horizontally, what is the rate of change expressed in percentage?
   A. 0.55%
   B. 0.56%
   C. 5.55%
   D. 5.57%

6. Determine the square root of 0.6935, showing the result to the nearest five decimal places.

   A. 0.832776
   B. 0.83276
   C. 0.83277
   D. 0.832766

7. 24.91 expressed in ft and in, equals:

   A. 24 ft, 10-7/8 in
   B. 24 ft, 10-3/8 in
   C. 24 ft, 10-1/4 in
   D. 24 ft, 11 in

8. 4,178.309 meters equals _______ United States survey ft.

   A. 1,273.56 survey ft
   B. 1,273.55 survey ft
   C. 13,708.20 survey ft
   D. 13,708.34 survey ft

9. 6,172.98 United States survey ft equals _______ meters.

   A. 1,881.528 m
   B. 1,881.547 m
   C. 20,252.313 m
   D. 20,252.519 m

10. When converted to survey ft, 3,421.381 meters equals ________ survey ft.

    A. 1,042.84 survey ft
    B. 1,042.85 survey ft
    C. 11,224.87 survey ft
    D. 11,224.98 survey ft

11. 21.56 chains converts to ______ survey ft.

    A. 1,422.36 survey ft
    B. 1,386.00 survey ft
    C. 1293.60 survey ft
    D. 1,422.96 survey ft

12. 14° 34' 37" converted to radian measurement is ______?

    A. 0.250345 rad
    B. 0.254416 rad
    C. 0.250351 rad
    D. 0.250337 rad

13. 0.758612 rad, when converted to degrees, minutes, and seconds is ______.

    A. 43° 46' 52"
    B. 43° 41' 35"
    C. 43° 33' 12"
    D. 43° 27' 55"

14. How many hectares are contained in a rectangular parcel that measures 19.23 ch. x 40.63 ch.?

    A. 78.131 hec
    B. 781.315 hec
    C. 31.619 hec
    D. 193.063 hec

15. An angle has been measured six individual times with the following results: a.) 46° 21' 45"; b.) 46° 22' 10"; c.) 46° 22' 05"; d.) 46° 22' 00"; e.) 46° 21' 45"; f.) 46° 21' 55". What is the most probable value of the angle?

    A. 46° 21' 45"
    B. 46° 21' 50"
    C. 46° 21' 57"
    D. 46° 22' 00"

16. Determine the standard error for the following group of six measurements: a.) 11,249.71 ft; b.) 11,250.06 ft; c.) 11,249.86 ft; d.) 11,249.99 ft; e.) 11,250.01 ft; f.) 11,249.98 ft.

    A. ±0.13 ft
    B. ±0.12 ft
    C. ±0.10 ft
    D. ±0.08 ft

17. Determine the standard error of the mean for the measurement set in problem #16.

    A. ±0.21 ft
    B. ±0.13 ft
    C. ±0.05 ft
    D. ±0.03 ft

18. A rectangular parcel of land was surveyed. The measurement for side X was 339.21 ft with an error of ±0.05 ft. Side Y was measured as 563.67 ft, with an error of ±0.09 ft. What is the area of the parcel and what is the expected error in the area?

    A. Area = 191,202 ft^2 or 4.389 ac.; standard error = ±41.7 ft^2
    B. Area = 191,202 ft^2 or 4.389 ac.; standard error = ±41.5 ft^2
    C. Area = 191,202 ft^2 or 4.389 ac.; standard error = ±24.1 ft^2
    D. Area = 191,202 ft^2 or 4.389 ac.; standard error = ±53.5 ft^2

19. The total length for a highway centerline was measured in four different segments using different equipment and different methods of measurement on different days. The total length of the line was found by totaling the length of each segment. Standard error for each segment was determined to be:

Standard Error of Segment #1 =±0.04 ft
Standard Error of Segment #2 =±0.03 ft
Standard Error of Segment #3 =±0.08 ft
Standard Error of Segment #4 =±0.11 ft

The standard error of the total distance of the centerline is _______?

A. Standard error of the sum = ±0.14 ft
B. Standard error of the sum = ±0.26 ft
C. Standard error of the sum = ±0.02 ft
D. Standard error of the sum = ±0.07 ft

20. What is the sum of the following five numbers: (-230.67); (+517.39); (+100.26): (-311.47); and (-481.28)?

A. 405.77
B. 1,641.07
C. -1,641.07
D. -405.77

21. The remainder after -146.11 has been subtracted from -37.82 is ______ ?

A. -108.29
B. 108.29
C. -183.93
D. 183.93

22. Write an equation based on the following word statement: "three times a number, plus the number cubed, minus the number multiplied by 87." In the algebraic equation, let b stand for the number referred to in the problem statement.

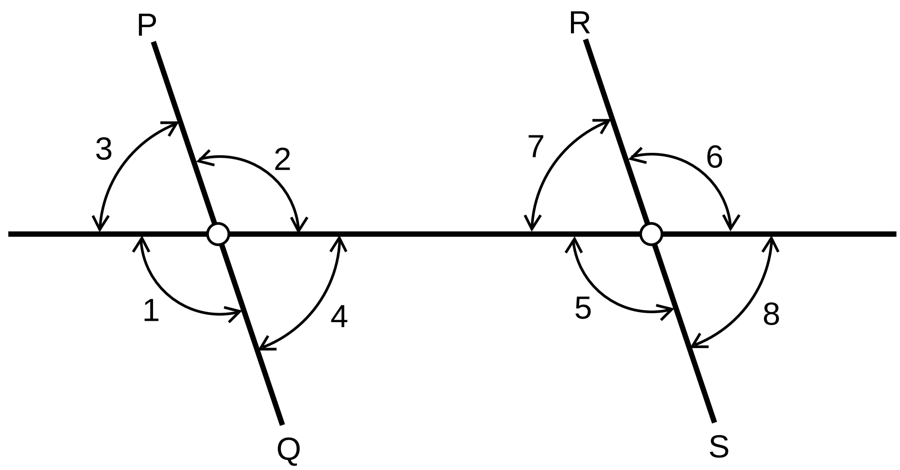
A. $3\ (b + b^{3)} - (87b)$
B. $3\ (b + b^3) - 87b$
C. $3\ b + b^3 - 87b$
D. $(3b) + b^3 - (87b)$

23. Letting w = 12 and z = 3, evaluate the following equation:
$5w + (21 - w)\ 14z + (z - 23)$.

A. 418
B. 2,878
C. 2,881
D. 19,162

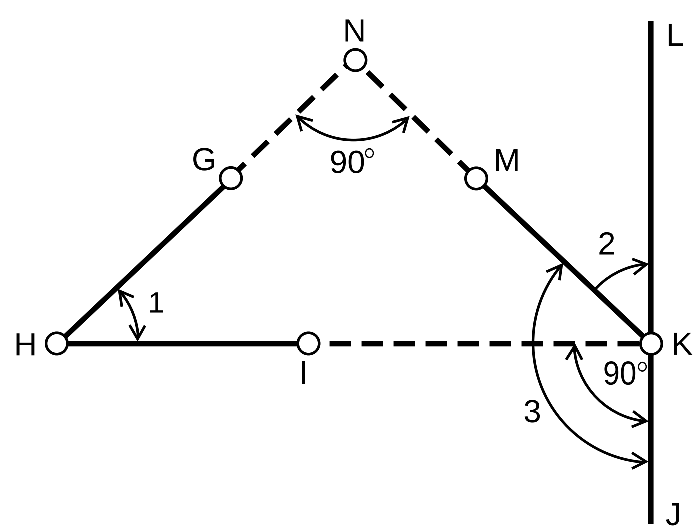
24. If angle 3 in the sketch below is 71° 39' 12", calculate the values of angles 1, 2, 5, and 8. Assume lines P-Q and R-S are parallel.

A. <1 = 108° 20' 48"; <2 = 108° 20' 48"; <5 = 108° 20' 48"; <8 = 71° 39' 12"
B. <1 = 108° 20' 48"; <2 = 71° 39' 12"; <5 = 71° 39' 12"; <8 = 108° 20' 48"
C. <1 = 108° 20' 48"; <2 = 71° 39' 12"; <5 = 108° 20' 48"; <8 = 71° 39' 12"
D. <1 = 108° 20' 48"; <2 = 108° 20' 48"; <5 = 71° 39' 12"; <8 = 71° 39' 12"



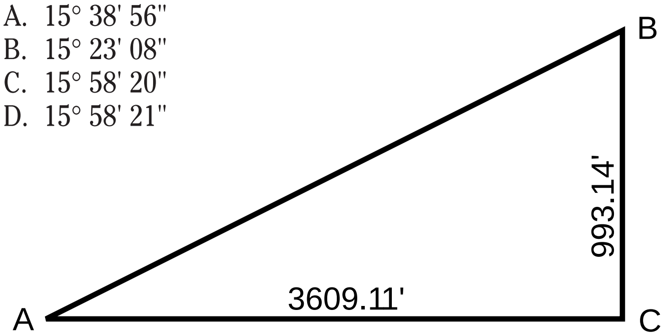
25. If angle 1 in the sketch below is 46° 11' 20", calculate the values of angles 2 and 3.

A. <2 = 46° 11' 20"; <3 = 133° 48' 40"
B. <2 = 43° 48' 40"; <3 = 133° 48' 40"
C. <2 = 46° 11' 20"; <3 = 136° 11' 20"
D. <2 = 43° 48' 40"; <3 = 136° 11' 20"



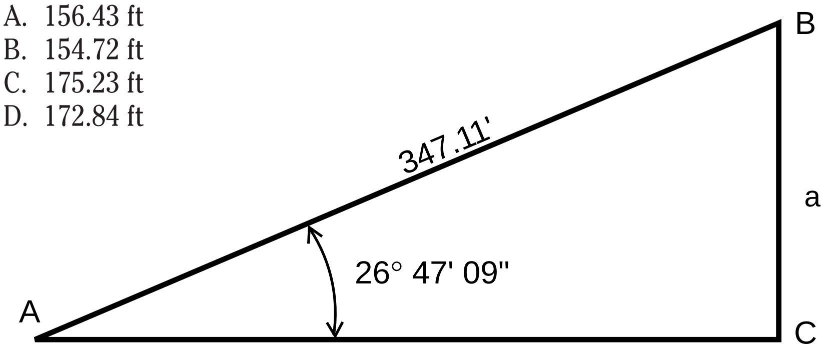
26. Solve for angle A in the triangle shown below:

A. 15° 38' 56"
B. 15° 23' 08"
C. 15° 58' 20"
D. 15° 58' 21"



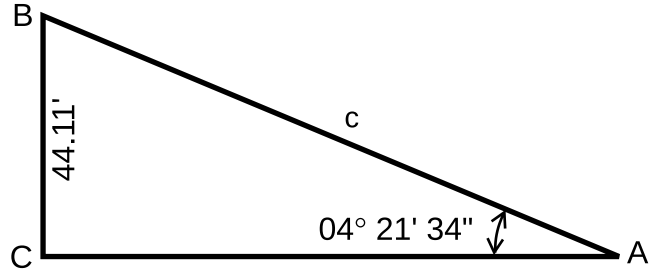
27. Solve for the missing side “a” of the triangle in the sketch below.

A. 156.43 ft
B. 154.72 ft
C. 175.23 ft
D. 172.84 ft



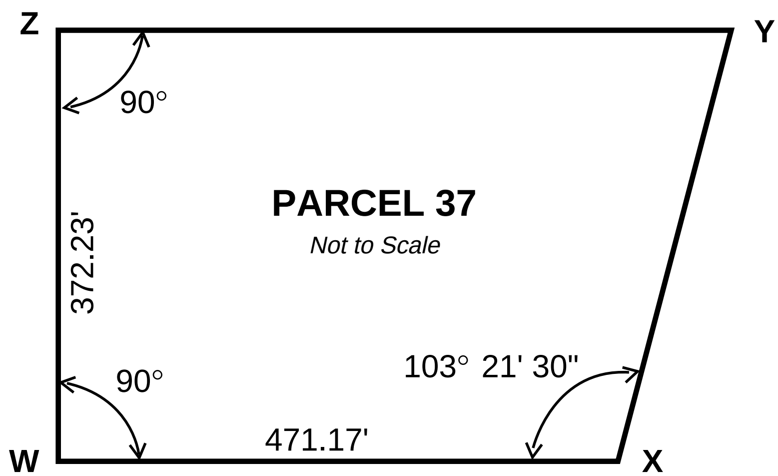
28. What is length of side “c” in the triangle shown in the sketch below?

A. 578.61 ft
B. 598.75 ft
C. 600.36 ft
D. 580.29 ft



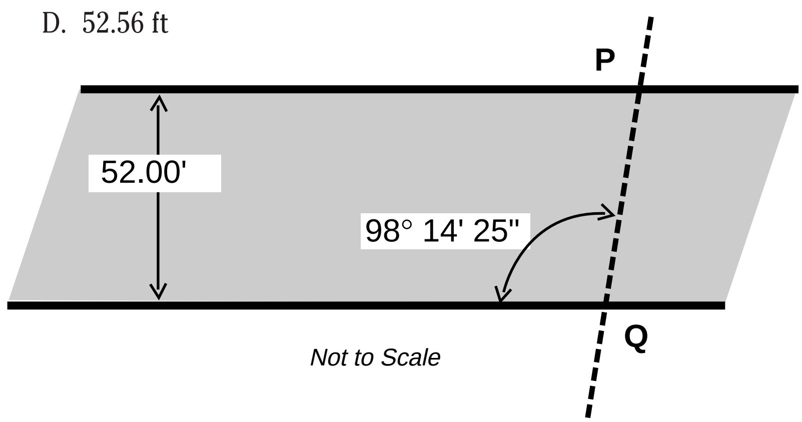
29. What is the length of side Y to Z in the sketch of parcel #37 shown below?

A. 557.17 ft
B. 559.56 ft
C. 558.56 ft
D. 556.25 ft



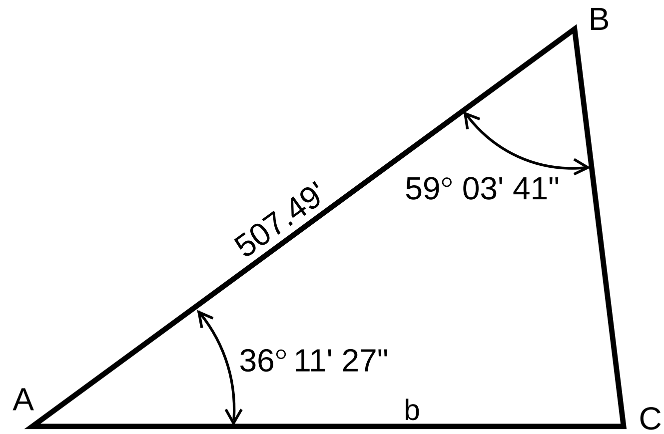
30. Line P to Q intersects the street alignment as shown in the sketch below. What is the length of line P-Q?

A. 52.50 ft
B. 52.52 ft
C. 52.54 ft
D. 52.56 ft



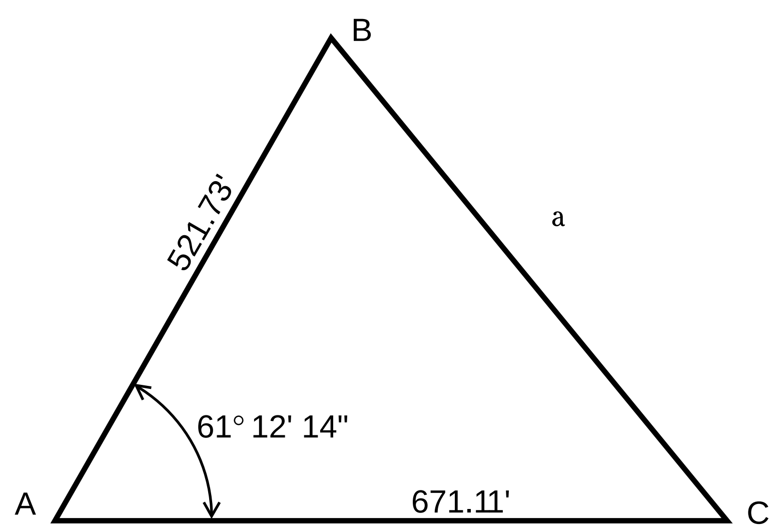
31. Given the following dimensions shown for the oblique triangle in the sketch below, solve for the length of side “b.”

    A. 855.58 ft
    B. 857.02 ft
    C. 438.34 ft
    D. 37.12 ft



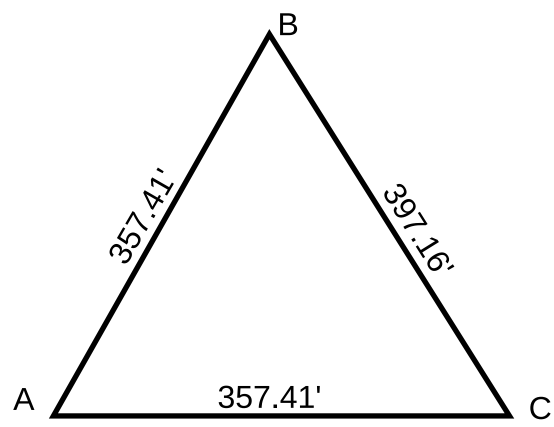
32. Solve for side “a” using the elements given for the oblique triangle shown in the sketch below.

    A. 751.27 ft
    B. 618.43 ft
    C. 744.27 ft
    D. 620.70 ft



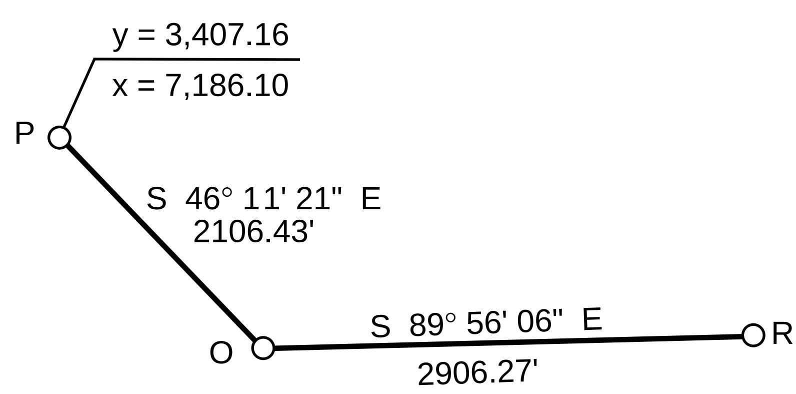
33. From the elements of the oblique triangle given in the sketch below, solve for angle A.

A. 68° 06' 10"
B. 67° 54' 41"
C. 67° 50' 53"
D. 67° 30' 19"



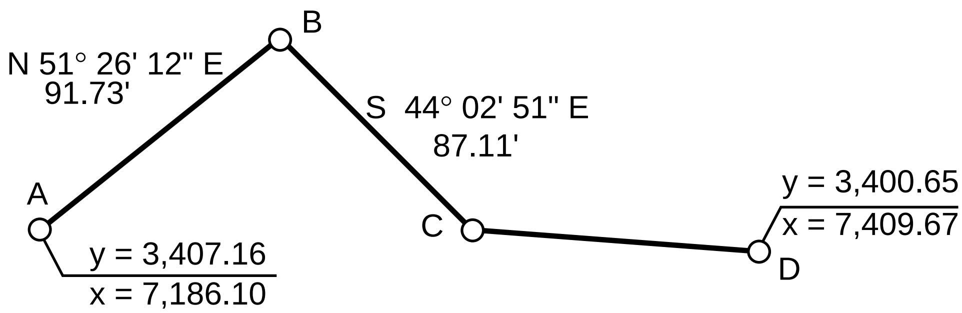
34. Using information given in the sketch below, calculate the coordinates for point R.

A. y = 1,945.62; x = 11,612.43
B. y = 1,945.73; x = 11,612.39
C. y = 1,945.61; x = 11,612.33
D. y = 1,945.68; x = 11,612.39



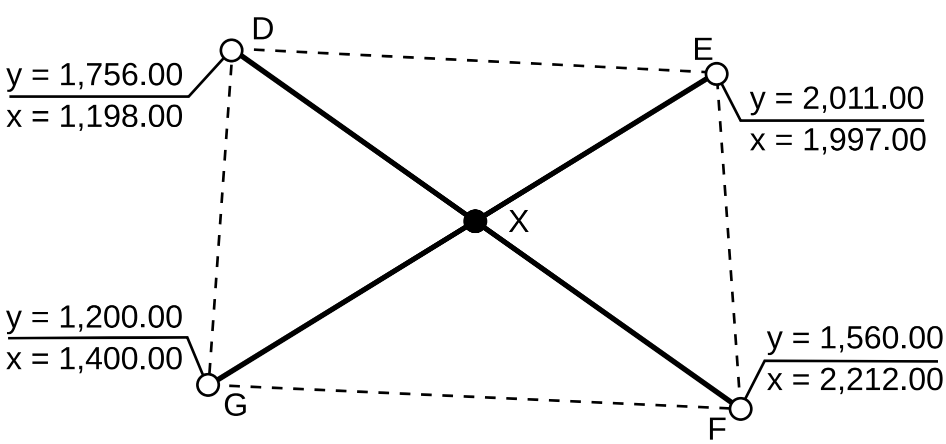
35. After looking at the sketch below, determine the bearing and distance of the line from point C to point D.

A. N 89° 19' 27" W; 91.33 ft
B. S 89° 49' 06" W; 91.31 ft
C. S 89° 19' 27" E; 91.33 ft
D. S 89° 19' 17" E; 91.29 ft



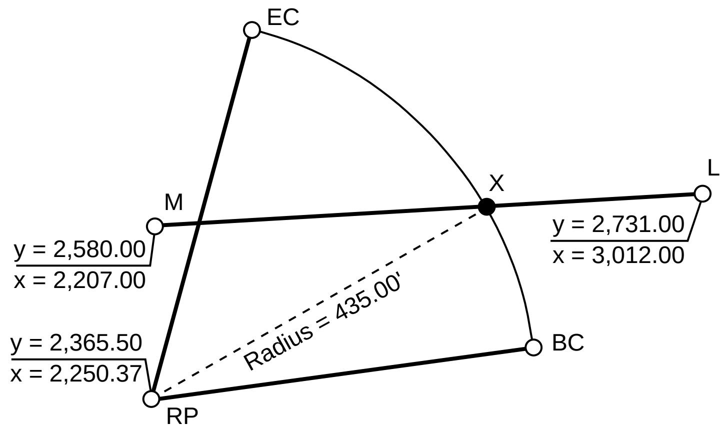
36. Determine the coordinates of the point "x"where the two lines intersect as shown in the sketch below.

A. y = 1,652.56; x = 1,733.14
B. y = 1,652.47; x = 1,732.99
C. y = 1,652.64; x = 1,733.24
D. y = 1,652.53; x = 1,733.19



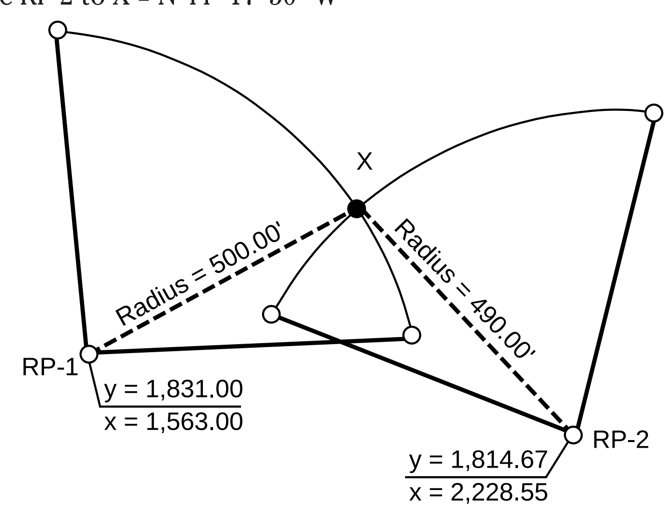
37. Using information given in the sketch, calculate the bearing of the line from the "RP" to point X. Also determine the distance from L to X.

A. Bearing "RP" to X = N 48° 51' 10" E; Distance L to X= 438.71 ft
B. Bearing "RP" to X = N 50° 51' 50" E; Distance L to X= 439.85 ft
C. Bearing "RP" to X = N 49° 10' 30" E; Distance L to X= 440.00 ft
D. Bearing "RP" to X = N 49° 12' 40" E; Distance L to X= 441.07 ft



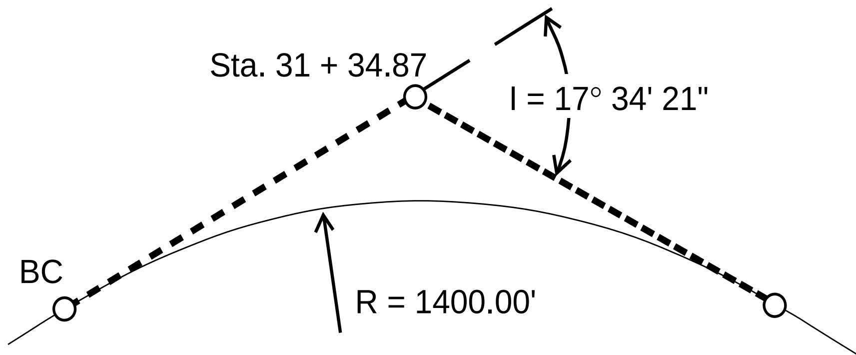
38. Determine the bearing of lines RP-1 to X and line RP-2 to X from the survey data shown in the sketch below.

A. Bearing Line RP-1 to X = N 44° 17' 50" W;
Bearing Line RP-2 to X = N 40° 12' 45" E
B. Bearing Line RP-1 to X = 40° 12' 45" E;
Bearing Line RP-2 to X = N 40° 12' 45" W
C. Bearing Line RP-1 to X = N 44° 17' 50" E;
Bearing Line RP-2 to X = N 40° 12' 45" W
D. Bearing Line RP-1 to X = N 40° 12' 45" W;
Bearing Line RP-2 to X = N 44° 17' 50" W



39. Determine the deflection angle and the sub-chord length (from beginning of curve) required to locate sta. 30+74.50 on its correct position on the arc using data given in the sketch below.

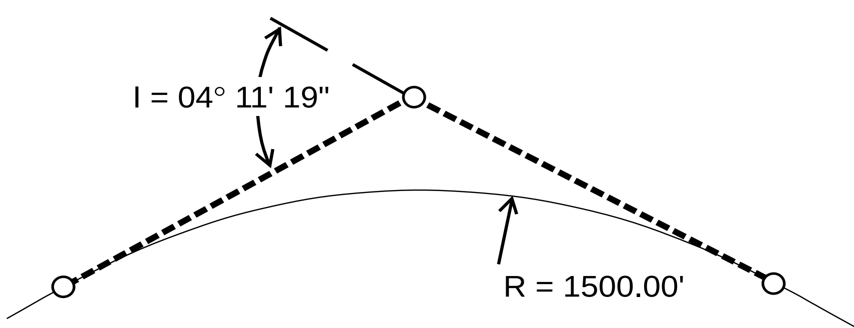
A. Deflection = 03° 11' 33"; Sub-chord = 155.94 ft
B. Deflection = 06° 23' 06"; Sub-chord = 155.69 ft
C. Deflection = 06° 23' 06"; Sub-chord = 311.48 ft
D. Deflection = 03° 11' 33"; Sub-chord = 152.07 ft



Not to Scale

40. Using curve information given in the sketch, calculate the external and mid-ordinate distances for the curve.

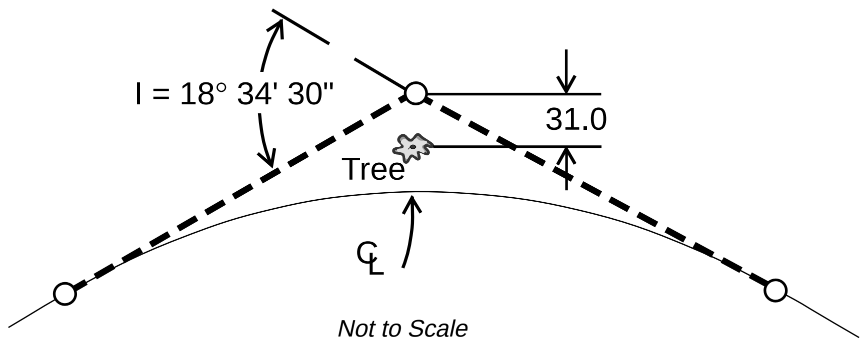
A. External = 0.37 ft; Mid-ordinate = 0.37 ft
B. External = 1.00 ft; Mid-ordinate = 1.00 ft
C. External = 1.04 ft; Mid-ordinate = 1.04 ft
D. External = 4.15 ft; Mid-ordinate = 4.14 ft



Not to Scale

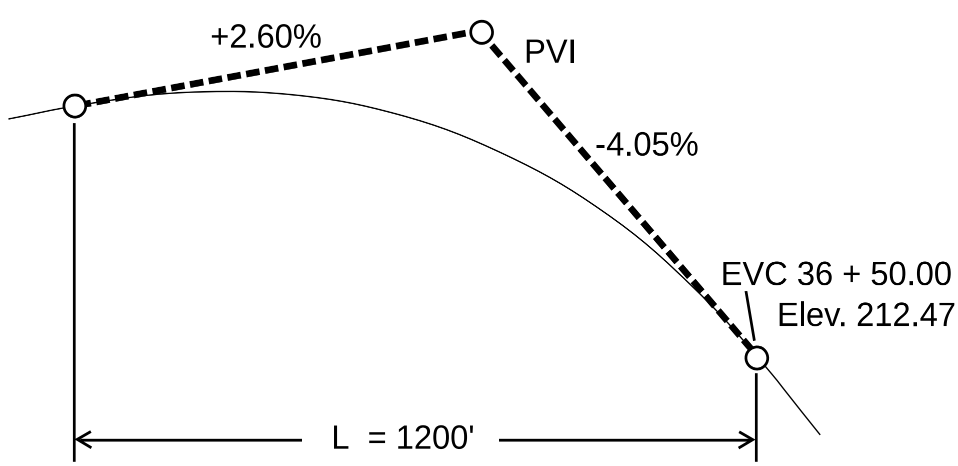
41. Using the data given in the sketch below, calculate the centerline radius that will allow the outside edge of a 42-ft roadway (overall width) to clear the center of the tree by 6 ft.

A. C/L radius = 4,366.42 ft
B. C/L radius = 4,478.80 ft
C. C/L radius = 4,424.92 ft
D. C/L radius = 4,436.80 ft



42. From data given on the sketch of the vertical curve below, calculate the elevation at station 31+56.

A. Elev. @ sta. 31+56 = 225.02 ft
B. Elev. @ sta. 31+56 = 225.25 ft
C. Elev. @ sta. 31+56 = 225.47 ft
D. Elev. @ sta. 31+56 = 225.72 ft

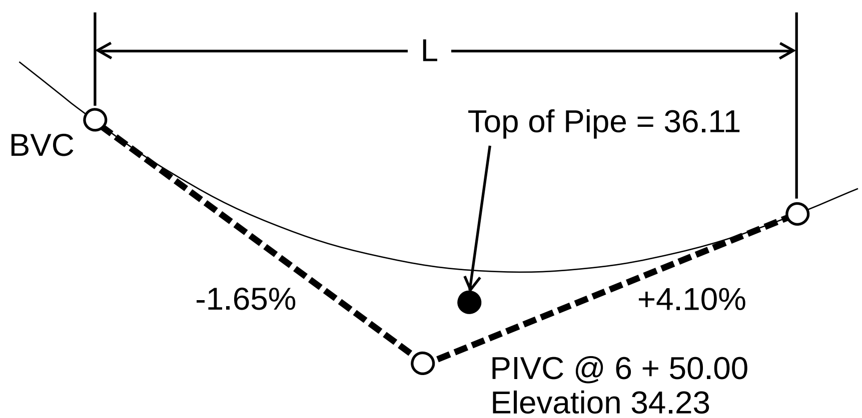


43. Referring back to Problem #42 (above), calculate the station and elevation of the high point of the curve.

A. Sta. = 29+19; Elev. = 205.74 ft
B. Sta. = 29+19; Elev. = 227.27 ft
C. Sta. = 30+50; Elev. = 226.80 ft
D. Sta. = 33+88; Elev. = 221.17 ft

44. Using the information given in the diagram below, calculate the station and elevation of the BVC of curve designed to provide a minimum of 3.0 ft clearance at the top of pipe located at station 6+87. Determine "L" to the nearest half station.

A. Sta. @ BVC = 3+50; Elev. = 39.18 ft
B. Sta. @ BVC = 4+00; Elev. = 38.36 ft
C. Sta. @ BVC = 4+25; Elev. = 37.94 ft
D. Sta. @ BVC = 3+25; Elev. = 39.59 ft



Answer Key

1. C. 87,932.24
2. D. 0.19
3. A. 0.50941
4. C. +12.64 ft
5. D. -5.57%
6. C. 0.83277
7. A. 24 ft – 10 7/8 in
8. D. 13,708.34 survey ft
9. A. 1,881.528 meters
10. D. 11,224.98 survey ft
11. D. 1,422.96 survey ft
12. B. 0.254416 rad
13. D. 43° 27' 55"
14. C. 31.619 hec
15. C. 46° 21' 57"
16. B. ±0.12 ft
17. C. ±0.05 ft
18. B. 191,202 ft^2 or 4.389 ac.; std. error = ±41.5 ft^2
19. A. ±0.14 ft
20. D. -405.77
21. B. 108.29
22. D. 3b – 87b
23. A. 418
24. A. <1 = 108° 20' 48"; <2 = 108° 20' 48"; <5 = 108° 20' 48"; <8 = 71° 39' 12"
25. A. <2 = 46° 11' 20"; <3 = 133° 48' 40"
26. B. 15° 23' 08"
27. A. 156.43 ft
28. D. 580.29 ft
29. B. 559.56 ft
30. C. 52.54 ft
31. D. 437.12 ft
32. D. 620.70 ft
33. D. 67° 30' 19"
34. A. y = 1,945.62; x = 11,612.43
35. D. S 89° 19' 17" E; 91.29 ft
36. A. y = 1,652.56; x = 1,733.14
37. C. Bearing “RP” to X = N 49° 10' 30" E; Distance L to X= 440.00 ft
38. C. Bearing Line RP-1 to X = N 44° 17' 50" E;
    Bearing Line RP-2 to X = N 40° 12' 45" W
39. A. Deflection = 03° 11' 33"; Sub-chord = 155.94 ft
40. C. External = 1.00 ft; Mid-ordinate = 1.00 ft
41. A. C/L radius = 4,366.42 ft
42. D. Elev. @ sta. 31+56 = 225.72 ft
43. B. Sta. = 29+19; Elev. = 227.27 ft
44. D. Sta. @ BVC = 3+25; Elev. = 39.59 ft

References

Brinker, Russell, and Minnick, Roy, Editors, *The Surveying Handbook,* Van Nostrand Reinhold, Co., New York, 1987. (Very comprehensive for all surveying operations and math formats)

Kavanagh, Barry F., *Surveying With Construction Applications, Second Edition,* Prentice Hall, New Jersey, 1992. (Good application of math concepts to survey calculations)

Minnick, Roy, *Land Survey Test Training Manual,* Landmark Enterprises, Rancho Cordova, CA, 1972.

Smith, Robert, *Applied General Mathematics,* Delmar, Inc., New York, 1982.

Wolf, Paul R., & Brinker, Russell C., *Elementary Surveying, Eighth Edition,* Van Nostrand Reinhold, Co., New York, 1987. (Very good presentation.)

Zimmerman, Edward, *Basic Surveying Calculations,* Landmark Enterprises, Rancho Cordova, CA, 1991. (Easily understood theory and operations.)

UNIT 3

BASIC MEASUREMENTS

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Introduction

Philip Kissum in his book, *Surveying Practice,* defines surveying as "the art, science and technology of making such measurements as are necessary to determine the relative position of points above, on or beneath the surface of the earth, or to establish such points in a specified position." The key terms in the above definition are art, science, technology and measurements. The science and technology of surveying are easy to recognize. The science includes the laws of mathematics and physics that surveyors use every day while making measurements. Technology includes the instrumentation, calculation devices, and mapping technologies (CADD) that have become necessities in current survey practice. The art of surveying is a little harder to define, but includes the use of judgment, gained through experience, that allows surveyors to choose the technologies and procedures to do a project correctly and efficiently.

Measurements are the cornerstones upon which the surveyor builds experience. Without a thorough understanding of the basic survey measurements, a surveyor cannot expect to move onto the more complicated technical issues nor onto the professional issues. This unit of the training program will deal with the basic survey measurements of distance, direction and elevation. The unit will also concentrate on measurement analysis which includes the study of errors and how they affect measurements.

Performance Expected on the Exams

Given sample measurements, determine the most probable value, the standard error and the standard error of the mean.

Given the standard error for certain measurements, equipment, and procedures, predict the outcome of a survey using these measurements.

Explain the difference between slope and horizontal distance. Calculate horizontal distances from slope distances and vertical angles or differences in elevation.

Explain the effects of systematic errors in distance measurements (i.e., temperature correction, sag correction, etc.) and be able to calculate their value.

Explain the fundamental principles of taping and Electronic Distance Measurement (EDM) operation.

Explain the basic methods and procedures for using a theodolite.

Explain the differences between angles, bearings and azimuths and be able to convert between them.

Key Terms

Measurements
Systematic errors
Mistakes
Accuracy
Mean
Standard error
Slope distance
Vertical angle
Taping
Elevation
Direction
Bearings
Magnetic azimuths
Double centering
Closing the horizon
Significant figures
Random errors
Precision
Probability
Residuals
Error of the mean
Horizontal distance
Vertical distance
Electronic distance measurement (EDM)
Leveling
Angles
Azimuths
Theodolite
Wiggling in

Video Presentation Outline

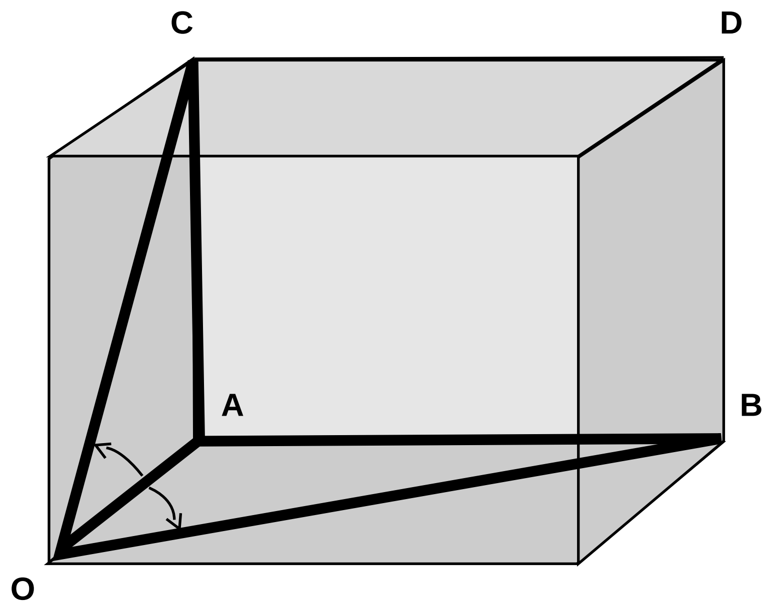


Figure 3-1. *Measurements in three dimensions.*

Types of Measurements

- Horizontal angles
- Horizontal distances
- Vertical angles
- Vertical distances
- Slope distances

Units of Measurements

- Length
- Angle
- Area
- Volume

Significant Figures

Numerical Value	Significant Figures
49.00	2
1600.00	2 or 4
.1284	4
0.21	2
00.000213	3
129.85	5
11.00	2 or 4
10,000.0001	9
5,280.00 ft/mile	infinite

- Number of significant figures
- Significant figures in addition and subtraction
- Significant figures in multiplication and division
- Problems relating to significant figures in surveying

Significant Figure Problem

Answer the following problems with the correct number of significant figures.

- Sum of 19.27, 0.0000345, 121 and 8.6
- Product of 28953.844 and 1.34
- Sum of 0.025, 1456.3, 18.466 and 438.6
- Quotient of 9430.445 divided by 9.45

Direct and Indirect Measurements

- Direct measurements
- Indirect measurements

Error Theory

- Errors in measurements:
  1. Error(E) = Measured Value - True Value

- Sources of error in making measurements:
  1. Natural error
  2. Instrumental error
  3. Personal error

- Types of errors:
  1. Systematic errors
  2. Random errors

- Precision and accuracy:

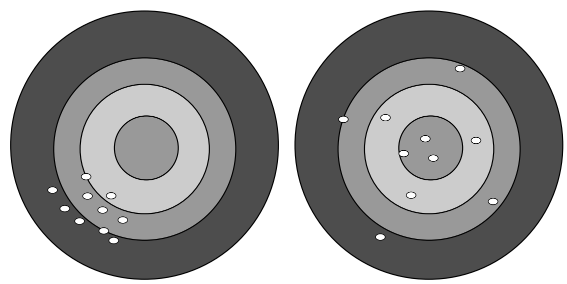


Figure 3-2. *Accuracy and Precision.*

- Eliminating mistakes and systematic errors
- Probability:

Measurement	r	r^2
1. 56° 23' 45"		
2. 56° 23' 46"		
3. 56° 23' 45"		
4. 56° 23' 43"		
5. 56° 23' 44"		
6. 56° 23' 45"		
7. 56° 23' 46"		
8. 56° 23' 45"		
9. 56° 23' 44"		
10. 56° 23' 46"		
11. 56° 23' 47"		
12. 56° 23' 44"		
Σ		

Most Probable Value

- Mean

Measures of Precision

- Residual(r) = measurement - most probable value
- Histogram
- Standard deviation (standard error)

$$= \pm \sqrt{\frac{\sum r^2}{n-1}}$$

- 50%, 90% and 95% errors:

Probable error (E_{50}) = ±0.6745 (SD)

90% Error (E_{90}) = ±1.6449 (SD)

95% Error (E_{95}) = ±1.9599 (SD)

Where:

SD = Standard deviation

Measures of Accuracy

- Standard error of the mean:

$$E_m = \pm \frac{SD}{\sqrt{n}}$$

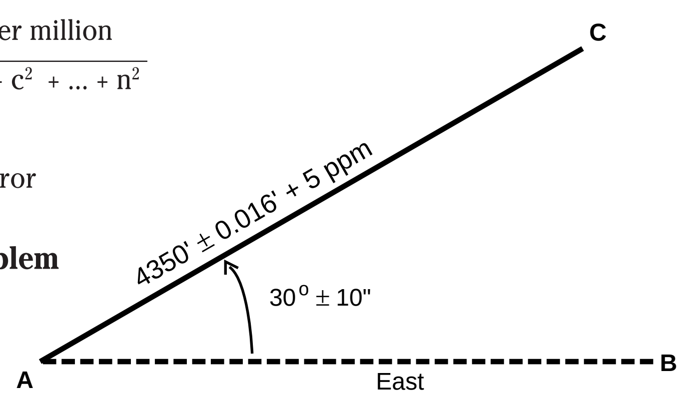
Error Propagation

- $E_{series} = \pm E \sqrt{n}$
- $E_{edm} = \pm E$ + Parts per million
- $E_{sum} = \pm \sqrt{a^2 + b^2 + c^2 + \ldots + n^2}$

Where:

E = Constant error

Point Location Problem



Determine the error in the position of Point C given the data as shown.

Estimated Traverse Closure Problem

Determine the estimated and actual closure of the traverse data shown below:

Side	Azimuth	Error	Distance	Error
A-B	90° 00' 00"	± 27"	1152.66'	±0.15'
B-C	184° 15' 16"	± 44"	501.92'	±0.14'
C-D	274° 49' 14"	± 18"	1192.10'	±0.10'
D-A	10° 15' 30"	± 58"	406.55'	±0.20'

Classifications of Traverse Accuracy Standards

	2nd Order (Modified)	3rd Order
Maximum number of courses between checks for azimuth	15	25
Azimuth closure not to exceed*	$10''\sqrt{N}$ or $3''$ per station	$30''\sqrt{N}$ or $8''$ per station
Position closure (after azimuth) adjustment not to exceed*	$1.67'\sqrt{M}$ or 1:10,000	$3.34'\sqrt{M}$ or 1:5000
Distance Measurement accurate within	1:15,000	1:7,500
Minimum distance to be measured with EDMs	0.1 mile	0.05 mile
Minimum number of Angle Observations:		
a) One-second theodolite	4 pos.	2 pos.
b) One-minute or 20-second theodolite	1 set of 6D, 6R	1 set of 2D, 2R

Notes:
1. N is the number of stations for carrying azimuth
2. M is the distance in miles

* *The expressions for closing errors in traverse surveys are given in two forms. The formula which gives the smaller permissible closure should be used.*

Table 3-1. *Table 4-04-B, "Caltrans Surveys Manual."*

Adjustments

- Traditional adjustments (proportional/linear adjustments)
- Least squares

Distance Measurements

Methods of Measuring Horizontal Distances

- Pacing
- Odometer
- Optical rangefinders
- Tacheometry (stadia)
- Subtense bars
- Taping
- EDM

Distance Measurements by Taping

- Basic procedures
- Types of tapes

Sources of Errors in Taping

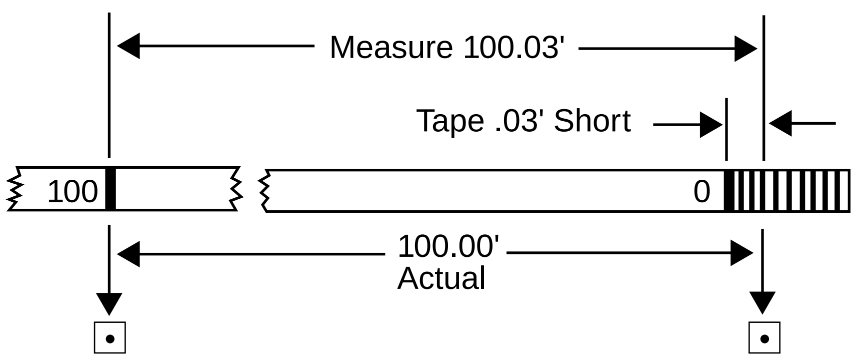


Figure 3-3. *Measuring with a short tape.*

- Incorrect length of tape

$$C_\ell = \left(\frac{\ell - \ell'}{\ell'}\right) L'$$

$$L = L' + C_\ell$$

Where:

L = Length of line

ℓ = Actual tape length

ℓ' = Nominal tape length

L' = Measured length of line

C_ℓ = Correction to the measured length of line

- Temperature other than standard

$$L = L' + k(T' - T)L'$$

Where:

L = Length of line

L' = Measured (recorded) length of line

k = Coefficient of thermal expansion and contraction of the tape

T' = Tape temperature at the time of measurement

T = Standardized tape temperature

Temperature Correction Problem

A field crew uses a tape that has been standardized at 68° F in the field at a temperature of 45° F. What is the corrected distance if the observed distance is 850.44 ft?

- Tension other than standard

$$L = L' + (P' - P)\frac{L'}{A\,E}$$

Where:

L = Length of line

L' = Measured (recorded) length of line

P' = Tension applied to tape in lb

P = Standardized tension of tape in lb

A = Cross-sectional area of tape in square in

E = Modulus of elasticity of steel in lb per square in

Tension Correction Problem

A field crew uses a tape that has been standardized at a tension of 12 lb. In the field, the tape is used at a tension of 20 lb to measure a distance of 650.45 ft. What is the corrected distance?

- Sag

$$C_s = -\frac{W^2 (L_s)}{24 (P')^2} = \frac{w^2 L_s^3}{24(P')^2}$$

$$L = L' + \sum C_s$$

Where:

L = Length of line

L' = Measured (recorded) length of line

C_s = Correction for sag

W = Total weight of tape between supports in lb

w = Weight of tape per ft in lb

L_s = Unsupported length of tape

P' = Tension in lb

Sag Correction Problem

A field crew uses a 100' tape weighing 1.5 lb that has been standardized for tension on a flat surface. In the field, the crew uses the tape supported at the ends only at a tension of 12 lb and measures a distance of 350.42 ft. What is the corrected distance?

- Poor alignment

$$C_A = -\frac{d^2}{ZL_T}$$

Where:

d = Distance tape off line

L_T = Length of tape

L = $L' + \sum C_A$

L = Actual length of line

L' = Measured length of line

$\sum C_A$ = Sum of alignment corrections for line

Poor Alignment Correction Problem

When a field crew measures a distance AB, the end of the first tape length is offline by 1.26 ft. The end of the second tape length is offline by an additional 0.98 ft. The end of the third tape length is offline by an additional 2.54 ft and the end of the final tape length is offline by an additional 1.66 ft. What is the correction for poor alignment if the measured distance is 350.56 ft? Assume four taped lengths, the first three are 100 ft each.

Corrections when measuring between points

Corrections when laying out points

Distance Measurement by EDM

- Fundamental principle of EDM operation
- Reduction of slope distance to horizontal distance

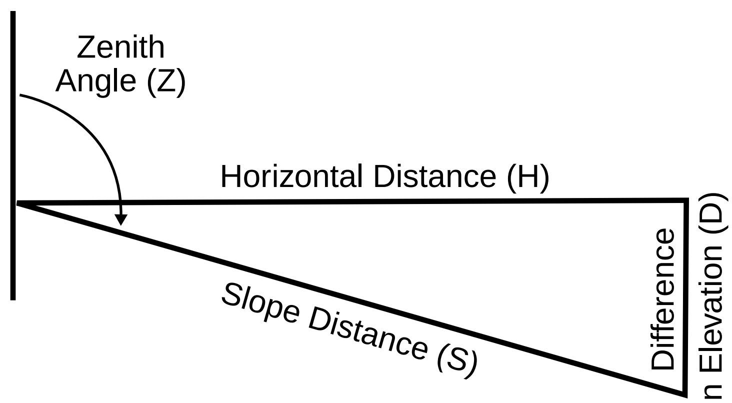


Figure 3-4. *Slope distance reduction.*

For measured Zenith angle:

$H = S \sin Z$

For known difference in elevation:

$H = \sqrt{S^2 - d^2}$

Sources of Errors in EDM Measurement

- Standard error of the instrument
- Temperature, pressure and humidity

Elevation Measurements

Error Sources

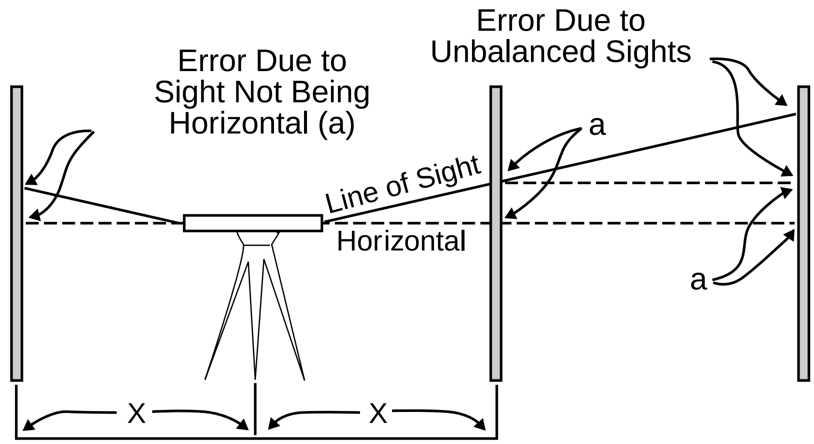


Figure 3-5. *Error due to unbalanced sights.*

Angular (Direction) Measurements

Introduction

- Horizontal vs. vertical
- Three basic requirements to determine angle
- Units of measure

Bearings and Azimuths

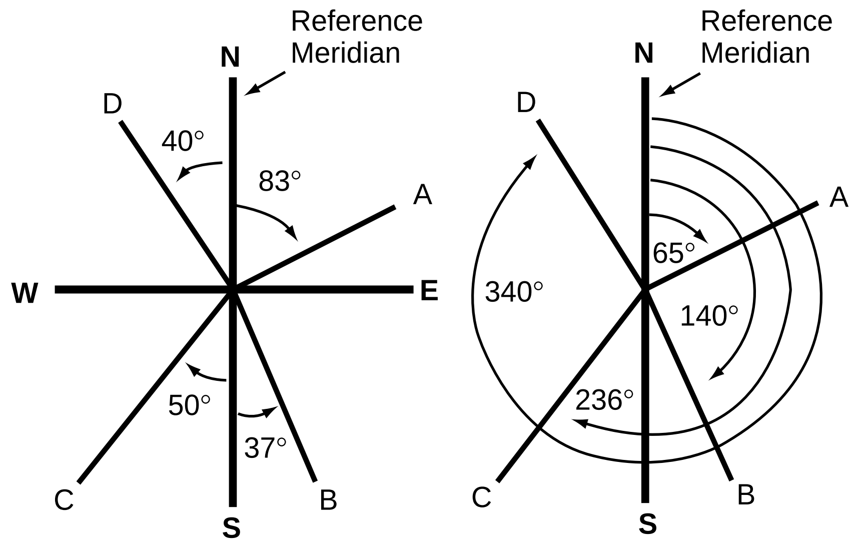


Figure 3-6. *Measurement of bearings and azimuths.*

Magnetic Declination and Bearings

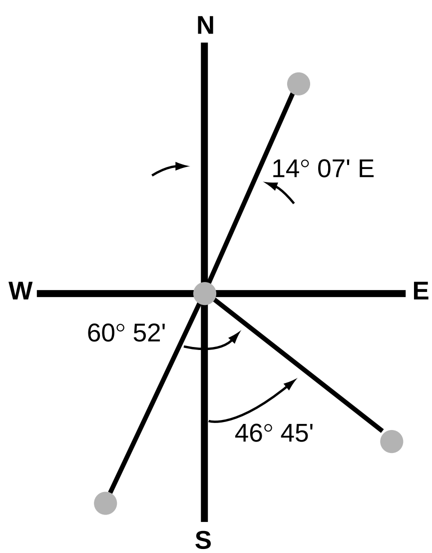


Figure 3-7. *Relationship between magnetic and astronomic bearings.*

Equipment Types

Basic Theodolite/Transit Operations

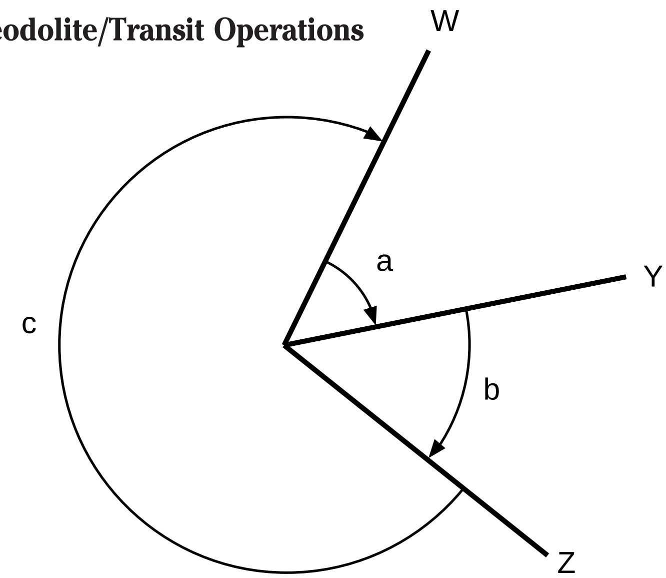


Figure 3-8. *Closing the horizon.*

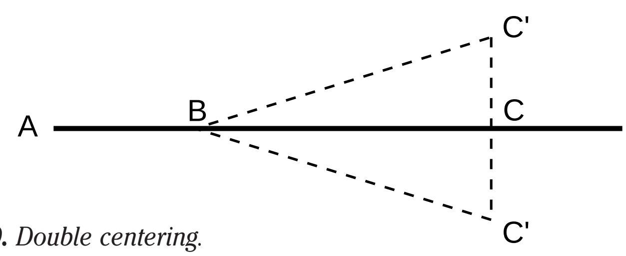


Figure 3-9. *Double centering.*

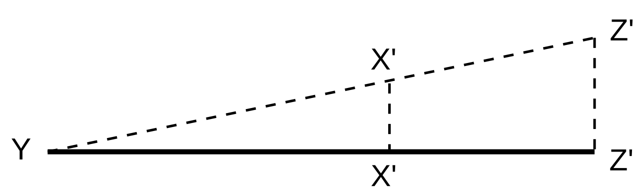
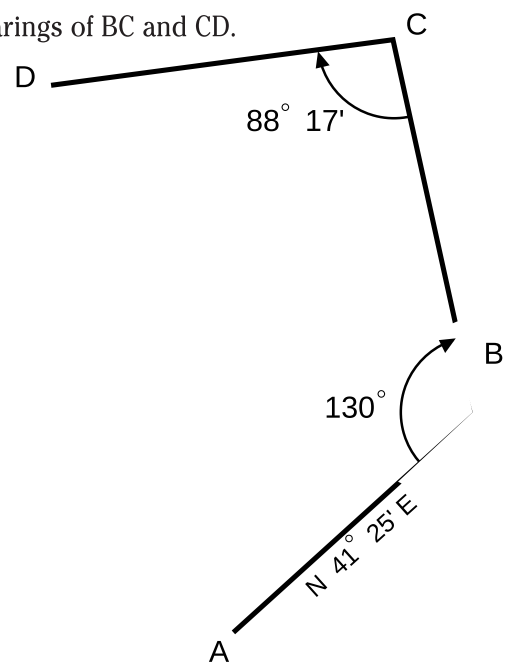


Figure 3-10. *Balancing in.*

Example Problems

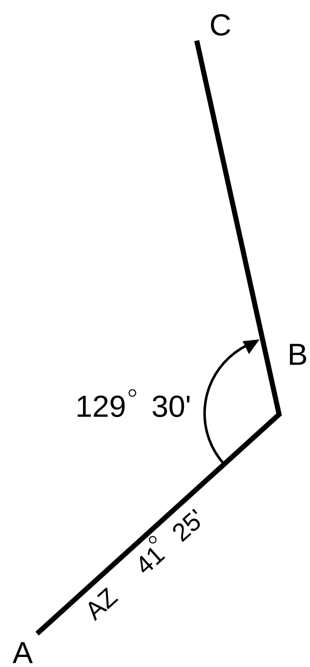
Problem 1.

Determine the bearings of BC and CD.



Problem 2.

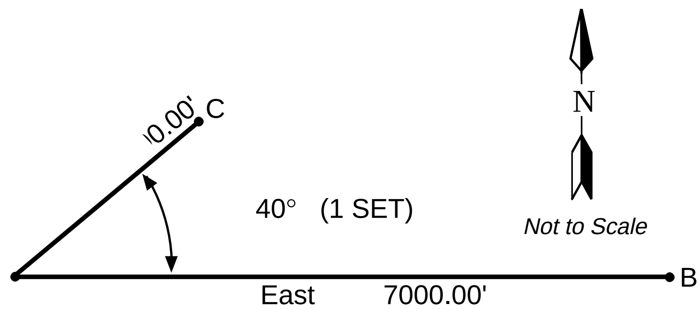
Determine the azimuth of BC.



Sample Test Questions

1. A-4 1991 LS

The measurements to Point C are shown below. A 6" theodolite with a measured standard error of ± 20" per angle set (direct and reverse) from all sources and an EDM with a standard error of ± .02 + 5 PPM was used.



Required:

A. Compute and sketch the standard error ellipse for point C. Label and dimension the semimajor axis, semiminor axis, and the orientation angle, U. Assume uncorrelated measurements.

B. What is the probability that the measured point is within or on the standard ellipse?

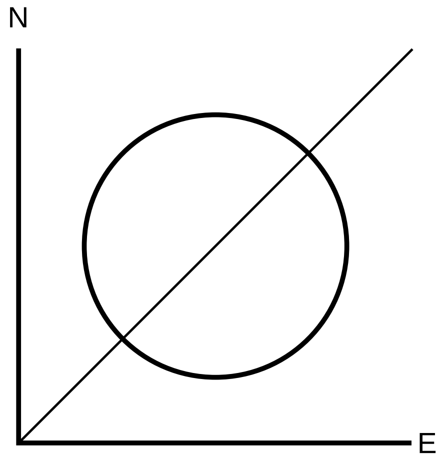
C. What are the dimensions of the semimajor and semiminor axis if probability of 95% is wanted?

D. What is the minimum number of angle sets needed to decrease the semimajor axis to 0.12' on the standard error ellipse?

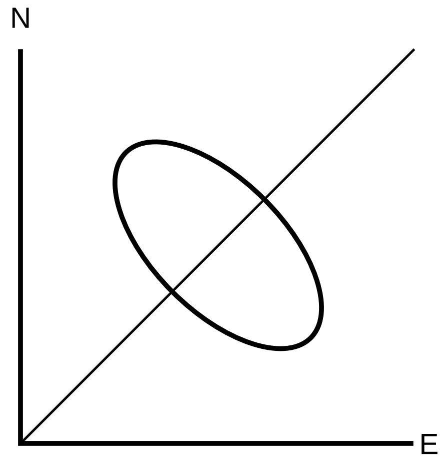
E. Each of the error ellipses shown on the following page indicates the relative comparative accuracy of establishing a point location with one of the following instrument combinations, A through F. In the spaces provided in the solution booklet, indicate the letter corresponding to the instrument combination that best works with the ellipses.

Angle	Angle Measured With:	Distance Measured With:
A	Transit	EDM
B	Compass	Steel Tape
C	Theodolite	EDM
D	Compass	Gunter’s Chain
E	Transit	Steel Tape
F	Theodolite	Steel Tape

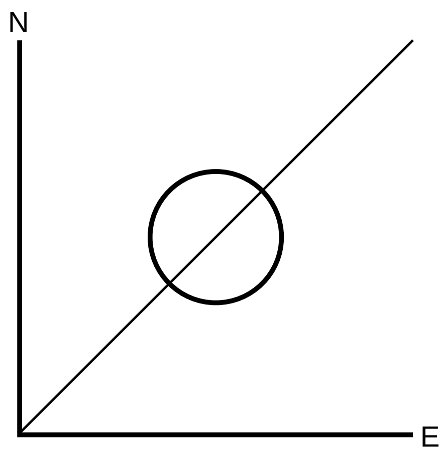
Assume the line is 2000' at an azimuth of 45°.



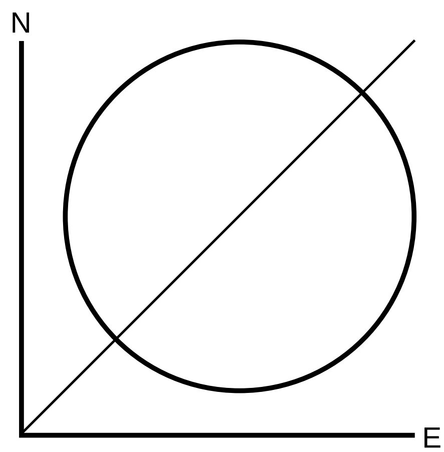
1. _____



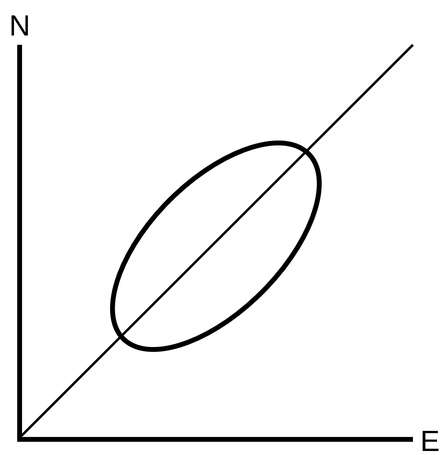
2. _____



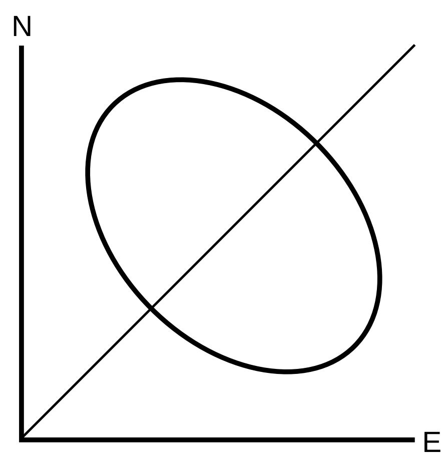
3. _____



4. _____



5. _____



6. _____

2. A steel tape standardized at 68° F. and supported throughout under a tension of 12 lb was found to be 99.991 ft long. This tape was used to lay out a horizontal distance AB of 650.23 ft. The ground was on a smooth 4% grade; thus, the tape was used fully supported. Determine the slope distance if a pull of 25 lb was used and the temperature was 42° F. (The cross sectional area of the tape = 0.005 sq. in. and the elasticity of steel = 29,000,000 lb/sq. in)

3. From the bearings below, compute the azimuths (measured from the North) and determine the angle, measured clockwise, between successive bearings.

   N 73° 34' 22"E, S 32° 15' 45"E, N 32° 12' 24"E, N 89° 59' 17"W.

4. A line AB is prolonged to point C by double centering. Two foresight points C' and C" are set. What is the the angular error introduced in a single plunging if the length BC is 900.44 ft and C' C" is 0.69 ft?

Answer Key

1. A. Semiminor axis = error in distance

$$= \pm\ .02 + \left(\frac{2000}{1000000}(5)\right)$$

$$= \pm 0.03'$$

or

$$= \pm \sqrt{(.02)^2 + \left(\frac{2000}{1000000}(5)\right)^2}$$

$$= \pm\ 0.022$$

Semimajor axis = error in angle

$$= \frac{20''}{206265} \times 2000$$

$$= \pm\ 0.194$$

Orientation angle (U) is angle between semimajor axis and the x or easting axis

$$\therefore 40° + 90° = 130°$$

B. Probability P [U≤ C^2] is represented by the volume under the bivariate normal density surface within the region defined by the error ellipse. For the standard error ellipse C 1, the probability is 0.394 or 39.4%.

C. For $P[U \leq C^2] = 0.95$, $C = 2.447$

= data from published tables

∴ semimajor axis = 0.194 x 2.447 = 0.475

or = 0.030 x 2.447 = 0.073

semiminor axis = 0.022 x 2.447 = 0.054

D. Number of angle sets needed = $\frac{0.194}{\sqrt{n}} = 0.12$

n = 2.61, therefore 3 sets

E.

Ellipse No.	Instrument Combination
1	E
2	A
3	C
4	D
5	F
6	B

2. True slope distance to be laid out:

4% of 650.23' = 26.009' (which equals the vertical distance)

Slope distance = $\sqrt{650.23^2 + 26.009^2}$

= 650.75'

Correction for incorrect length:

((99.991 - 100) / 100) 650.75 = -0.059'

Correction for temperature:

(0.0000065 (42-68)) 650.75 = -0.110'

Correction for tension:

(25-12) (650.75 / (.005 (29,000,000)) = 0.058

Slope distance to lay out:

650.75 - (-0.059) - (-0.110) - 0.058 = 650.86'

3. Bearings to azimuths:
   N 73° 34' 22"E = 73° 34' 22"
   S 32° 15' 45"E = 180° - 32° 15' 45" = 147° 44' 15"
   N 32° 12' 24"E = 32° 12' 24"
   N 89° 59' 17"W = 360° - 89° 59' 17" = 270° 00' 43"

   Clockwise angles:
   147° 44' 15" - 73° 34' 22" = 74° 09' 53"
   360° + 32° 12' 24" - 147° 44' 15" = 244° 28' 09"
   270° 00' 43" - 32° 12' 24" = 237° 48' 19"

4. 0.69' / 2 = 0.345' = The error in a single plunging
   Tangent of the angle = 0.345 / 900.44
   Tangent of the angle = 0.0003831
   Angle = 0° 01' 19"

References

California Department of Transportation, *Surveys Manual*, Chapters 2, 4, 5 and 6.

Buckner, R.B., *Surveying Measurements and Their Analysis,* First Edition, Landmark Enterprises, Rancho Cordova, California, 1983.

Mikhail, Edward M., and Gordon Gracie, *Analysis and Adjustment of Survey Measurements*, Van Nostrand Reinhold, New York, 1981.

Kissum, Philip, *Surveying Practice,* Fourth Edition, McGraw Hill, New York, 1988.

Wolf, Paul R. and Russel C. Brinker, *Elementary Surveying,* Eighth Edition, Harper and Row, New York, 1989.

UNIT 4

AZIMUTH DETERMINATION BY CELESTIAL OBSERVATION

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County of San Diego

Introduction

The interaction of the earth with the sun and other stars is a three-dimensional happening best explained using a three-dimensional model. The relationship of the earth to the other celestial bodies is described in relation to the PZS or astronomical triangle and its six parts.

Azimuth determinations are generally made by observing either the sun or Polaris, although other stars may be used. The two methods for determining azimuth from the sun are the hour angle method and the altitude method. Accuracy requirements for the work, as well as accuracy of the data used in the calculations, will determine the method to be used.

Performance Expected on the Exams

Describe the interaction of the earth and the sun and other stars in relation to the PZS triangle.

Compute the astronomic azimuth of a line from a solar observation by either hour angle or altitude method.

Compute the astronomic azimuth of a line from a Polaris observation.

State the relative strengths and weaknesses of the hour angle and altitude methods of solar observation.

State the reasons for choosing a Polaris observation rather than a sun observation.

Key Terms

Celestial sphere
Astronomical triangle
Latitude
Longitude
Altitude
Declination
Greenwich hour angle
Local hour angle
Sidereal hour angle
Apparent time
Equation of time
Moment of time
True meridian time
Parallax
Refraction
Elongation
Culmination
Zenith
Polar distance
Vernal equinox
Autumnal equinox
Right ascension
Civil time
Elapsed time
Universal time
Atomic time
Sidereal time
Daylight saving time
The first point of Aries
Standard time
Atomic second

Video Presentation Outline

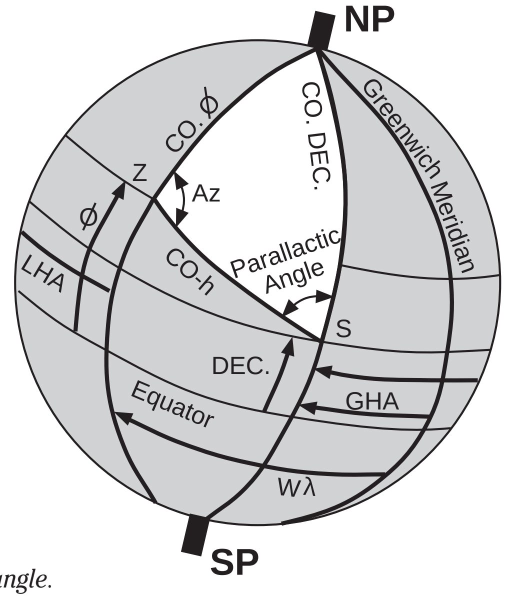


Figure 4-1. *The PZS triangle.*

The PZS Triangle

Co Ø	=	90° - LAT
CoDEC	=	90° - DEC
Co h	=	90° - h
NP	=	North Pole
Z	=	Zenith
S	=	Sun or star

- The three corners
- Time, latitude, longitude
- The six parts of the PZS triangle

Azimuth by Solar Observation

- Choosing between hour angle method and altitude method
- Hour angle method

$$\text{AZIMUTH} = \tan^{-1} \frac{\sin \text{LHA}}{\sin \text{LAT} \cos \text{LHA} - \cos \text{LAT} \tan \text{DEC}}$$

Where:

LHA = local hour angle of sun

Dec = declination of sun

LAT = latitude of observe

AZ = Azimuth of sun

To normalize AZ, add correction		
LHA	corr	corr
0-180	180(+AZ)	360(-AZ)
180-360	0(+AZ)	180(-AZ)

- Altitude method

$$Z = \cos^{-1} \frac{\sin \text{DEC} - \sin \text{LAT} \sin h}{\cos \text{LAT} \cos h}$$

Where:

AZ = Z (when sun is east of the local meridian)

AZ = 360 - Z (when sun is west of the local meridian)

DEC = declination of sun

LAT = latitude of observer

h = vertical angle to the sun corrected for parallax and refraction

Example Problem

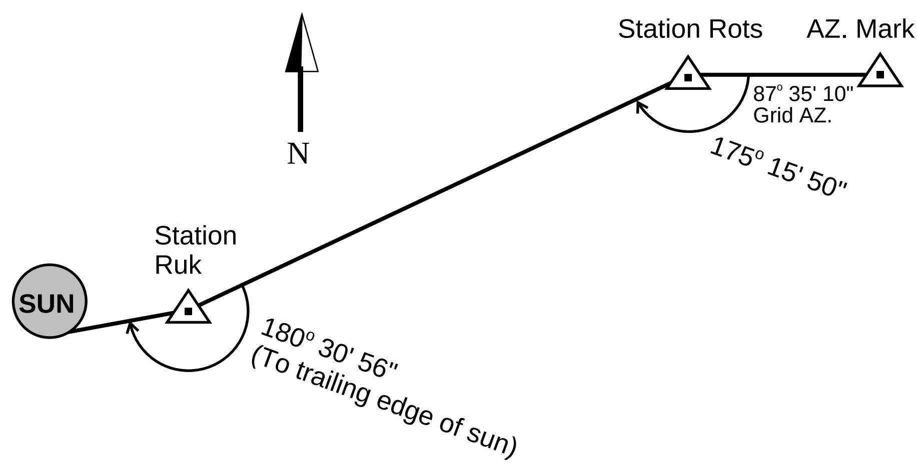
1989 LS Problem

(Portions of this problem have been deleted.)

California coordinates and basis of bearings for Station ROTS are shown on the sketch along with data for a solar observation at Station RUK.

Use the data provided and the diagram to reduce the solar observation either by the hour angle method or the altitude method. Show your work.

Determine the angle of closure at station RUK.



Solar Observation

Date	=	May 5, 1988
Time	=	5:23:35.0 PDST
		Watch is 0.5" fast
D.U.T.	=	- 0.3"
Vert. angle to center of sun	=	28° 05' 49" (Corrected for parallax and refraction)
LAT. RUK	=	36° 48' 57.0"
LONG. RUK	=	119° 46' 54.5"
Θ RUK	=	- 0° 27' 59"

Solar Ephemeris Table

	GHA	Declination	Semidiameter
May 4 W	180° 48' 22.7"	15° 58' 04.3"	15' 53.2"
May 5 Th	180° 49' 44.8"	16° 15' 19.0"	15' 53.0"
May 6 F	180° 50' 58.4"	16° 32' 17.5"	15' 52.7"
May 7 S	180° 52' 03.3"	16° 48' 59.7"	15' 52.5"

Azimuth by Observation of Polaris

The equation given for hour angle method for the sun can be used, or the following:

$$Z = \frac{p \ \sin \ LHA}{\cos h}$$

Where:

h = (true altitude of Polaris)

p = 90° - declination

Z is west of north when $0^{\circ} \leq LHA \leq 180^{\circ}$

Z is east of north when $180^{\circ} \leq LHA \leq 360^{\circ}$

AZ = Z (when Z is east of north)

AZ = 360 - Z (when Z is west of north)

Sample Test Questions

The following questions are from Problem B-3, 1988 LS:

Answer the following questions T (True) or F (False).
NOTE: False statements require a brief explanation.

1. Astronomic azimuth is based on true north.

2. Grid azimuth is based on true north.

3. Geodetic azimuth is astronomic azimuth - Θ + 2nd term.

4. To determine true north from observations on Polaris, the latitude of the observer must be known.

5. GHA = LHA - west longitude.

6. The best time to observe the sun to determine azimuth using the altitude method is 1/2 hour after sunrise or 1/2 hour before sunset.

7. Exact time is more important when using the hour angle method versus the altitude method.

8. The hour angle method requires both horizontal and vertical observations to determine the azimuth.

9. It is not necessary to know the latitude of the observer when using the hour angle method.

10. The best time to observe the sun to determine azimuth using the hour angle method is just after sunrise or just before sunset.

11. Standard time must be converted to local time to determine Greenwich time.

12. Local time is increased (in California) by eight hours to determine Greenwich time.

13. True solar time is local civil time minus the equation of time.

14. A level line at sea level is parallel with a level line at 8000 ft.

15. One sidereal day is longer than one solar day measured in civil time.

Which of the following is the best answer:

16. At what time will the effects of a small error in the determination of observer's latitude be minimized when making azimuth observations on Polaris?
    - A. 12:00 midnight
    - B. when Polaris is at elongation
    - C. when Polaris is at culmination
    - D. when the LHA is 90 degrees
    - E. when the GHA is 90 degrees

17. At what time will the effects of a small error in the determination of the time of observation be minimized when making azimuth observations on Polaris?
    - A. 12:00 midnight
    - B. when Polaris is at elongation
    - C. when Polaris is at culmination
    - D. when LHA is 180 degrees
    - E. when GHA is 180 degrees

The following questions are from Problem B-3, 1991 LS:

There are two methods by which azimuth can be determined by observation of the sun. Answer the following questions concerning these methods.

18. Name the two methods that can be used to determine azimuth by observations of the sun.

19. Which method is more accurate? Explain your answer.

20. The following two questions concern the method that uses vertical angle observations:
    - A. How would inconsistencies of the angular (vertical and horizontal) observations be detected?
    - B. How would calculations for the effect of the semidiameter of the sun be eliminated?

21. For each method, indicate whether parallax and refraction are taken into account. Explain your answer.

22. When using the method that uses only the horizontal angle, what is the single most important area where errors, excluding time and angular measurements, would most likely occur?

23. What is an appropriate source for accurate time determination?

24. When using the method that uses only the horizontal angle, if observations are made on the trailing limb of the sun, how does that affect angular calculations?

25. For each method, describe how the time of day of the observations affects azimuth determination.

26. For each method, describe how averaging the observations for calculation purposes would affect the final azimuth determination.

Use the solar ephemeris table from the sample problem in the video presentation outline to solve problems 27 and 28.

27. What is the local hour angle for a solar observation at 4:01:37.2 p.m. PDST on May 4, 1988, at a longitude of 120° 39' 15" west?

28. What is the sun's declination at 3:36:44 p.m. PDST May 6, 1988?

The following questions are from Problem B-4, 1992 LS:

Problem Statement: The following questions are independent and require a demonstration of your knowledge in using the sun or a star to determine a bearing basis for a survey.

Problem Requirements: Answer the questions with a brief sentence and show your calculations if any are required.

29. An observation on the sun is taken on Thursday, April 2, at 5:05:30.2 p.m. Pacific Standard Time (120 degrees west longitude). Afterward, the observer turns to the radio time station and determines that the observation clock is 2.6 seconds fast and also notes three double ticks between the ninth and fifteenth second after the minute tone.
    A. What is the Coordinated Universal Time (UCT) time and date?
    B. What is the correct UT1 time after applying the DUT correction?

30. Precise time is not available. Name a stellar body and time to observe to obtain an astronomic bearing within 10 seconds of true north in North America.

31. The following observations are recorded. Determine which one is inconsistent with the remaining observations.

	Time	Clockwise Horizontal Angle
(a)	9:05:01	40° 10' 15"
(b)	9:06:20	40° 12' 05"
(c)	9:07:10	40° 13' 17"
(d)	9:08:40	40° 14' 18"

32. The star Sirius is observed for a basis of bearing using the altitude method. The measured vertical angle is +45° 10' 10". The elevation is 1000 ft above sea level and the temperature is 60° F. What is the correction for parallax to be applied to the vertical angle?

33. A horizontal clockwise angle of 135° 30' 30" is measured to the trailing edge of the sun. The vertical angle to the center is +38° 25' above the horizon. The sun's semidiameter is 16' 45". What is the correct horizontal angle to the center of the sun?

34. At a latitude of 40° 42' north, a Polaris observation is taken from which a traverse is run due west 6000 ft. A second Polaris observation is then taken. Compute the correction of meridian convergence for the second observation.

36. A solar observation is taken at 120° 15' 45" west longitude. Determine the Local Hour Angle of the sun at 12:00:00 UT1 on January 1, 1990, given:

Date	GHA (Sun)
January 1, 1990	179° 10' 39.5"
January 2, 1990	179° 03' 33.8"

37. Given the following calculated azimuths of a line, determine the answers to a and b below.

A. The standard deviation of the set

B. The 90% error of the mean

Azimuths
1. 40° 21' 10"
2. 40° 21' 12"
3. 40° 21' 05"
4. 40° 20' 55"
5. 40° 20' 58"

Answer Key

1. T
2. Astro Az = true Az
3. Geod AZ = Grid Az + Θ - 2nd term
4. $Z = P \dfrac{\text{Sin LHA}}{\cos h}$
5. GHA = LHA + west longitude
6. Altitude above 20° and two hours before or after apparent local noon.
7. T
8. No vertical angle needed.
9. $AZ = \tan^{-1} \dfrac{\sin LHA}{\sin LAT \cos LHA - \cos LAT \tan DEC}$
10. T
11. Standard time is converted to Greenwich time by adding eight (1 hour for every 15° of west longitude).
12. Local time must be converted to standard and then to Greenwich time.
13. TST = CT + ET
14. Level surfaces at higher altitudes are more elliptical than at lower altitudes.
15. One sidereal day = 23h 56m in civil time.
16. C. At culmination the bearing is 0 at all LAT.
17. B. Polaris has no visual movement right or left at elongation for ± 3 min.
18. Altitude method and hour angle method.
19. Hour angle method—because time can be very accurately determined, and inaccuracies in measuring the vertical angle and determining refraction make the altitude method less reliable.
20. A. By plotting the horizontal/vertical angles vs. time;
    B. Sighting the left and right edge of the sun or Roeloff's Prism.

21. A. Hour angle method—No. There is no vertical angle.
    B. Altitude method—Yes. It is applied only to the vertical angle.

22. In the determination of the latitude and longitude of the point. Examples: Scaling on a U.S.G.S. quad sheet or leveling the instrument.

23. Time cube method—monitoring WWV.

24. The method does not take account of the semidiameter of the sun.

25. Hour angle method—because a vertical angle is not being measured, the time of day of observation, except as an extreme, does not affect the determination of the azimuth. The lower the elevation, the less the effect of misleveling will be.

    Altitude method—best time is 8 a.m. - 10 a.m. and 2 p.m. - 4 p.m. Vertical angle not changing enough from 10 a.m. - 2 p.m. and refraction correction becomes very large and uncertain at low altitudes.

26. Altitude method—averaging of adjusted data would not have any measurable effect on the final azimuth.

    Hour angle method—averaging will give errors due to the fact that the sun travels in an apparent curved path and the semidiameter correction would be affected by the change in altitude.

27.

Time 5-4-88	4:01:37.2
PM Correction	+ 12:00:00
Zone Correction	+ 7:00:00
UT1 5-4-88	23:01:37.2

$$GHA = GHA\ 0^h + (GHA\ 24^h - GHA\ 0^h + 360)\frac{UT1}{24}$$

$$= 180° \ 48' \ 22.7'' + (180° \ 49' \ 44.8'' - 180° \ 48' \ 22.7'' + 360)\frac{23{:}01{:}37.2}{24}$$

$$= 526° \ 13' \ 59.5''$$

Normalize GHA by subtracting 360°

$$LHA = GHA - \text{west longitude}$$

$$= 166° \ 13' \ 59.5'' - 120° \ 39' \ 15''$$

$$LHA = 45° \ 34' \ 44.5''$$

28. Time 5-6-88 3:36:44 PMPDST

PM Correction 12:00:00

Zone Correction 7:00:00

UT1 5-6-88 22:36:44

$$\text{Dec} = \text{Dec } 0^h + (\text{Dec } 24^h - \text{Dec}0^h)\frac{\text{UT1}}{24}$$

$$= 16° \ 32' \ 17.5'' + (16° \ 48' \ 59.7'' - 163° \ 2' \ 17.5'')\frac{22:36:44}{24}$$

$$\text{Dec} = 16° \ 48' \ 01.7'' + \text{correction}$$

Correction for linear interp. $= (0.0000395)\ (\text{Dec}0^h)\ \text{Sin}\ (7.5\ \text{UT1})$

29. Time 5:05:30.2 P.M. PST April 2

PM +12

Zone +8

Watch -2.6

25:05:27.6

A. UCT 1:05:27.6 April 3

DUT -0.3

B. UT1 1:05:27.3 April 3

1. Polaris at elongation (the star has no noticeable movement for approximately 10 minutes of time at elongation).

2.

	Time	D	Angle	D
(a)	9:05:01		40° 10' 15"	
		79"		110"
(b)	9:06:20		40° 12' 05"	
		50"		72"
(c)	9:07:10		40° 13' 17	
		90"		61"
(d)	9:08:40		40° 14' 18"	

The (d) observation is inconsistent with the others because the ratio of time to angle of (c) to (d) does not match the ratio of (a) to (b) or (b) to (c). The ratios of (a) to (b) and (b) to (c) are nearly the same.

3. There is no parallax correction for stars. They are light years away.

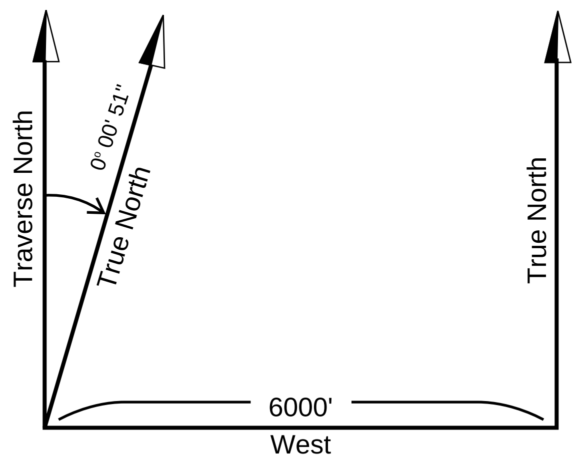
4. The semi-diameter (SD) has to be divided by the cosine of the vertical angle before it can be added to the trailing edge of the sun.

$$\frac{SD}{\cos h} = 0° 21' 22.7''$$

$$135° 30' 30'' + 0° 21' 22.7'' = 135° 51' 52.7''$$

5. There are approximately 6080 feet in 1 minute of longitude at the equator.

6080' x cos LAT = 4609



There are approximately 4609 feet in 1 minute of longitude at LAT 40° 42".

$$\text{DIF LONG} = 60'' \frac{6000}{4609}$$

$$= 78.1''$$

$$\text{Convergence} = \text{DIF LONG} \times \sin \text{LAT}$$

$$= 50.9''$$

6.

$$\text{GHA} = \text{GHA } 0^h + (\text{GHA } 24^h - \text{GHA } 0^h + 360) \frac{\text{UT1}}{24}$$

$$= 179° 10' 39.5'' + (179° 03' 33.8'' - 179° 10' 39.5'' + 360) \frac{12}{24}$$

$$\text{GHA} = 359° 07' 06.6''$$

$$\text{LHA} = \text{GHA} - \text{west longitude}$$

$$\text{LHA} = 238° 51' 21.6''$$

7.

	DIFF	$DIFF^2$
40° 21' 10'	+06"	36"
40° 21' 12"	+08"	64"
40° 21' 05"	+01"	01"
40° 20' 55"	-09"	81"
40° 20' 58"	-06"	36"

Average = 40° 21" 04" $\sum DIFF^2$ = 218"

$$a.\ s_s = \sqrt{\frac{\sum DIFF^2}{(n - 1)}} = \pm 7.38''$$

$$b.\ E_{90} = 1.6449 \sqrt{\frac{\sum DIFF^2}{n(n - 1)}} = \pm 5.43$$

References

Buckner, R. B., *Astronomic and Grid Azimuth,* Landmark Enterprises, Rancho Cordova, CA 1984. (An excellent book for solar azimuth instruction.)

Chelapati, C. V., *Land Surveyors License Examination Review Manual,* California State University, Long Beach, 1987.

Elgin, Richard L., David R. Knowles and Joseph H. Senne, *Celestial Observation Handbook and Ephemeris,* Lietz Co., Overland Park, Kansas. (A lot of good information in a little book.)

Mackie, J. B., *The Elements of Astronomy for Surveyors,* Charles Griffin House, England, 1985.

UNIT 5

TRAVERSING

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Introduction

Traversing is the method of using lengths and directions of lines between points to determine positions of the points. Traversing is normally associated with the field work of measuring angles and distances between points on the ground. Closed traverses provide the primary method used in checking surveying field work. Traverse closure and adjustment procedures are used to distribute error in measurements. Mathematical traverses performed on a computer are used to check surveying work such as mapping and legal descriptions.

Performance Expected on the Exams

Explain the difference between the precision and accuracy of a traverse.

Identify the sources of error in traversing.

Compute angular misclosure in a traverse and distribute the error.

Compute adjusted coordinates for a traverse given angles and distances measured in the field.

Key Terms

Traverse
Closed figure traverse
Closed linear traverse
Open traverse
Radial traverse
Direct angles
Deflection angles
Ordered surveys
Precision
Accuracy
Collimation error
Systematic error
Random error
Blunder
NAD 1927
NAD 1983
Basis of bearings
Ground distance
Grid distance
Combination factor
Latitudes
Departures
Closure
Balancing angles
Transit rule
Crandall rule
Compass rule
Least squares adjustment

Video Presentation Outline

Purpose and Types of Traverses

- The use and purpose of traversing
- Closed traverses
- Open traverses

Traverse Basics

- Angle and distance measurement
- Basis of bearings
- Coordinate datums
- Standards of accuracy
- Accuracy/precision
- Traverse errors

Traverse Computations

- Sum of angles in closed figures
  $\sum$ interior angles = (n-2) 180°
  $\sum$ exterior angles = (n+2) 180°

  Where:
  n = number of sides

- Distance measurements
  Conversion factors:

  $\frac{12}{39.37}$ U.S. survey feet = meters

  $\frac{39.37}{12}$ meters = U.S. survey feet

- Computing latitudes and departures
  Latitude = cos bearing x length of course
  Departure = sin bearing x length of course

  $\text{tan bearing} = \frac{\text{Dep}}{\text{Lat}}$

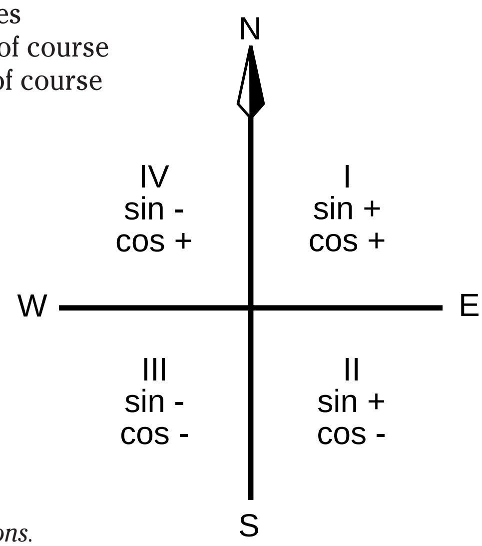


Figure 5-1. *Signs of cosine and sine functions.*

Traverse Closure and Adjustment

- Balancing angles
- Slope reduction of distances

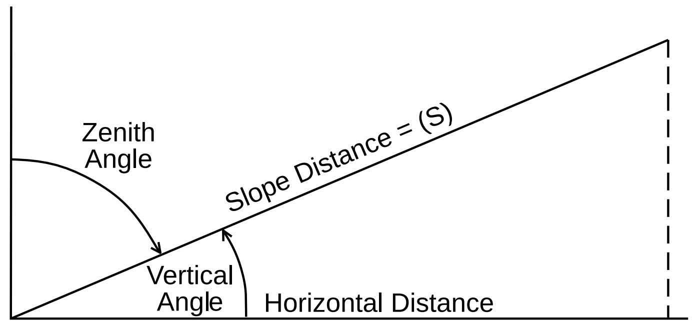


Figure 5-2. *Slope reductions.*

Hor. Dist. = S (cos vertical angle)
or
= S (sin zenith angle)

- Adjustment methods
- Compass rule example

Field Angles and Distances

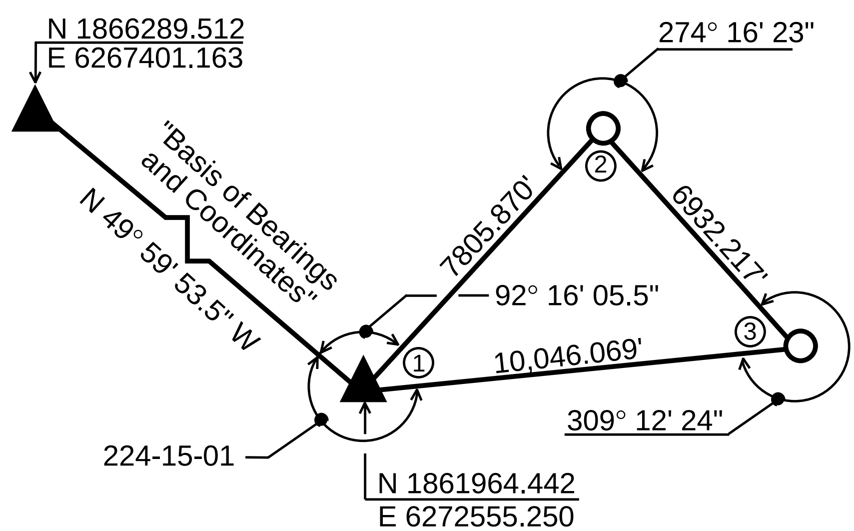


Figure 5-3. *Traverse example.*

Station	Dist.	Bearing	Lat.	Dep.
1	7805.87	N42° 16' 12.5"E	5776.20	5250.44
2	6932.22	S43° 27' 22.4"E	-5032.10	4767.98
3	10046.07	S85° 45' 04.0"W	-744.30	-10018.46
Close	∑ 24,784.16		-0.20	0.04

$$\text{Linear Misclosure} = \sqrt{-0.20^2 + -0.04^2}$$
$$= 0.204$$

- Accuracy (expressed as ratio of closure error):

  0.204/24784.16 = 1/121,491

- Adjustment to latitude of course =

  $$\text{Traverse lat misclosure} \times \left(\frac{\text{length of course}}{\text{length of traverse}}\right)$$

- Adjustment to departure of course =

  $$\text{Traverse dep misclosure} \times \left(\frac{\text{length of course}}{\text{length of traverse}}\right)$$

Sample Test Questions

1. Answer the following questions true or false.

   A. The terms *precision* and *accuracy* mean the same thing.

   B. A deflection angle is turned from the backsight clockwise to the foresight.

   C. A Record Map is the only valid reference for a basis of bearings.

   D. The compass adjustment method presumes that the angles in a traverse are more accurate than the distances.

   E. The sum of the external angles for a seven-sided figure is 1420 degrees.

   F. To compute the traverse closure accuracy ratio, divide the square root of the sum of the squares of the latitude and departure misclosures by the sum of the horizontal distances of the traverse.

   G. To balance the angles of a traverse, distribute the angular error of closure equally to all the traverse angles.

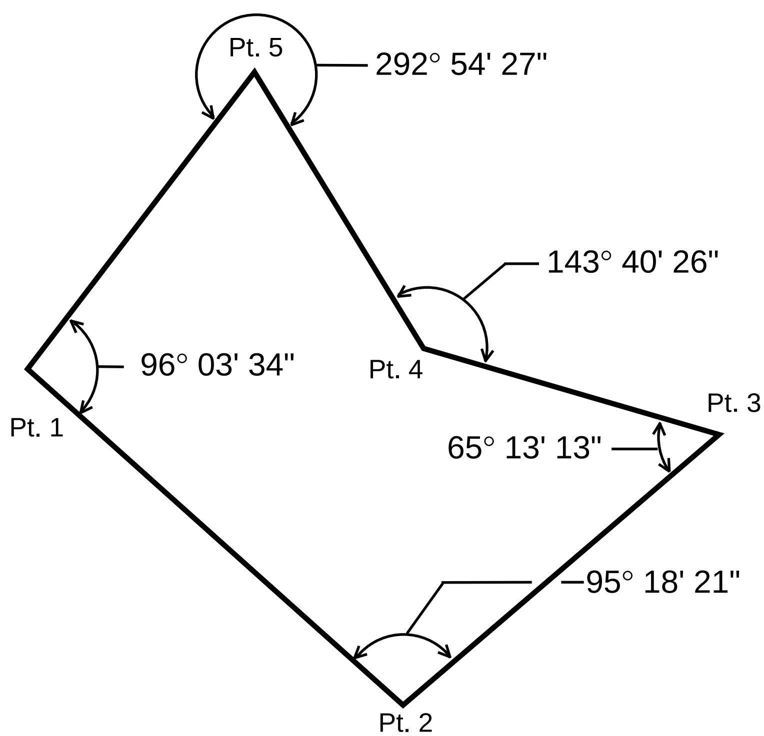
H. According to FGCC standards for Horizontal Traverse Control, a Second Order, Class I Traverse, performed in a metropolitan area, must have a minimum angular closure of not more than 2" per traverse angle, and a minimum linear precision closure of not more than 1:20,000.

I. To convert U.S. Survey Feet to meters, multiply the distance in feet by 12/39.37.

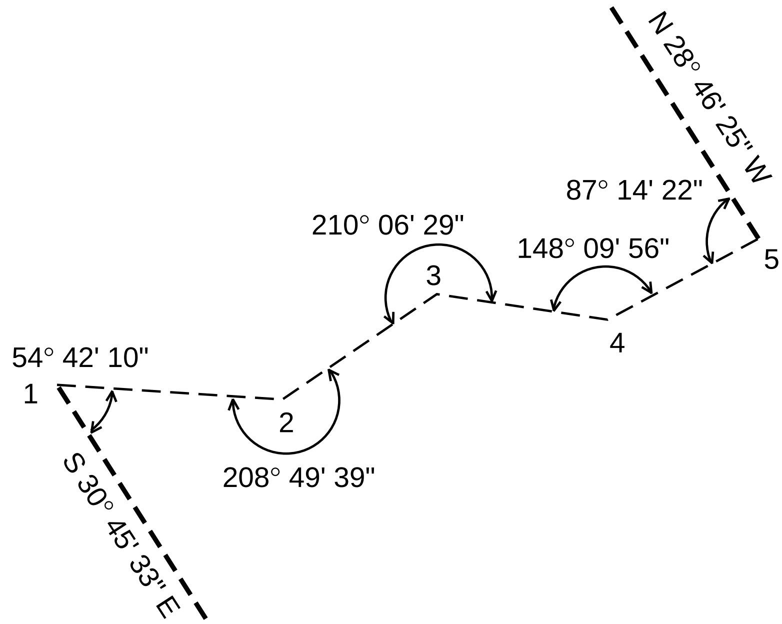
J. Ideally, the algebraic sum of the latitudes of a traverse should equal the algebraic sum of the departures.

K. The latitude of a traverse course is equal to its length, multiplied by the cosine of the bearing of the course.

2. In the sample traverse figure below, calculate the angular error of closure, and balance the traverse angles. Angles shown are unadjusted.

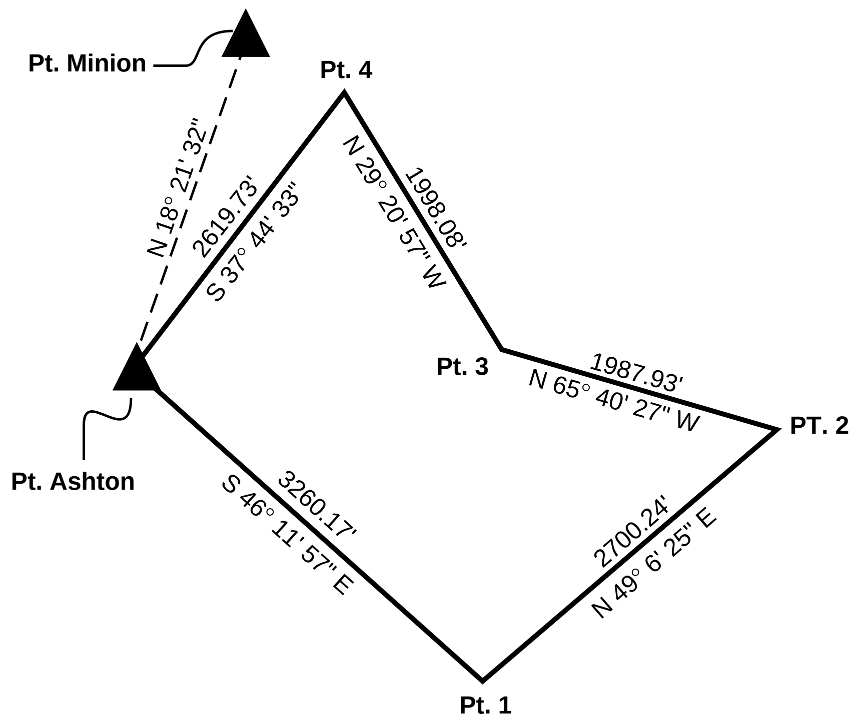


3. Calculate ADJUSTED bearings from field angles:



4. The latitudes of a closed traverse failed to close by -0.27', and the departures failed to close by +0.55'. The sum of the horizontal traverse distances is 8930.27'. What is the error of closure? Express the error of closure as a ratio. Determine the bearing of the error of closure.

5. Calculate the latitudes and departures for each course in this traverse. Bearings shown are from balanced angles and distances are grid. Coordinates for Ashton: E = 1,861,964.442E, N = 6,272,555.250.



6. Perform a compass rule adjustment on the latitudes and departures in problem 5, and list the balanced latitude and departure for each course.

7. Calculate adjusted bearings, distances, and coordinates for the traverse in problem 5.

Answer Key

1.

A.	False	G.	True
B.	False	H.	False
C.	False	I.	True
D.	False	J.	True
E.	False	K.	True
F.	False		

2.

∑ Interior angles = 540° 00' 15"
Angular misclosure = (5-2) 180° - 540° 00' 15"
= -00° 00' 15"

Adj. per interior angle = $\frac{-00° \, 00' \, 15'}{5}$ = -00° 00' 03"

Adjusted Angles

Pt. 1	96° 03' 31"
Pt. 2	95° 18' 18"
Pt. 3	65° 13' 10"
Pt. 4	143° 40' 29"
Pt. 5	292° 54' 30"

3.

From – To:	Field Azimuth	Adjusted Azimuth	Bearing
1-2	94° 32' 17"	94° 32' 19"	S 85° 27' 41" E
2-3	65° 42' 38"	65° 42' 42"	N 65° 42' 42" E
3-4	95° 49' 07"	95° 49' 13"	S 84° 10' 47" E
4-5	63° 59' 03"	63° 59' 11"	N 63° 59' 11" E
5-AZ	331° 13' 25"	331° 13' 35"	N 28° 46' 25" W
	(331° 13' 35")		

Distribute 10" in angular closure error by rotating each field azimth by 02" clockwise.

4\.

Error of closure $= \sqrt{\sum lat^2 + \sum Dep^2}$

$= \sqrt{0.27^2 + 0.55^2}$

$= 0.61'$

Ratio of error

$\frac{0.61}{8930.27} = \frac{1}{x} = \frac{1}{14646}$

1:14640

cos bearing closing line $= \frac{\sum \text{Lat error}}{\sum \text{Dep error}}$

$= \frac{-.27}{.55}$

Bearing = N60° 35' 59" W

5\.

Station	Bearing	Dist.	Lat.	Dep.
Ashton				
	S 46° 11' 57" E	3260.17	-2256.539	2353.028
1				
	N 49° 06' 25" W	2700.24	1767.710	2041.200
2				
	N 65° 40' 27" W	1987.93	818.879	-1811.437
3				
	N 29° 20' 57" W	1998.08	1741.624	-979.320
4				
	S 37° 44' 33" W	2619.73	-2071.603	-1603.573
Ashton				
	Σ	12,566.15	0.071	-0.102

Closing Line S 34° 50' 27" E 0.124'

Closure 1:101113

6.

Station	Lat.	Dep.	Correction Lat.	Correction Dep.	Balanced Lat.	Balanced Dep.
Ashton						
	-2256.539	2353.028	-0.019	0.027	-2256.558	2353.055
1						
	1767.710	2041.200	-0.015	0.022	1767.695	2041.222
2						
	818.879	-1811.437	-0.011	0.016	818.868	-1811.421
3						
	1741.624	-979.320	-0.011	0.016	1741.613	-979.304
4						
	-2071.603	-1603.573	-0.015	0.021	-2071.618	-1603.552
Ashton						
∑	0.071	-0.102	-0.071	0.102	0.00	0.00

7.

Station	Adjusted Bearings	Dist.	Adjusted N	Adjusted E
Ashton				
	S 46° 11' 57" E	3260.20	6,272555.250	1,861,964.442
1				
	N 49° 06' 27" E	2700.25	6,270,298.692	1,864,317.497
2				
	N 65° 40' 27" W	1987.91	6,272,066.387	1,866,358.719
3				
	N 29° 20' 57" W	1998.06	6,272,885.255	1,864,547.298
4				
	S 37° 44' 31" W	2619.73	6,274,626.868	1,863,567.994
Ashton			6,272,555.250	1,861,964.442

References

_________, *Definitions of Surveying and Associated Terms*, A.C.S.M., Bethesda, Maryland, 1978.

Brinker, Russell C., *4567 Review Questions for Surveyors*, 1978. (Very helpful.)

Brinker, Russell; Barry, Austin; and Minnick, Roy, *Noteforms for Surveying Measurements*, Second Edition, Landmark Enterprises, Rancho Cordova, CA, 1981.

Brinker, Russell and Minnick, Roy, editors, *The Surveying Handbook*, Van Nostrand Reinhold, Co., New York, 1987.

Chelapati, C. V., ed., *Land Surveyor License Examination Review Manual*, Professional Engineering Development Publications, Long Beach, 1987. (Highly recommended.)

Moffitt, F. H. and Bouchard, H., *Surveying*, Eighth Edition, Harper & Row, New York, 1987.

Wolf, Paul R., Brinker, Russell C., *Elementary Surveying*, Eighth Edition, Harper & Row, New York, 1989. (A very good presentation of the subject.)

UNIT 6

LEVELING

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Introduction

"Leveling" is a general term used in land surveying that applies to vertical measurements. Vertical measurements are made and referenced to datums, as elevations. The reference datum might be an arbitrary elevation chosen for convenience or a very precise value determined after lengthy studies. The standard reference datum used throughout California is mean sea level, based on the National Geodetic Vertical Datum (NGVD 1929).

Three methods used to measure differences in elevation are direct vertical measurement, trigonometric leveling, and differential leveling. It is important to understand the procedure, equipment and note keeping format used for each method.

Performance Expected on the Exams

Define the terms “curvature” and “refraction,” be able to calculate their combined effects and explain the procedure used to limit their effects.

Explain how to peg a level and calculate the collimation correction of a properly adjusted level.

Given field measurements, calculate the difference in elevation between two stations using trigonometric leveling method.

Explain, interpret, reduce and adjust differential leveling notes.

Explain, interpret, reduce and adjust three-wire leveling notes.

Obtain the difference in elevation between two stations by reciprocal leveling.

Plan and analyze the results of a leveling project with regard to NGS standards and specifications.

Key Terms

Altimetry

Automatic pendulum level

Backsight (+shot)

Bench mark

Curvature

Datum

Direct leveling

Differential leveling

Elevation

Foresight (-shot)

H.I.

Horizontal line

Horizontal plane

Intermediate foresight (side shot)

Leveling

Level surface

Level line

Lenker rod

Mean sea level

National Geodetic Vertical datum (NGVD 1929)

North American Vertical Datum (NAVD 1988)

Pegging a level

Philadelphia rod

Plumb line

Profile leveling

Reciprocal leveling

Refraction

Three-wire leveling

Trigonometric leveling

Turning point (TP)

Vertical difference

Vertical line

Video Presentation Outline

Basic Concepts

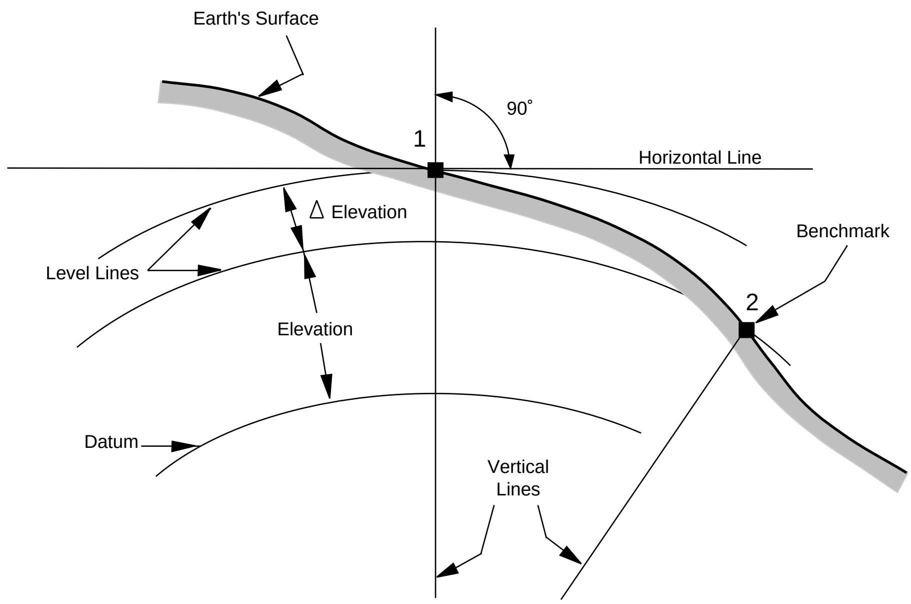


Figure 6-1. *Leveling concepts.*

- Level line
- Horizontal line
- Vertical line
- Datum

Curvature and Refraction

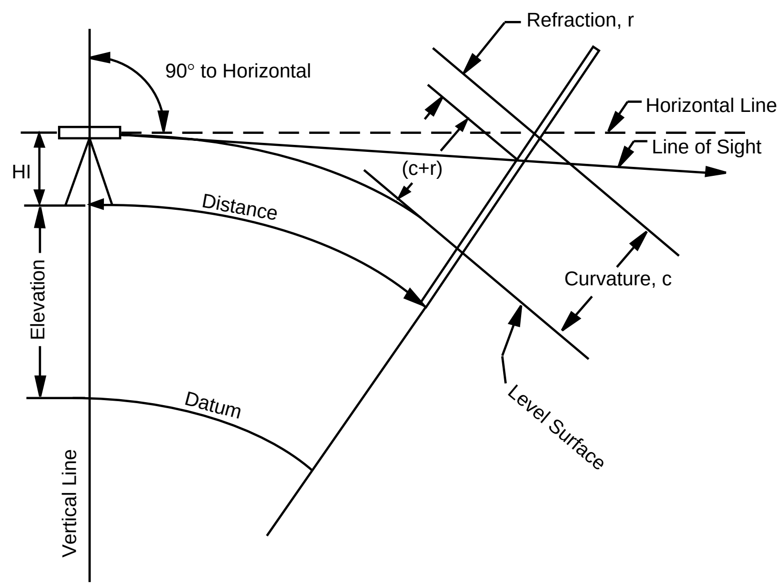


Figure 6-2. *Curvature and refraction.*

- Curvature
- Refraction
- The formula for computing the combined effect of curvature and refraction is:

  $C + R = 0.021K^2$

  C = correction for curvature

  R = correction for refraction

  K = sighting distance in thousands of feet

- Corrections for various distances

Distance	Correction
100'	0.00021'
200'	0.00082'
500'	0.0052'
700'	0.01'
1 mile	0.574'

Three Methods of Vertical Measurement Leveling

Direct Vertical Measurement Leveling

- Altimetry
- Direct elevation rods
- Lasers
- G.P.S.

Trigonometric Leveling

- Equipment
- Method
- Calculation

Elev. "B" = Elev. "A" + H.I. + (cos Z) (S) - rod - (c+r)

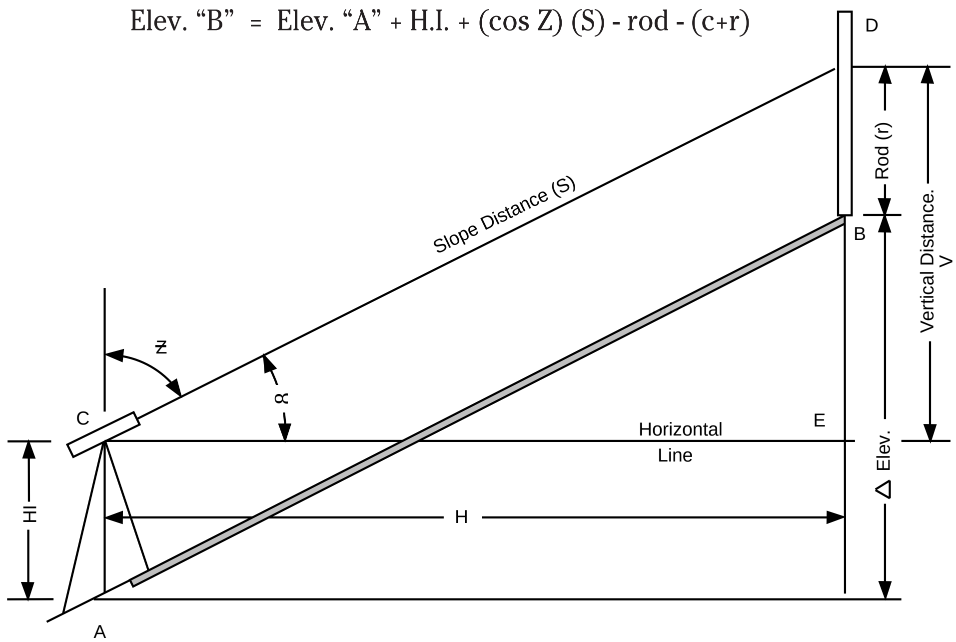


Figure 6-3. *Trigonometric leveling.*

Differential Leveling

- Equipment
- Method
- Calculations

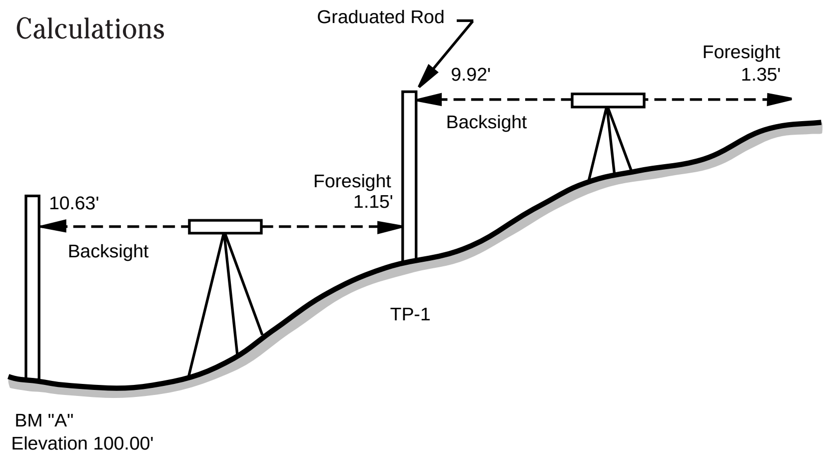


Figure 6-4. *Differential leveling.*

Notekeeping for Differential Leveling

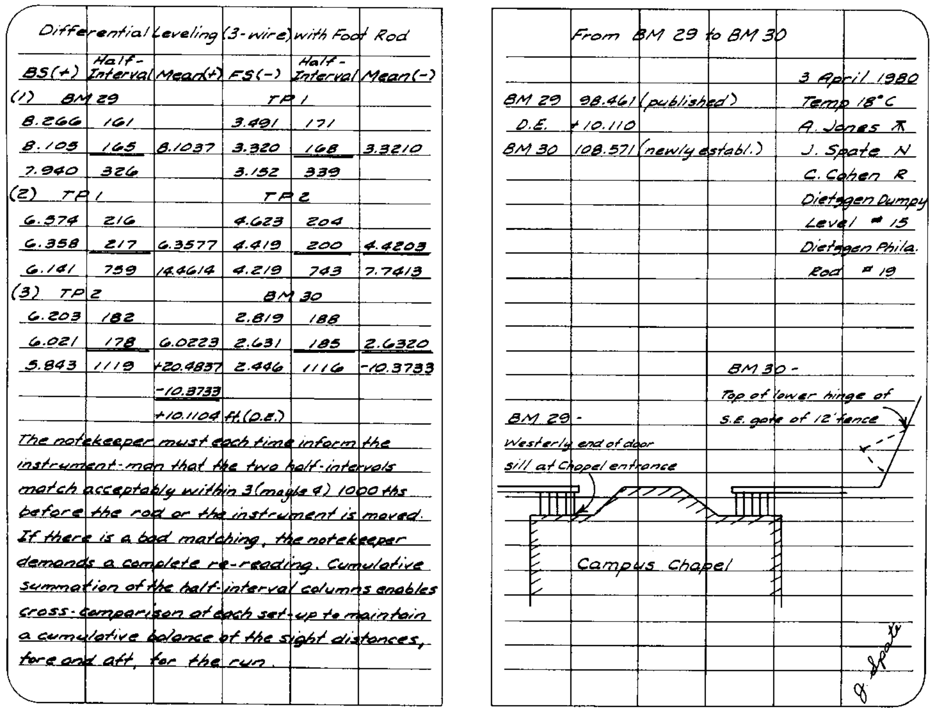
- Standard notekeeping

	+	$\overline{\wedge}$	–	Elev.
				(412.011)
B.M. #2			5.090	412.005
	7.330	417.095		
T.P.			4.765	409.765
	6.995	414.530		
B.M. #1				407.535

Figure 6-5. *Profile leveling noteform.*

Three-Wire Leveling Method

Foot Rod



Differential Leveling (3-wire) with Foot Rod

BS(+)	Half-Interval	Mean(+)	FS(−)	Half-Interval	Mean(−)
(1) BM 29			TP 1		
8.266	161		3.491	171	
8.105	165	8.1037	3.320	168	3.3210
7.940	326		3.152	339	
(2) TP 1			TP 2		
6.574	216		4.623	204	
6.358	217	6.3577	4.419	200	4.4203
6.141	759	14.4614	4.219	743	7.7413
(3) TP 2			BM 30		
6.203	182		2.819	188	
6.021	178	6.0223	2.631	185	2.6320
5.843	1119	+20.4837	2.446	1116	−10.3733
		−10.3733			
		+10.1104	ft. (D.E.)		

The notekeeper must each time inform the instrument-man that the two half-intervals match acceptably within 3 (maybe 4) 1000 ths before the rod or the instrument is moved. If there is a bad matching, the notekeeper demands a complete re-reading. Cumulative summation of the half-interval columns enables cross-comparison at each set-up to maintain a cumulative balance of the sight distances, fore and aft, for the run.

Figure 6-6. *Level notes for foot rod from noteforms for surveying measurements.*

(Reproduced with permission from Landmark Enterprises.)

Meter Rod

Figure 6-7. *Level notes for meter rod from noteforms.*
(Reproduced with permission from Landmark Enterprises.)

Special Leveling Procedures

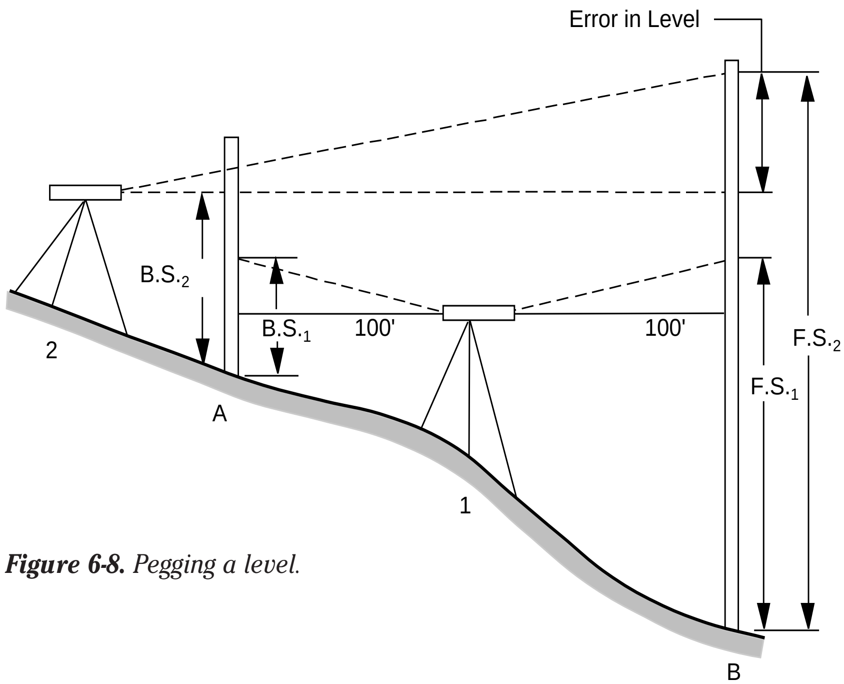


Figure 6-8. *Pegging a level.*

Sample Peg Test

Station	Backsight (+)	H.I.	Foresight	Elevation
A (assumed)	5.10	105.10		100.00'
B			4.96	100.14'
A	5.51	105.51		100.00'
B			5.35	100.16'

Adjustment = elevation of B from setup 1 - elevation of B from setup 2.

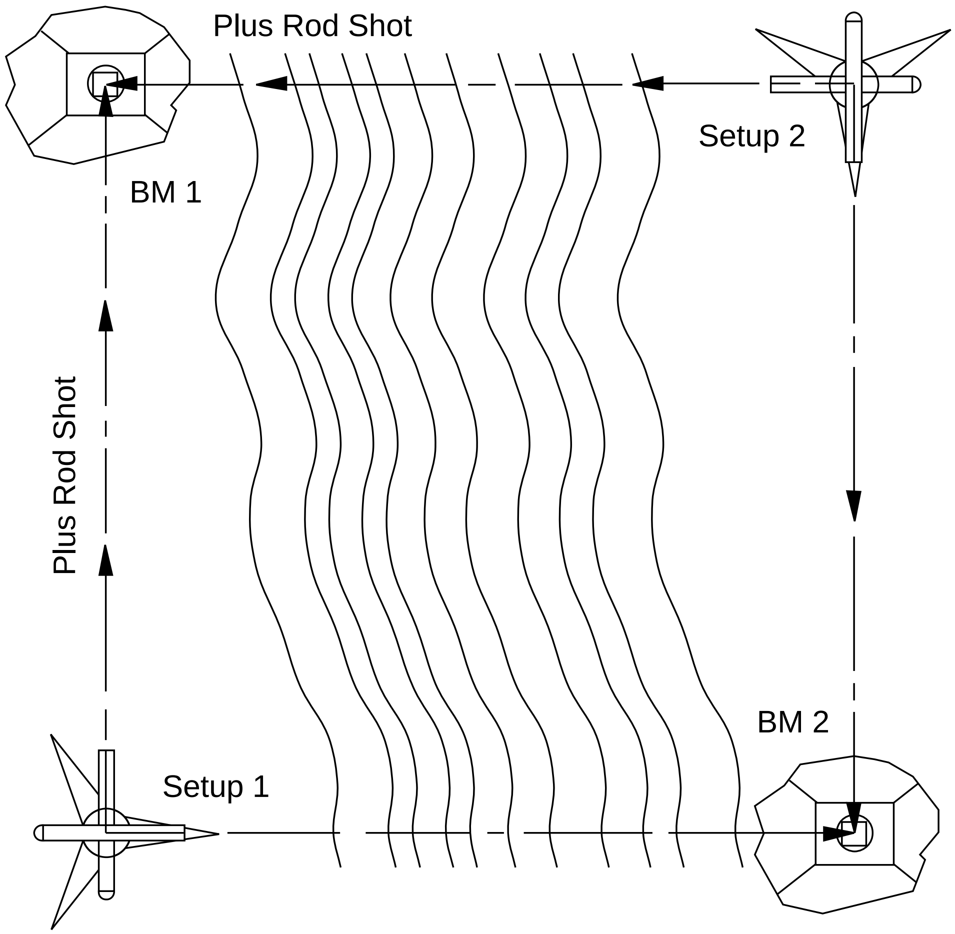


Figure 6-9. *Reciprocal leveling.*

Classification of Accuracy Standards and Adjustments

General Specifications for Vertical Control Field Procedures

Order Class	First I	First II	Second I	Second II	Third
Minimal Observation Method	Micrometer	Micrometer	Micrometer or Three-Wire	Three-Wire	Center Wire
Section Running	DR, DS, or MDS	DR, DS, or MDS	DR	DR	DR
Difference of forward and backward sight lengths never to exceed:					
per setup (m)	2	5	5	10	10
per section (m)	4	10	10	10	10
Maximum sight length (m)	50	60	60	70	90
Minumum ground clearance of line of sight (m)	0.5	0.5	0.5	0.5	0.5
Even no. of setups when not using leveling rods with detailed calibration	yes	yes	yes	yes	——
Determine temp. gradient for vert. range of line of sight for each setup	yes	yes	yes	——	——
Maximum section misclosure (mm)	$3\sqrt{D}$	$4\sqrt{D}$	$6\sqrt{D}$	$8\sqrt{D}$	$12\sqrt{D}$
Maximum loop misclosure (mm)	$4\sqrt{E}$	$5\sqrt{E}$	$6\sqrt{E}$	$8\sqrt{E}$	$12\sqrt{E}$
Single-Run Methods					
Reverse direction of single runs every half day	yes	yes	yes	——	——
Non-reversible compensator leveling instruments					
Off-level/relevel instrument bet. observing the high and low rod scales	yes	yes	yes	——	——
Three-Wire Method					
Reading check (difference between top and bottom intervals)					
for one setup not to exceed (tenths of rod units)	——	——	2	2	3
Read rod first in alternate setup method	——	——	yes	yes	yes
Double Scale Rods					
Low-high scale elevation difference for one setup not to exceed (mm)					
With reversible compensator	0.40	1.00	1.00	2.00	2.00
Other instrument types:					
Half-centimeter rods	0.25	0.30	0.60	0.70	1.30
Full-centimeter rods	0.30	0.30	0.60	0.70	1.30

DS-Double Simultaneous procedure
DR-Double-Run
MDS-Modified Double Simultaneous
D-shortest length of section (one-way) in km.
E-perimeter of loop in km.
Must double-run when using three-wire method
* May single-run if line length between network control points is less than 25 km and may single-run if line length between network control points is less then 10 km.

NOTE: *See Geometric Geodetic Accuracy Standards and Specifications for Using GPS Relative Positioning Techniques, Federal Geodetic Control Committee for latest specifications.*

Figure 6-10. *"General Specifications for Vertical Control," National Geodetic Survey.*

Adjustments to Level Runs

- Length of lines methods
- Number of turning points method
- Least squares method

Sample Test Questions

1. When pegging a level the surveyor reads 5.25 on the backsight rod and 5.38 on the foresight rod. After moving the level adjacent to the backsight rod, a reading of 5.18 is taken on the near rod. What should be on the far rod?

2. When pegging a level, how far apart should the rod readings be taken?

3. The effects eliminated by keeping backsights and foresights equal are ___________ and __________________.

4. Does an instrument in perfect adjustment sight a level line to a distant object? Explain.

5. Fill in the missing data in the sample differential level run below.

Station	B.S. (+)	H.I.	F.S. (–)	Elevation
B.M. A				256.18
	4.05	a.		
T.P. 1			6.48	253.75
			10.26	b. (top Pipe)
	5.26	259.01		
T.P. 2			7.56	c.
	2.56	254.01		
T.P. 3			d.	245.56
	e.	250.45		
B.M. B			7.08	f. (243.39)

6. What is the misclosure in the sample differential level run in problem 5? What is the adjusted elevation of T.P. 2?

7. To meet Class II, Second Order accuracies, what would be the maximum misclosure of a level run of two kilometers?

8. What is the recommended leveling method for meeting the Class II Second Order Standard?

9. Is it necessary to balance the foresights and backsights to achieve the necessary accuracies in question 7? If yes, what is the maximum difference allowed per setup? What is the maximum length allowed per sight?

10. A theodolite is set up over Point 123 with an H.I. of 5.59 feet. The elevation of Point 123 is 2105.67 feet. The measured zenith angle to a target with an H.I. of 4.77 at Point 124, is 94° 35' 46". The slope distance measured from the theodolite to the target is 2145.89 feet. What is the difference in elevation between Point 123 and Point 124? What field procedure could you use that would allow you to discount the effects of curvature and refraction on the results?

11. Problem D-6, 1978 LS

    Problem Statement: A collection of rod readings is shown below. These readings were taken over a section of line of three-wire levels run in both directions using a precision self-leveling level and invar-faced-rods graduated in centimeters with readings estimated to the nearest millimetre. The C-factor of the instrument is -0.150, the stadia constant is 0.335 and the average rod temperature is 30° C.

    Required:

    A. Arrange these notes in the workbook paper as would be done in a field book, showing all data normally shown in field notes for precise leveling.
    B. Reduce and analyze the notes, showing all intermediate steps and checks. Note any deviations from acceptable practice and/or limits, and proceed to a determination of the mean difference in elevation for the section and assumptions you make. Compute and apply all applicable corrections for systematic errors. Express the difference in elevation in meters.
    C. Determine and state the highest order of leveling for which this run would qualify. Use the latest published standards for vertical control surveys.

D. Discuss the concept of orthometric corrections: What it is, what it does, when it is used, and the kind of work to which it is normally applied. Would it be likely to be applied to the data in this problem? Why?

Rod Readings		**Three Wire Levels**	
Forward Run		**Backward Run**	
+	−	+	−
234	2392	1831	721
198	2359	1802	693
162	2327	1775	666
1455	1629	3077	171
1384	1557	3057	151
1313	1484	3037	131
158	3250	3332	298
155	3227	3310	227
112	3203	3288	257
126	3125	1854	1811
117	3106	1784	1742
088	3087	1714	1672
808	1832	2603	240
777	1795	2563	202
747	1779	2542	164

Answer Key

1. 5.31 ft

2. 200 ft

3. Curvature, refraction

4. No, the line of sight is a curved line due to atmospheric refraction.

5. A. 260.23
   B. 249.97
   C. 251.45
   D. 8.45
   E. 4.89
   F. 243.37

6. Misclosure = -.02 ft., t.p.#2 = 251.46

7. 11.31 mm

8. Three-wire, double-run or single-run double simultaneous procedure

9. Yes, 10 m, maximum sight length is 70 m

10. Change in
    Elevation = H.I. PT. 123 - H.I. PT 124 + ((cos Z) (slope dist.) - (c+r))
    = 5.59 - 4.77 + (cos 94° 35' 46" x 2145.89) - (.021 x 2.14589^2)
    = -171.23 ft

    The effect of curvature and refraction will be canceled if measurements are made from each end of the line, and the mean of the results of the two sets of measurements is used.

11. A. Refer to Figure 6-7 for field noteform for three-wire level notes using a meter rod.

B. Assume K = 100

Forward run distance leveled = B.S. Intervals + F.S. Intervals + 10 (Stadia Constant)

= 359 + 367 + 10 (.335)

= 729.4M

= .7294 km

Corrected elev. = F.S. + B.S.

= 2.6013 + (-12.0440)

= -9.4427 m

Correction for interval imbalance = Difference between F.S. and B.S. Intervals x - 0.15

= 8 x -0.15 mm

= -1.2 mm

Elev. = -9.4427 - .0012 = -9.4439

Backward run distance leveled = 712 M + 10 (.335) = .7154 km

Corrected elev. = $\frac{9.4510 - 10X.15}{1000}$

= 9.4495 m

There are two bad rod readings in the notes, both in the forward run.

1. The foresight middle wire reading at STA. 4 should be 107 rather than 117.
2. The backsight low wire reading at STA. 5 should be 1759 rather than 1779.

C. Average length of section = Mean of forward and backward runs

= .7224 km

Average difference in elevation = 9.4467

Difference in elevation between forward and backward runs = 9.4439 - 9.4495

= .0056 m

= 5.6 mm

Difference in forward and backward sight lengths < 10 m and maximum section misclosures from Figure 6-10 show this run meets Second Order, Class II requirements.

D. Reference: *The Surveying Handbook*, Brinker and Minnick and *Caltrans Surveys Manual.*

Level surfaces are perpendicular to the direction of gravity. Gravity is affected by the variation of centrifugal force which increases with altitude and decreasing latitude. In geodetic leveling, this variation in gravity accounts for nonparallel level surfaces. Orthometric correction is applied to account for convergence of level surfaces for long level runs in north-south directions or runs at high elevations.

Orthometric correction would not be applied to the data in this problem because required information for application of orthometric correction such as latitude and elevation are not given, and short level runs of this order and class would be unaffected by orthometric correction.

References

________, *Surveys Manual*, California Department of Transportation, Chapters 2, 4, and 5.

________, *Definitions of Surveying and Associated Terms*, A.C.S.M., Bethesda, Maryland, 1978.

Brinker, Russell C., *4567 Review Questions for Surveyors*, P. E. Publisher, 1978. (Very helpful.)

Brinker, Russell; Barry, Austin; and Minnick, Roy, *Noteforms for Surveying Measurements*, Second Edition, Landmark Enterprises, Rancho Cordova, CA, 1981.

Brinker, Russell and Minnick, Roy, Editors, *The Surveying Handbook*, Van Nostrand Reinhold, Co., New York, 1987.

Chelapati, C. V., Editor, *Land Surveyor License Examination Review Manual*, California State University, Long Beach, 1987. (Highly recommended.)

Moffitt, F. H. and Bouchard, H., *Surveying*, Eighth Edition, Harper & Row, New York, 1987.

Wolf, Paul R., and Brinker, Russell C., *Elementary Surveying*, Eighth Edition, Harper & Row, New York, 1989. (A very good presentation of the subject.)

UNIT 7

ROUTE SURVEYING

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Introduction

Route surveying is comprised of all survey operations required for design and construction of engineering works such as highways, pipelines, canals, or railroads. At Caltrans a route surveying system is generally associated with highway design and construction.

A route surveying system usually contains four separate but interrelated processes:

- Reconnaissance and planning
- Works design
- Right of way acquisition
- Construction of works

This video unit presents principles and techniques of route surveying and calculation operations related to these four processes.

Performance Expected on the Exams

Solve various exam problems related to principles, design and application of vertical curves.

Solve various exam problems related to principles, design and application of horizontal curves.

Compute and interpret grades for roads or streets from design information.

Compute and interpret grades, cuts and fills and positions for placement of slope stakes.

Key Terms

Arc
Centerline
Chord
Circular curve
Contours
Degree of curve
Elevation
Grade
Length of arc
Offset
Percent of slope
Radius
Rate of change
Sag curve
Stationing
Tangent
Vertical curves
Arc definition
Central angle
Chord definition
Construction
Deflection angle
Direction of travel
External distance
Horizontal curve
Mid-ordinate distance
Parabola
Radian
Radius point
Right of way
Slope stakes
Summit curve
Tangent offsets

Video Presentation Outline

The Route Surveying System

Linear Projects

- Highways
- Pipelines
- Canals
- Railroads

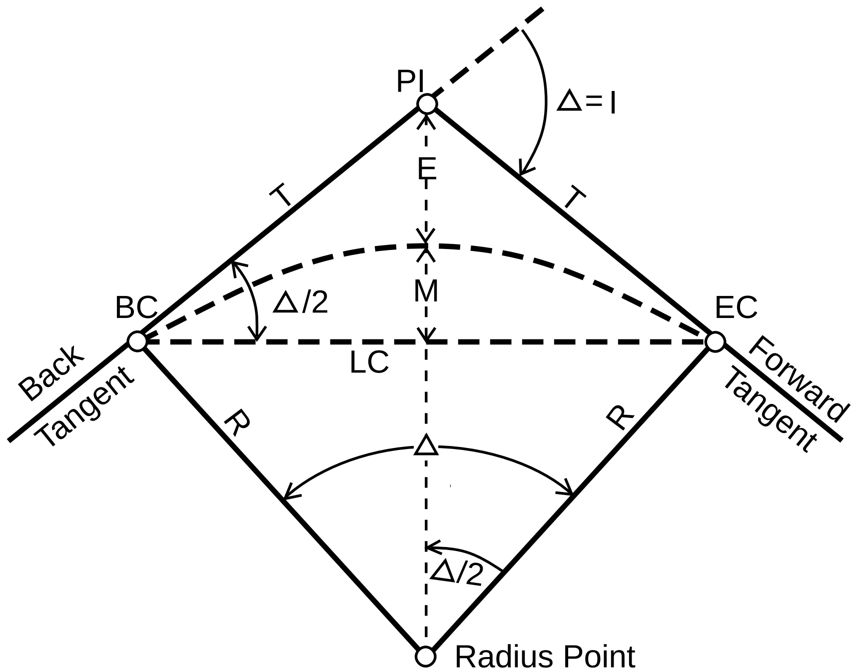
Major Elements of Route Surveying Systems

- Reconnaissance and planning
- Design
- Right-of-way acquisition
- Construction

Major Components of Route Surveying Systems

- Stationing
- Offset distance
- Profile grades (slope percentage)
- Horizontal curves
- Vertical curves
- Cross sections
- Slope staking

Components of Horizontal Circular Curves



Terminology for Horizontal Circular Curves

L = length of curve (arc)

LC = long chord

T = tangent length

E = external distance

Δ or I = central angle

R = radius

PI = intersection of tangents

M = middle ordinate distance

Formulae for Horizontal Circular Curves

$$T = R\tan\frac{\Delta}{2}$$

$$LC = 2R \times \sin\frac{\Delta}{2}$$

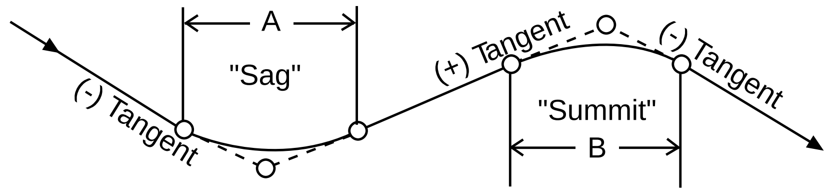
$$L = R \times \Delta \text{ (exp. in radians)}$$

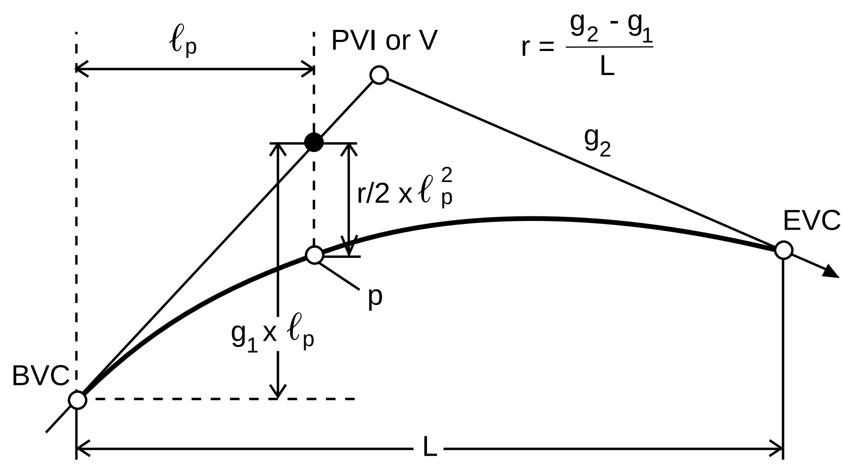
$$M = R\left(1-\cos\frac{\Delta}{2}\right)$$

$$E = R\left(\frac{1}{\cos\frac{\Delta}{2}} - 1\right)$$

$$L = 2\pi R\left(\frac{\Delta}{360^\circ}\right)$$

Components of Vertical Parabolic Curves





Terminology for Vertical Parabolic Curves

BVC = beginning of vertical curve

EVC = end of vertical curve

PVI = point of intersection

r = rate of change

g_1 = slope of back tangent (in %)

g_2 = slope of forward tangent (in %)

L = length of curve, in stations

ℓ = length of sub-curve, in stations, to point on curve from BVC

$$\text{Elev.}_p = \left(\frac{r}{2}\right) \ell^2_p + g_1 (\ell_p) + \text{Elev}_{BVC}$$

$$\ell_{\text{high/low point}} = \left| \frac{g_1}{r} \right|$$

Example Problem

Problem B-5 1990 LS

You have been provided design criteria shown in the diagrams 1, 2, and 3, below and on the next page.

Answer the following questions using the information provided in the diagrams.

1. Determine the ground elevation of the back of the sidewalk at the following locations:
   A. Driveway centerline
   B. Southeasterly property corner
   C. Southwesterly property corner

2. Provide the grade percentage between Point C and the building pad. Show all calculations.
3. What is the slope ratio from Point A to the toe of slope?
4. Calculate the cut from the back of the sidewalk to the sewer lateral invert at the property line.
5. Calculate the distance from the north property line to the toe of slope at Point B.

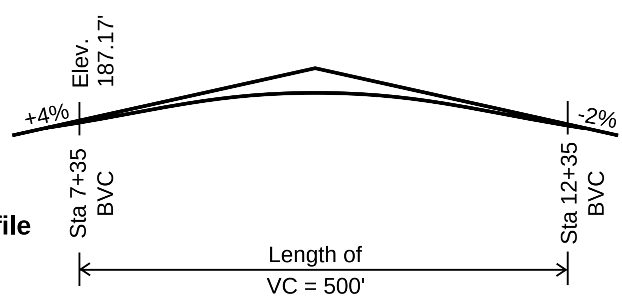


Diagram 1
Street ℄ Profile

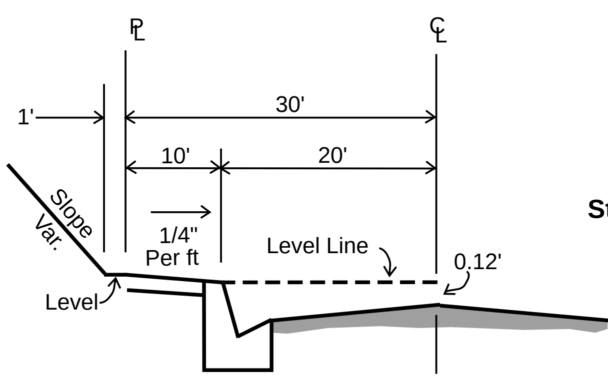


Diagram 2
Street Section

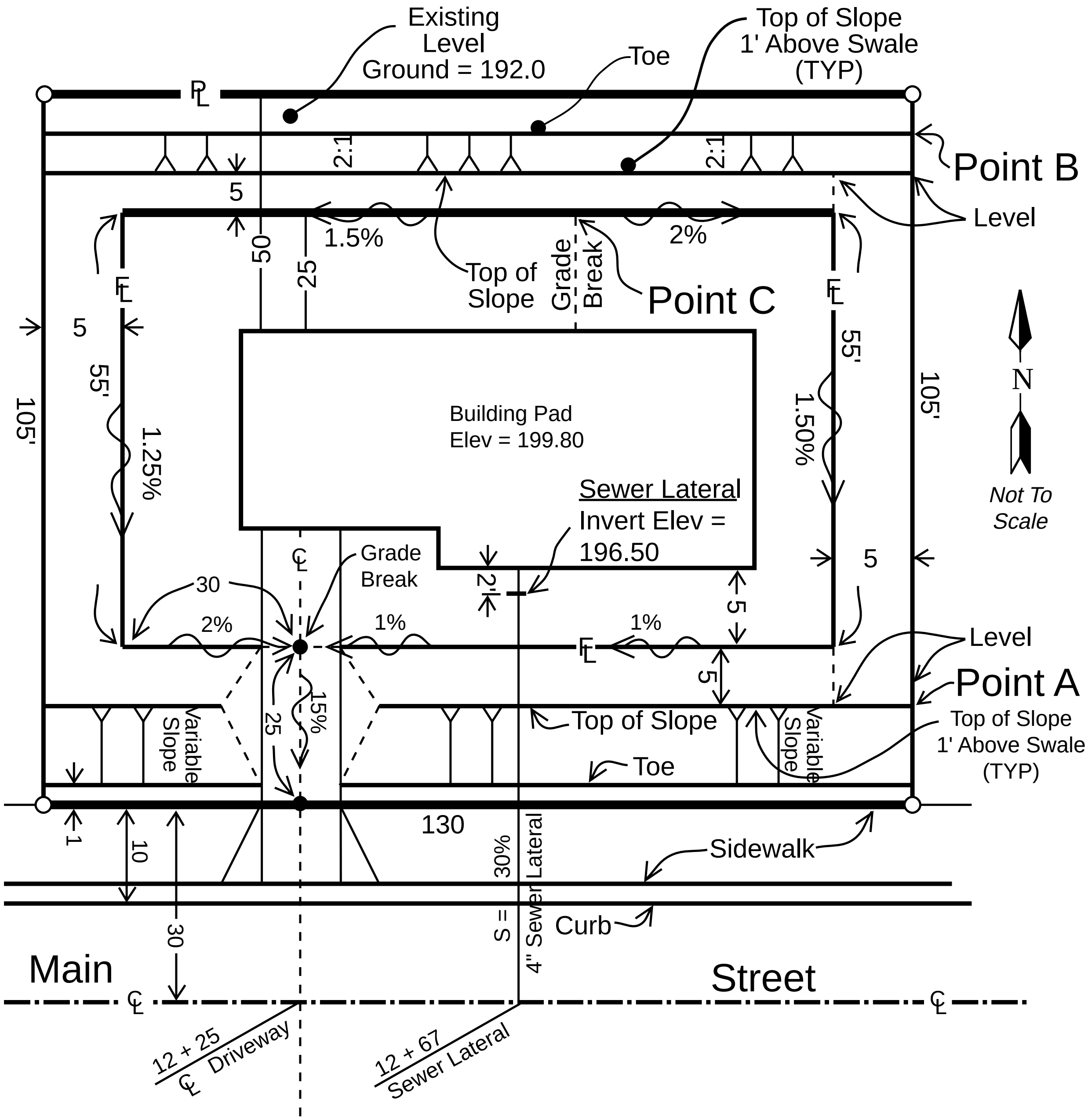


Diagram 3
Plan View
(all dimensions are in ft)

Solution of 1990 California LS Examination Problem B-5

NOTE: See video for solution methodology; ±0.02' is acceptable for all answers.

1. Calculation of the elevation of the back of sidewalk at the:

   A. Centerline of the driveway: **192.69'**
   B. Southeasterly property corner: **190.80'**
   C. Southwesterly property corner: **193.28'**

2. Grade percentage of the slope between Point C and the top of building pad: **3.40%**
3. Slope ratio from Point A to toe of slope opposite Point A: **39.68% or 2.52/1**
4. Cut from the back of sidewalk to the invert of sewer lateral at the property line: **C-3** $^{\underline{76}}$'
5. Distance from the north property line to toe of slope at point B: **5.66'**

Sample Test Questions

1. What is the “station” of the ending point of a surveyed line originating at “sta. 23+45.50" that has a measured length of 412.91 ft?

   A. Sta. 19+32.59
   B. Sta. 27+58.41
   C. Sta. 19+32.41
   D. Sta. 27+58.59

2. How far apart is survey point “G” (sta. 61+56.81) and survey point “H” (sta. 24+12.93)?

   A. 8,569.26
   B. 3,743.22
   C. 3,743.88 ft
   D. 8,569.74 ft

3. A section of road rises 18.50 ft in 435 ft (horizontal run). What is the percentage of slope for this section of the road (nearest two decimal places)?

   A. +4.25%
   B. -4.25%
   C. +4.05%
   D. -4.05%

4. The percentage of slope for a proposed ramp is -2.65%. What is the change in elevation of this ramp for a horizontal length of 412 ft?

   A. +10.92
   B. +100.92
   C. -10.92 ft
   D. -100.92

5. A distance measured perpendicularly from the center or base line of a survey project is called:

   A. agonic line
   B. an offset
   C. tangent correction line
   D. secant correction line

6. Stationing and offsets may define a ______________ system.

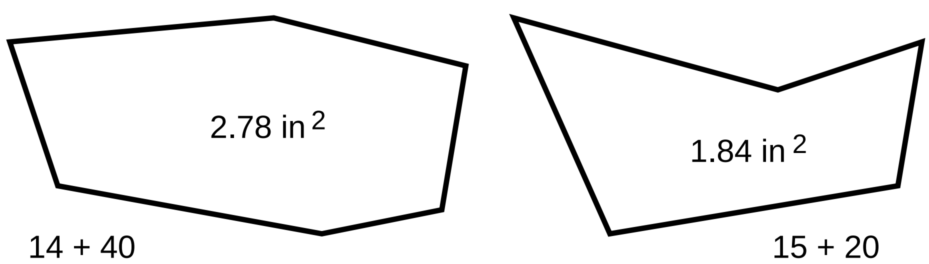
   A. solar correction
   B. plane coordinate
   C. cadastral
   D. construction

7. NGVD 1929 is one of the control systems that ___________ are referenced to.

   A. elevations
   B. solar positions
   C. horizontal positions
   D. GIS data bases

8. Two cross sections, plotted at a horizontal scale: 1 in=40 ft, and vertical scale: 1 in=10 ft, along with their areas are shown in the sketch below. Compute the volume (in cubic yards) contained between the two stations.

   A. 147,840
   B. 73,920
   C. 16,427
   D. 2,738

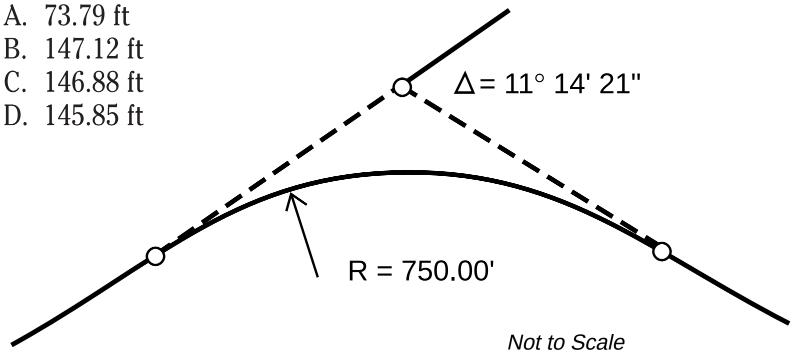


9. In laying out a highway for construction, slope stakes are needed. A fill is required at station 14+40; centerline elevation 48.75. The full width of the road from top of slope to top of slope is 36 ft and is level. The road design specifies side slopes of 2.5/1. A trial shot is taken 40.0 ft from the centerline at elevation 42.3. Assuming natural ground is fairly level, how far and in which direction from the trial shot should the "catch point" be located and staked?

   A. 19.4 ft toward centerline
   B. 5.9 ft away from centerline
   C. 5.9 ft toward centerline
   D. 19.4 ft away from centerline

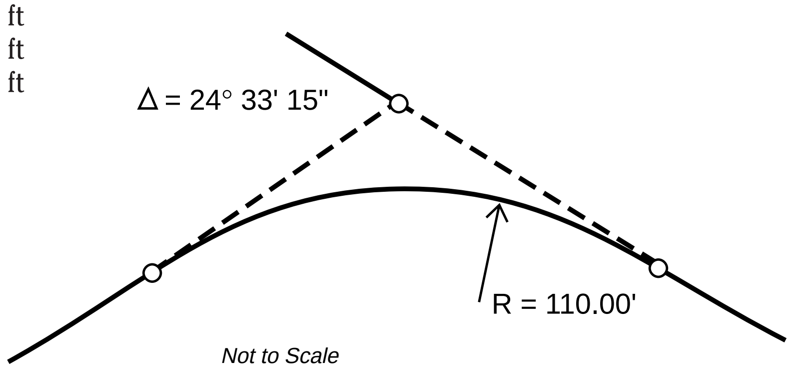
10. What is the length of arc for the horizontal curve shown in the sketch below?

   A. 73.79 ft
   B. 147.12 ft
   C. 146.88 ft
   D. 145.85 ft



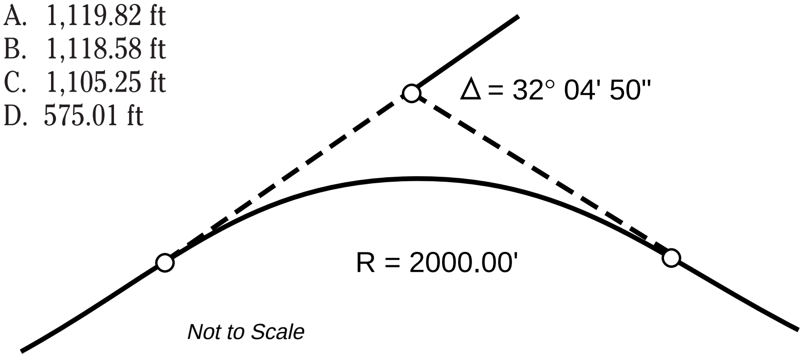
11. What is the length of tangent for the horizontal curve shown in the sketch below?

   A. 47.14 ft
   B. 46.78 ft
   C. 46.71 ft
   D. 23.94 ft



12. What is the length of long chord for the horizontal curve shown in the sketch below?

    A. 1,119.82 ft
    B. 1,118.58 ft
    C. 1,105.25 ft
    D. 575.01 ft



13. For a highway curve with a degree of curve of 06° 30', what is the length of chord between sta.16+32.09 and sta. 17+51.86?

    A. 119.65 ft
    B. 119.68 ft
    C. 119.77 ft
    D. 119.81 ft

14. From the given curve design data, calculate the stations of the BC and EC of this horizontal curve.

    R = 1270.00 ft

    Sta. @ PI = 34+21.89

    I = 26° 14' 11"

    A. BC = 31+25.93; EC = 37+17.85
    B. BC = 31+31.13; EC = 37+12.67
    C. BC = 31+26.42; EC = 37+10.48
    D. BC = 31+25.93; EC = 37+07.48

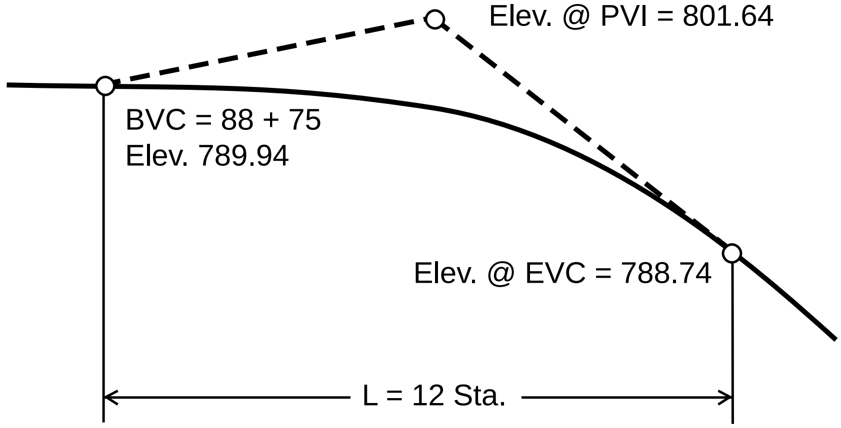
15. From the following design information for a circular curve:

    R = 760.00 ft; delta = 12° 04' 15"; Sta @ BC = 9+63.04

    What is the deflection angle (from the BC) to sta. 10+80?

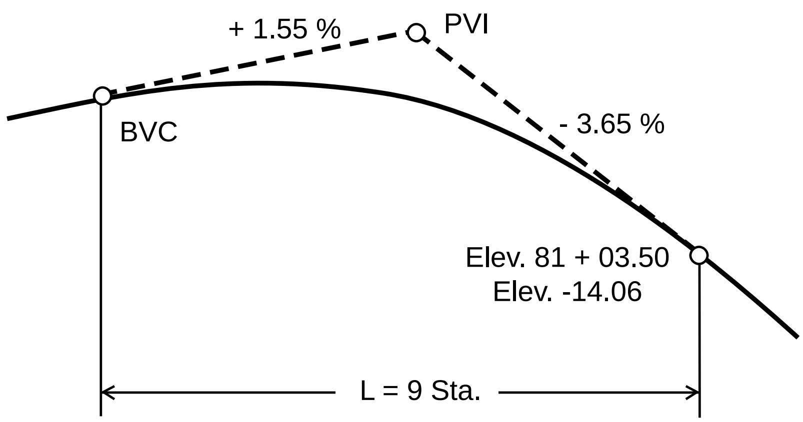
    A. Deflection angle = 04° 24' 32"
    B. Deflection angle = 04° 24' 03"
    C. Deflection angle = 08° 49' 06"
    D. Deflection angle = 08° 48' 06"

16. From the equal-tangent vertical curve data given in the sketch below, calculate the values of g_1 and g_2.



A. g_1 = -1.95%; g_2.= +2.15%
B. g_1 = +1.95%; g_2.= -2.15%
C. g_1 = -0.51%; g_2.= +0.46%
D. g_1 = +0.51%; g_2.= -0.46%

17. Using the equal-tangent vertical curve data given in the sketch below, calculate the station and elevation for the PVI and the BVC of the curve.



A. Sta. of BVC = 72+03.50; elev. = -9.34
Sta. of PVI = 21+45.50; elev. = -2.36

B. Sta. of BVC = 72+03.50; elev. = -4.62
Sta. of PVI = 76+53.50; elev. = 2.36

C. Sta. of BVC = 72+03.50; elev. = 4.62
Sta. of PVI = 21+45.50; elev. = 2.36

D. Sta. of BVC = 72+03.50; elev. = 9.34
Sta. of PVI = 21+45.50; elev. = 2.36

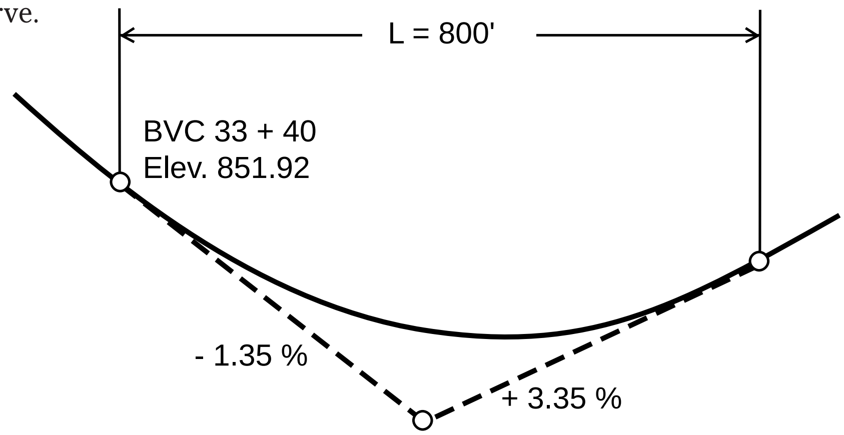
18. An equal-tangent vertical curve has the following design data:

g_1 = -3.65%; g_2 = -0.30%; L = 4 sta.

What is the rate of change for this curve?

A. +0.988% per full station
B. - 0.988% per full station
C. +0.838% per full station
D. - 0.838% per full station

19. Using the design data for the equal-tangent vertical curve shown in the sketch below, calculate the station and elevation for the low point of the curve.



A. Low point station = 35+65; elev. = 850.31
B. Low point station = 35+70; elev. = 850.37
C. Low point station = 35+70; elev. = 850.41
D. Low point station = 35+90; elev. = 850.37

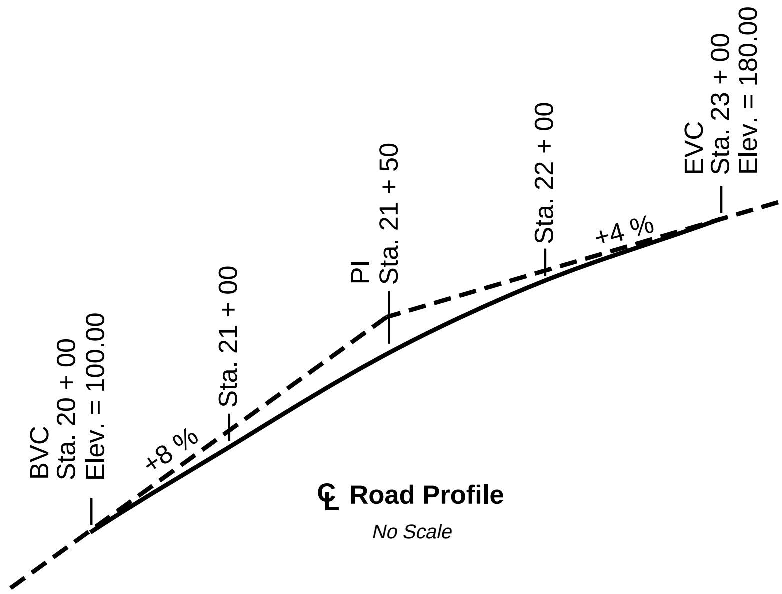
20. Problem A-6 1992 LS

A surveyor has been requested to set a slope stake at a five-ft offset as shown on the following data sheet. Place the slope stake for the daylight cut at Station 21+00. Assume the slope has a uniform grade between topo shots.

Requirements: Answer the following questions using the information provided on the data sheet.

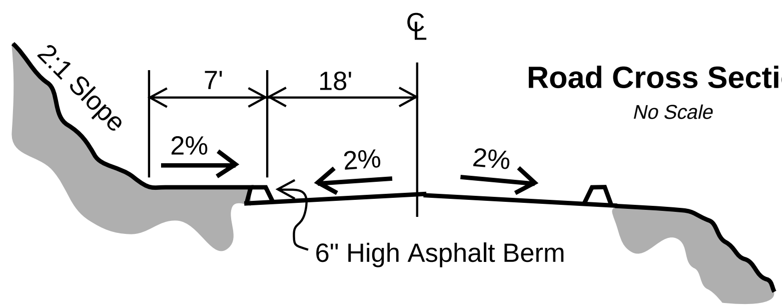
A. Determine the following for Station 21+00:
   1. Centerline elevation
   2. Hingepoint elevation (toe of 2:1 slope)
   3. Distance left from centerline for offset stake

B. What specific information should be put on the stake to construct the daylight cut and toe of slope as required above?



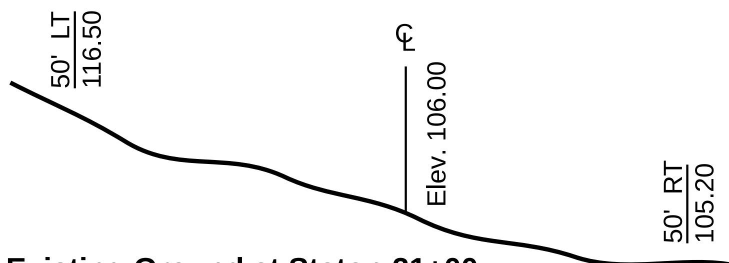
℄ **Road Profile**

No Scale



Road Cross Section

No Scale



Existing Ground at Staton 21+00

No Scale

NOTE: All units are in ft unless otherwise stated

21. Problem B-5 1991 LS

During the rough grading phase of construction, you discovered a 12-inch water pipe crossing the roadway at Station 18+50. The elevation on top of the pipe is 730.92 ft. You have communicated this information to the project engineer who has asked you to calculate and lay out an equal tangent vertical curve so that the top of pavement passes 36 in above the top of the water pipe with the following design elements:

Vertical curve beginning at Station 16+50 (Vertical Curve #2)

G_1 = +8.75%

G_2 = -1.50%

A drop inlet needs to be installed at the lowest possible elevation between the beginning and end of horizontal curve #1 along the flowline.

Required:
Show all work in completing the following requirements.

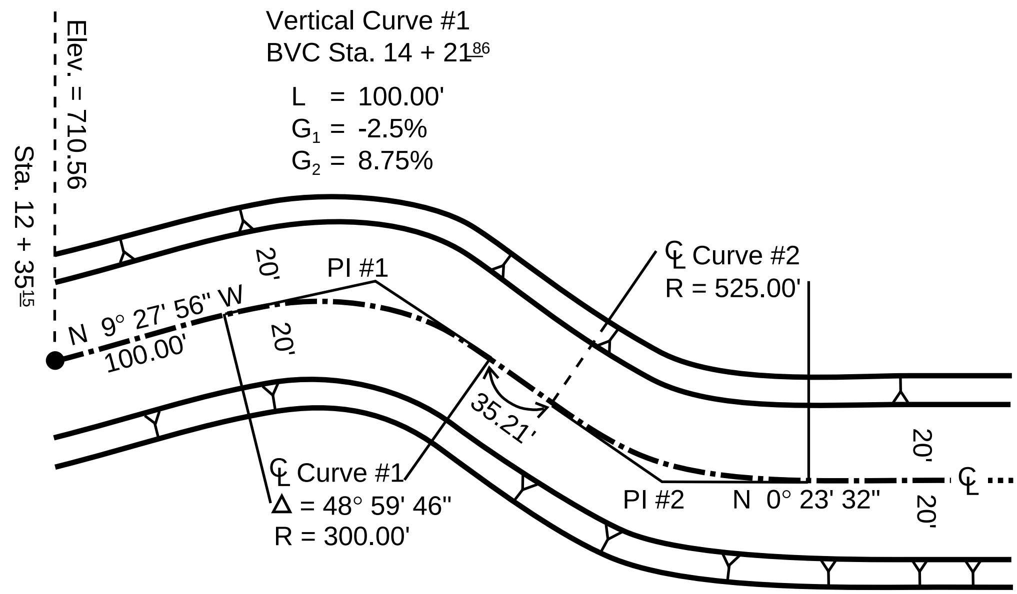
A. Calculate the following elements of horizontal curve #1:
   1. Tangent
   2. Length
   3. EC Station

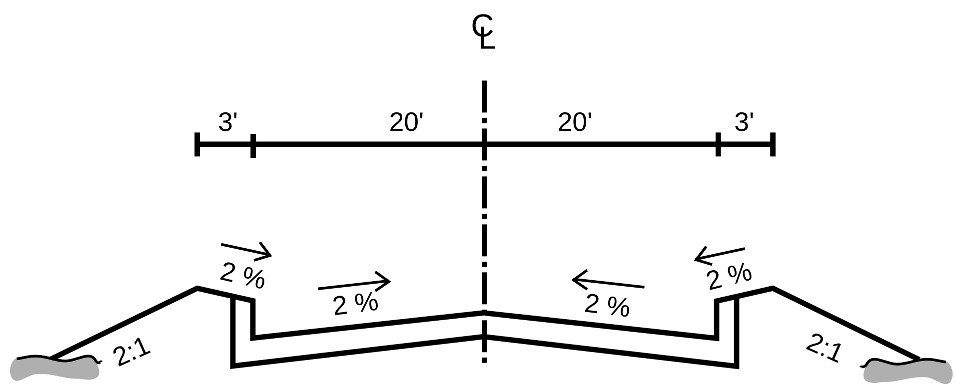
B. Calculate the delta of horizontal curve #2.

C. Calculate
   1. The station, and
   2. The elevation of the top of the drop inlet to be installedbetween the beginning and the end of horizontal curve #1.

D. Calculate the following elements of the equal tangent vertical curve #2:
   1. Total length
   2. Point of Vertical Intersection Station
   3. Pavement elevation at the intersection of the centerline and the water pipe

Plan View of Future Roadway





Typical Cross Section

Not to Scale

Answer Key

1. B. (sta. 27+58.41)
2. C. (3,743.88 ft)
3. A. (+4.25%)
4. C. (-10.92 ft)
5. B. (an offset)
6. B. (plane coordinate)
7. A. (elevations)
8. D. (2,738)
9. C. (5.9 ft towards the centerline)
10. B. (147.12 ft)
11. D. (23.94 ft)
12. C. (1,105.25 ft)
13. B. (119.68 ft)
14. D. (the sta. of BC is 31+25.93; the sta. of EC is 37+07.48)
15. A. (the deflection angle to sta. 10+80, is 04° 24' 32")
16. B. (G_1 = +1.95%; G_2 = -2.15%)
17. B. (Sta. @ BVC = 72+03.50; elev. = -4.62)
    (Sta. @ PVI = 76+53.50; elev. = 2.36)
18. C. (+0.838% per full station)
19. B. (sta. of "low point" = 35+70; elev. = 850.37)
20. NOTE: By using graph paper, the reader may solve or check certain elements of this problem by plotting and scaling.

20A. Calculation of the required elevations and offset distance:

1. Centerline grade of the road @ 21+00 by vertical curve computation: 107.33'
2. Elevation at hingepoint: 107.61'
3. Distance out from centerline for daylight stake: 42.6' ± 0.5'

20B.

1. Indicate that stake is set five ft offset from daylight at top of slope.
2. Indicate a $6^{\underline{3}'}$ from begin daylight or $7^{\underline{3}'}$ from offset stake.
3. Indicate that the design slope is 2:1
   OR a cut of $6^{\underline{3}}$ out $12^{\underline{5}}$ from daylight
   OR a cut of $7^{\underline{3}}$ out $17^{\underline{5}}$ from offset stake.

21A. Given: delta = 48° 59' 46", R = 300.00'

1. T = Rtan $\Delta/2$ = (300.00) (tan 24° 29' 53") = 136.71
2. $L = 2\pi R\Delta/360° = \dfrac{2\pi\ (300)\ 48° 59' 46''}{360°} = 256.54$
3. Station 12+35.15 (Station given)

Station 12+35.15	(Station given)
1+00.00	(Given tangent length to BC)
13+35.15	
2+56.54	(Length of arc)
15+91.69 EC	

21B.

First tangent	S 09° 27' 56" E
First delta	+ 48° 59' 46"
Second tangent	S 39° 31' 50" W
Second tangent	S 39° 31' 50" W
Last tangent	-S 00° 23' 32" W
Delta curve #2	39° 08' 18"

21C. 1.

$$\ell = \frac{g_1}{r}$$

$$r = \frac{g_2 - g_1}{L}$$

$$= \frac{8.75 - (-2.5)}{1}$$

$$= 11.25$$

$$\ell = \frac{-2.5}{11.25}$$

$$= 0.2222 \text{ station}$$

Given:	BVC station	14+21.86
		+ 22.22
	station of lowest point	14+44.08

2.

$$\text{Elev. p} = \left(\frac{r}{2}\right) \ell^2 p + g_1 (\ell_p) + \text{Elev}_{BVC}$$

$$= \frac{11.25}{2} \quad (.222^2) \;= (-2.5)\,(.222) + 705.89$$

$$= 705.61$$

$$\text{Elev.}_{\text{top inlet}} = 705.61 - (.02)\,(20)$$

$$= 705.21$$

21D. 1. Data given for Vertical Curve #2

g_1 = +8.75% g_2 = -1.50% $BVC_{sta.}$ = 16+50

Compute BVC $\#2_{elev.}$:

$PIVC\ \#1_{elev.}$ = (14+71.86 - 12+35.15)(-2.5%)+710.56 = 704.64

$BVC\ \#2_{elev.}$ = (16+50.00 - 14+71.86)(8.75%)+784.64 = 720.23

Elev. required @ 18+50 = 730.92+3.00+0.40 = 734.32

$$r = \frac{g_2 - g_1}{L}$$

ℓ = 2 stations

$$elev._p = \left(\frac{r}{2}\right)\ell^2_p + g_1 l_p + elev_{BVC}$$

$$734.32 = \frac{1}{2}\left(\frac{-1.50 + 8.75}{L}\right)(2^2) + 8.75(2) + 720.23$$

L = 6.01 sta. or round to 600'

2. $PIVC\ \#2_{sta.}$ = 16+50 + 300' = 19+50

3.

Existing pipe elev.	730.92
Plus 3.00	3.00
Plus crown (gutter to centerline)	0.40
Pavement centerline over top of pipe	734.32

References

________________, *California Department of Transportation Surveys Manual.*

Brinker, Russell, and Minnick, Roy, Editors, *The Surveying Handbook,* Van Nostrand Rienhold Co., New York, 1987. (Very comprehensive for all surveying operations.)

Kavanagh, Barry F., *Surveying With Construction Applications, Second Edition,* Prentice Hall, New Jersey, 1992.

Minnick, Roy, *Land Survey Test Training Manual,* Landmark Enterprises, Rancho Cordova, CA.

Wolf, Paul R., & Brinker, Russell C., *Elementary Surveying, Eighth Edition,* Van Nostrand Rienhold Co., New York, 1987. (Very good presentation.)

Zimmerman, Edward, *Basic Surveying Calculations,* Landmark Enterprises, Rancho Cordova, CA, 1991. (Easily understood theory and operations.)

UNIT 8

AREAS AND VOLUMES

Don Hunter, PLS
California Department of Transportation

Introduction

Area is the surface within a set of lines, expressed in square units, as in "square feet." The area within a triangle is the surface within three sides. The area of a circle is the surface within the circumference. One of the objectives of land surveys is to determine the area of a parcel of land. There are a number of methods used for calculating areas, including the use of double meridian distances and coordinates.

Volume is defined as the amount of material occupying a certain space, expressed in cubic units, as in "cubic yards." Data obtained from engineering design surveys are used to calculate earthwork quantities. The standard method for calculating earthwork quantities is the average end area method.

Performance Expected on the Exam

Calculate the areas of circles, triangles, quadrilaterals and regular polygons.

Calculate the areas of circular sectors and segments.

Calculate areas using the trapezoidal rule and Simpsons 1/3 rule.

Calculate the areas of irregular polygons by double meridian distance and coordinate methods.

Calculate volume by average end area method.

Key Terms

Sector

Segment

Polygon

Regular polygon

Quadrilateral

Parallelogram

Trapezoid

Trapezium

Trapezoidal rule

Simpson's 1/3 rule

Latitudes

Departures

Northings

Eastings

DMD

Prism

End area

Video Presentation Outline

Standard Geometric Figures and Area Formulas

- Circle, sector, segment
- Polygon
- Triangle
- Quadrilateral

Irregular Polygons with More than Four Sides and Curved Sides

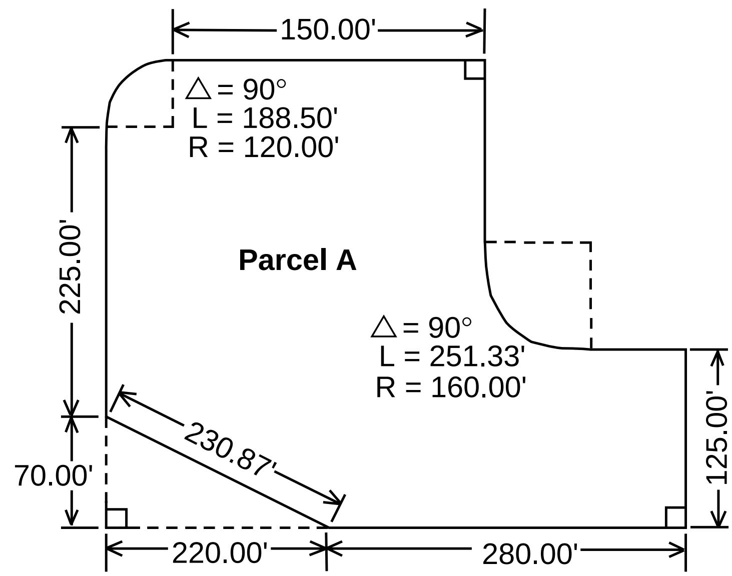


Figure 8-1. *Division into simple geometric shapes.*

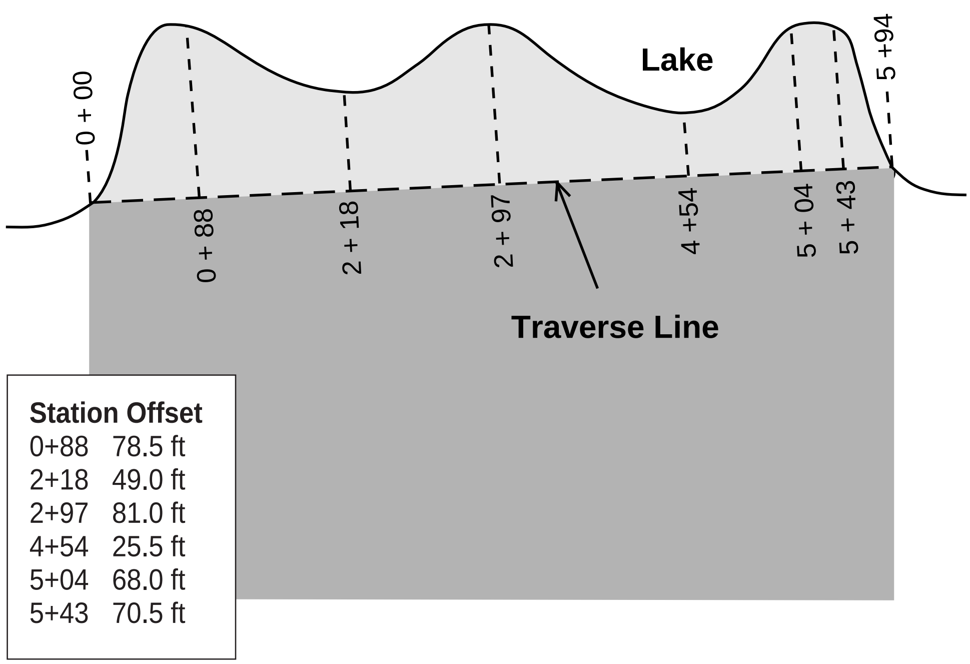
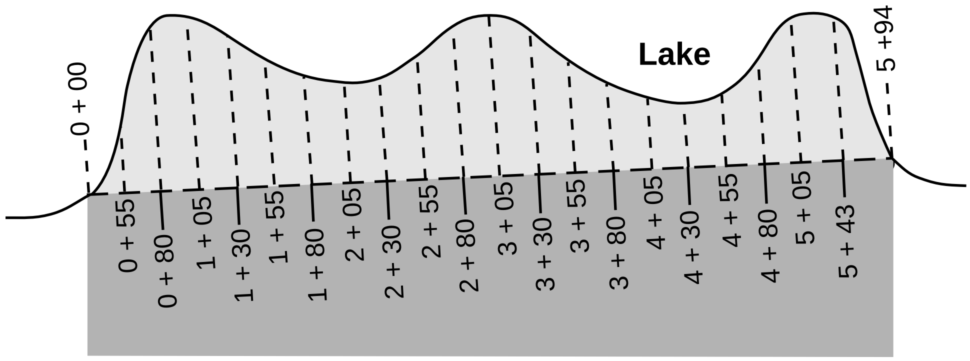


Figure 8-2. *Offsets to traverse line at irregular intervals.*



Station	Offset	Station	Offset
0+55	52.5 ft	3+05	79.5 ft
0+80	88.0 ft	3+30	69.0 ft
1+05	87.0 ft	3+55	58.5 ft
1+30	78.5 ft	3+80	50.0 ft
1+55	69.5 ft	4+05	40.5 ft
1+80	59.0 ft	4+30	29.5 ft
2+05	50.5 ft	4+55	27.0 ft
2+30	52.5 ft	4+80	46.0 ft
2+55	65.0 ft	5+05	68.0 ft
2+80	78.0 ft	5+43	71.0 ft

Figure 8-3. *Offsets to traverse line at regular intervals.*

- Trapezoidal rule

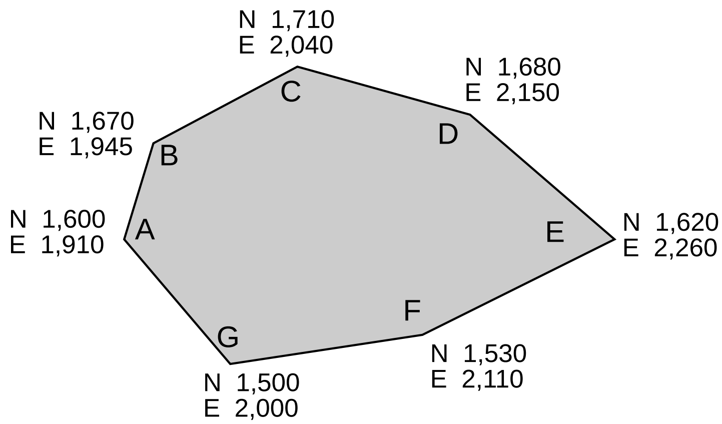
  $$A = S\left(\frac{O_1}{2} + O_2 + O_3 + \cdots + \frac{O_n}{2}\right)$$

- Simpson's 1/3 rule

  $$A = \frac{S}{3}\left[O_1 + O_n + 4\left(\sum O_{even}\right) + 2\left(\sum O_{odd}\right)\right]$$

Areas of Polygons Formed by Traverses

Determine the area of Parcel A, B, C, D, E, F, G.



- Area of a rectangle using maximum and minimum values of northings and eastings.
- Area by coordinates

  $A = \frac{1}{2} [N_1 (E_n - E_2) + N_2 (E_1 - E_3) + \cdots + N_n (E_{n-1} - E_1)]$

- Area by DMD

  $\frac{1}{2} A = \sum \text{DMD x LAT for each line}$

Area equals the absolute value of $\frac{1}{2}$ A

Rule:

1. Start DMD calculations from the most westerly point in the traverse.
2. The DMD of the first course is equal to the departure of the first course.
3. The DMD of any course is equal to the DMD of the preceding course, plus the departure of the preceding course, plus the departure of the course itself. Alegbraic signs must be considered.
4. The DMD of the last course is equal to the departure of the last course with the opposite sign.

Volumes

- Parrallelopipeds
- Prisms
- Volume by average end area

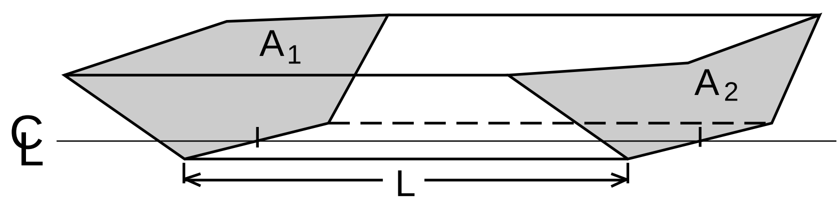
Volume in cu yd

$$2V = \frac{L\,(A_1 + A_2)}{27}$$

Where:

L = Distance between end areas, A_1 and A_2, in ft

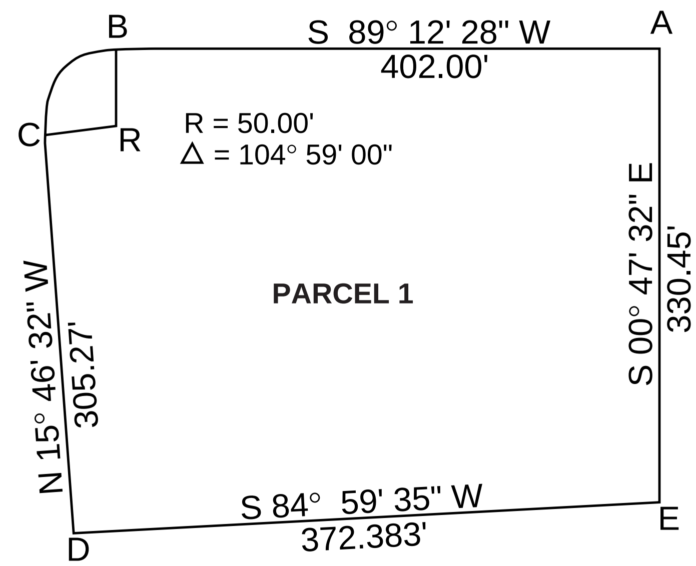
A_1 A_2 = end areas



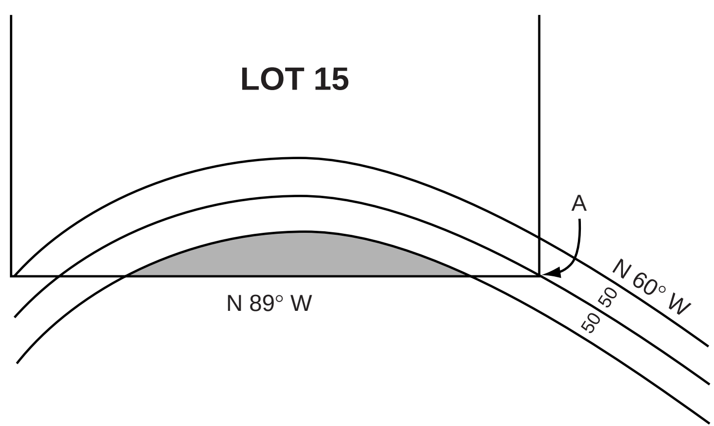
Sample Test Questions

1. The area of a parcel shown on a map is 6800 sq ft. The map has a note stating "the bearings distances and coordinates are based on California Coordinate System, 1927, Zone 2." The combination factor is O.999567. What is the actual ground area?
2. The corners of a parcel of land have the following coordinate values: (1) N 6472.88, E 7939.12; (2) N 6538.55, E 7802.41; (3) N 8447.65, E 8117.64 (4) N 8457.11, E 9190.96; (5) N 7128.07, E 9201.77; (6) N 7116.88, E 7933.88. What is the area, in acres, of the parcel?
3. The area on a map with a scale of 1:12000 as measured with a planimeter is 2.54 sq in. What is the area in acres?

4. Calculate the area of Parcel 1.



5. A highway, R/W 100 ft wide, is to be acquired from the owner of Lot 15 as shown on the sketch. Point "A" is the centerline B. C. of a curve with a radius of 1000 ft. Point "A" is also the SE corner of Lot 15. What is the area of that portion of Lot 15 south of the R/W?



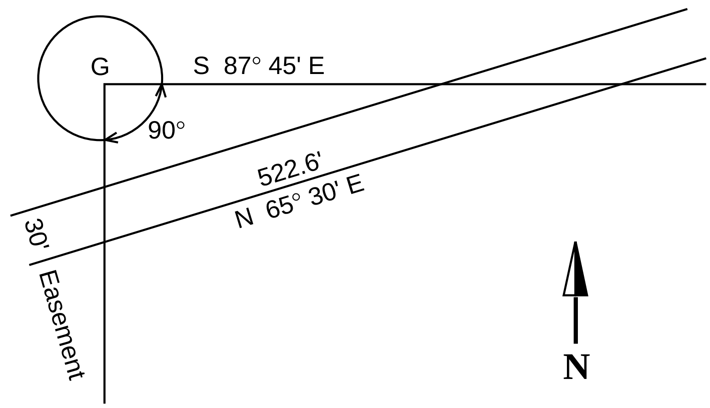
6. Problem C-4, 1983 LS

   A utility company proposes to acquire an easement across the land of a local property owner, as indicated in the plat shown below. The property owner has agreed to accept $7500 per acre in severance fees for the area that lies northwest of the northwest line of the easement. The settlement for the area within the easement is a separate consideration and it may be neglected.

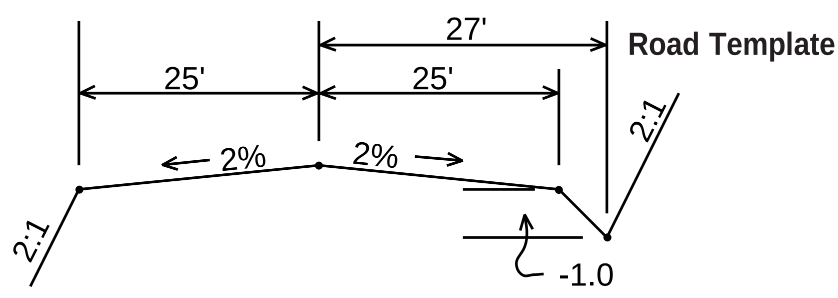
   Required:

   A. What is the area for which compensation to the landowner is appropriate?

   B. What fee is ultimately to be paid to the landowner?



7. Use the following road template and cross section notes to calculate the earth work quantity for the roadway between station 35 + 50 and 36 + 00. Centerline grade at 35 + 50 is 887.0 and at 36 + 00 is 888.5.



X-Section Notes

$\frac{93.0}{50}$	$\frac{91.6}{25}$	$\frac{890.4}{36+00}$	$\frac{90.0}{15}$	$\frac{89.0}{50}$
$\frac{92.3}{50}$	$\frac{90.0}{20}$	$\frac{889.5}{35+50}$	$\frac{90.0}{20}$	$\frac{88.0}{50}$

Answer Key

1. Divide the grid area by the combination factor squared.
   6806 sq ft

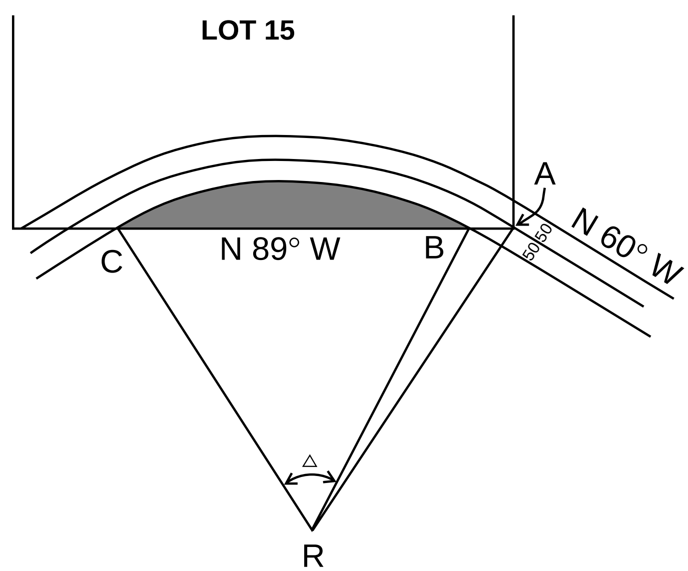
2. Use the formula for computing area by coordinates.
   37.513 acres

3. 1 in = 1000 ft scale, therefore, each sq in of the map is 1000 x 1000 ft or 1,000,000 sq ft per sq in.

   $\frac{1{,}000{,}000 \text{ x } 2.54}{43{,}560 \text{ ft}^2}$ = 58.3 acres

4. Calculate the area by either coordinate or DMD method for polygon ABRCDE and add the sector RBC.
   3.310 acres

5. Angle BAR = Difference in bearings between the south line of Lot 15 and the radial to A = 61°.



5. (Cont.)

By law of sines angle ABR = 112° 58' 42"

Angle ARB = 180 - (112° 58' 42" + 61°)

= 5° 21' 36"

Bearing of RB = Bearing RA - Angle ARB

= N24° 38' 24"E

Angle RBC = Difference in bearings between south line of Lot 15 and RB.

= 66° 21' 36"

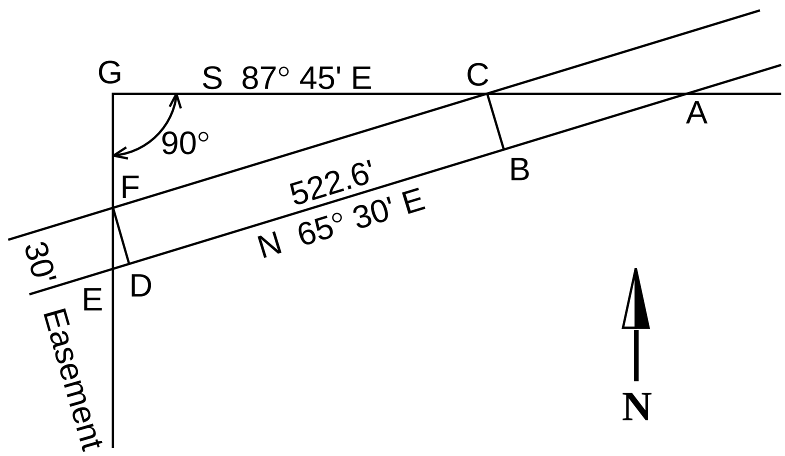
= 180° - 2 (66° 21' 36")

= 47° 16' 48"

Area of segment = $\frac{\pi r^2 \Delta}{360} - \frac{r^2 \sin^2 \Delta}{2}$

= .938 acres

6. Solve right triangles ABC and DEF for AB and DE



$$CF = 522.6 - AB - DE$$
$$= 522.6 - 15.12 - 59.52$$
$$= 447.96 \text{ ft}$$

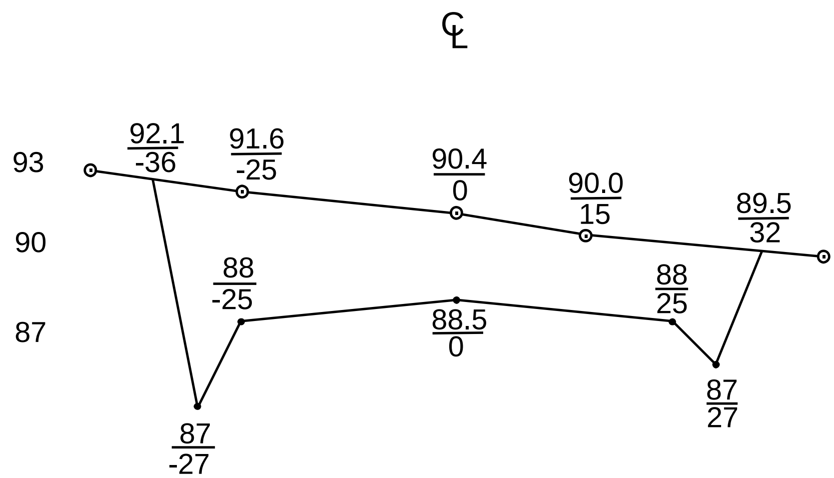
A. Solve right triangle CFG for base FG and height GC

$$\text{Area} = \frac{1}{2}\left(\frac{FG \times GC}{43560}\right)$$
$$= \frac{1}{2}\left(\frac{201.63 \times 400.02}{43560}\right)$$
$$= .926 \text{ acres}$$

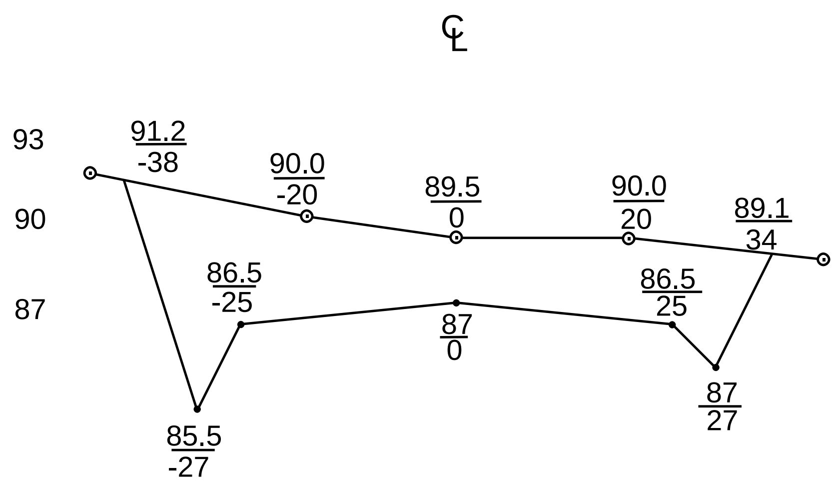
B. Payment = .926 x $7500

= $6943.37

7. Volume to be calculated by average end area.



36 + 00



35 + 50

7. (Cont.)

First, place the road template at the correct elevation on each cross section. Generate the intersection of the 2:1 side slopes with the ground line.

The two end areas can be calculated using the coordinate method for determining the area of a polygon. Coordinate pairs are composed of elevation (N) and distance left (-) or right (+) of the centerline of the template (E).

For the end area at station 35 + 50 (A_1)

$$
\begin{aligned}
87\,(25 - (25)) &= 4350.0 \\
86.5\,(0 - (-27)) &= 2335.5 \\
85.5\,(-25 - (-38)) &= 1111.5 \\
91.2\,(-27 - (-20)) &= -638.4 \\
90\,(-38 - 0) &= -3420.0 \\
89.5\,(-20 - 20) &= -3580.0 \\
90\,(0 - 34) &= -3060.0 \\
89.1\,(20 - 27) &= -623.0 \\
85.5\,(34 - 25) &= 783.0 \\
86.5\,(27 - 0) &= 2335.5 \\
\hline
\Sigma &= -405.90 \\
\text{Area} &= \frac{405.90}{2} \\
&= 203 \text{ sq ft}
\end{aligned}
$$

End area at 36 + 00 (A_2) by coordinate method = 154 sq ft

$$
\begin{aligned}
2 \text{ volume in cu yd} &= \frac{L\,(A_1 + A_2)}{27} \\
&= \frac{50\,(203 + 154)}{27} \\
2V &= 661 \\
V &= 330 \text{ cu yd cut}
\end{aligned}
$$

References

________________, *California Department of Transportation Survey Manual,* Appendix.

Davis, Raymond E., Francis S. Foote, et. al., *Surveying Theory and Practice,* Sixth Edition, McGraw Hill, New York, 1981.

Moffitt, Francis H. and Harry Bouchard, *Surveying,* Eighth Ed., International Textbook Co., Scranton, PA, 1987.

UNIT 9

PHOTOGRAMMETRY

Richard Burns, PLS
Caltrans Geometronics

Introduction

The *Manual of Photogrammetry* defines photogrammetry as, “the art, science, and technology of obtaining reliable information about physical objects and the environment through processes of recording, measuring, and interpreting photographic images and patterns of electromagnetic radiant energy and other phenomena.”

Land surveyors are primarily concerned with the photogrammetric applications involving aerial photography used for topographic mapping, including the ground location of physical objects, lines and points, and the determination of earthwork quantities.

The Land Surveyors Act includes within the definition of surveying practice the determination of the “configuration or contour of the earth’s surface or the position of fixed objects thereon” by photogrammetric methods. Land surveyors must be able to use the basic principles of photogrammetry necessary for flight and ground control planning to ensure that photogrammetric projects will meet National Map Accuracy Standards or other project-specific standards.

Performance Expected on the Exams

Change scales given as a ratio to scales given in ft per in. Explain the difference between large- and small-scale mapping.

Calculate the flying height necessary for a given photo scale.

Explain the concept of C-factor.

State National Map Accuracy Standards.

Given required mapping scale, contour interval, and project limits, determine the most efficient flight plan and ground control net for a photogrammetric project.

Key Terms

Focal length
Fiducial marks
Negative format
Photo scale
Map scale
Contour interval
Large scale
Small scale
Ratio scale
Flight height
C-factor
Terrain height
Datum height
Base sheets
National Map Accuracy Standards
Neat model
Overlap
Sidelap
Ground control
Premark
Flight line
Crab
Stereovision
Block
Halation
Spot elevations
Plannimetrics
Areotriangulation
Photo identification

Video Presentation Outline

Photography

- Cameras
- Focal length

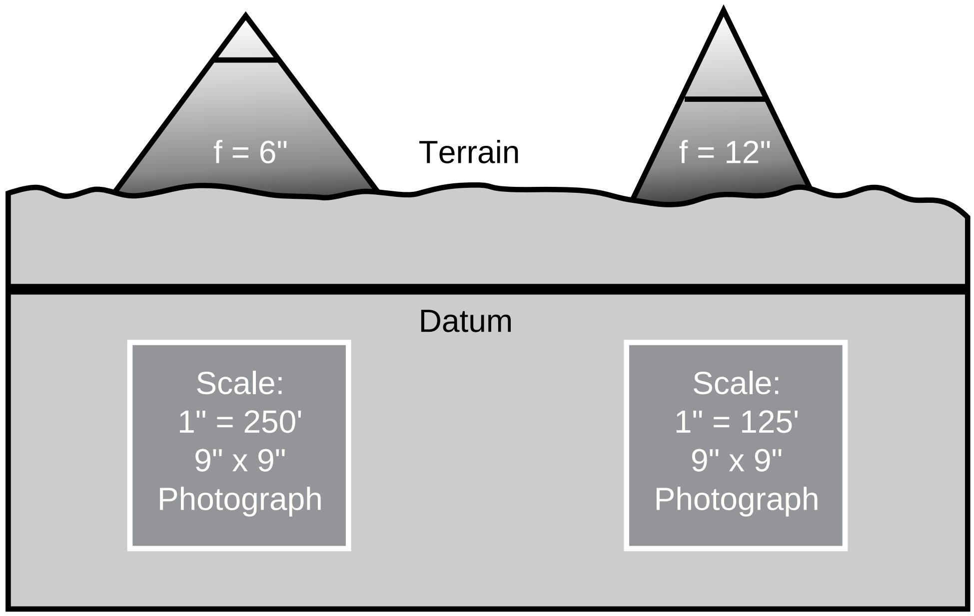


Figure 9-1. *Relation of focal length to area coverage.*

- Aerial photographs

Scale

- Expression of scale

  1 in = 250 ft or 1:3000
- Large scale/small scale

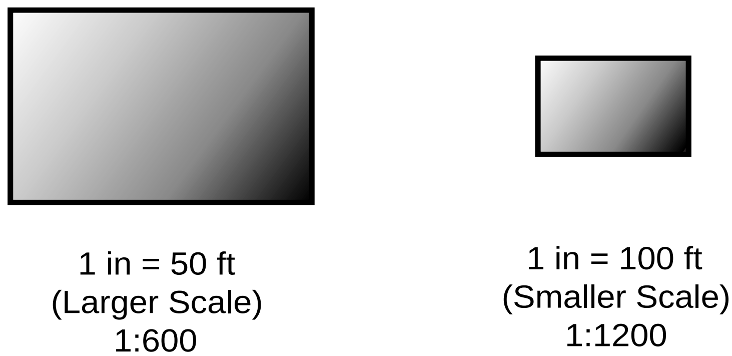


Figure 9-2. *Large scale/small scale.*

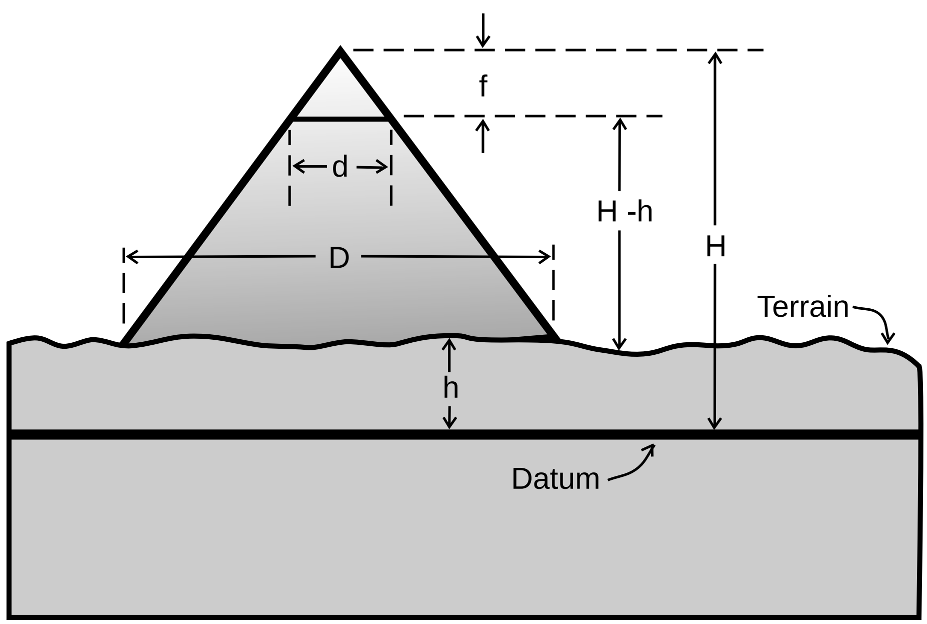


Figure 9-3. *Determining photo scale.*

- Basic formulas

$$S = \frac{f}{H-h}$$

Where:

S = scale given as in/ft or as a ratio

f = focal length of camera as in or as ft if scale is given as ratio

H = flying height above datum as ft

h = average terrain elevation as ft

$$S = \frac{d}{D}$$

Where:

S = scale given as in/ft

d = distance on photograph as in

D = corresponding distance on ground as ft

For ft per in $S = \frac{f}{H-h} = \frac{d}{D} = \frac{1}{x}$

Where:

x = ft/in on photograph

- C-factor

$$\text{C-factor} = \frac{H-h}{C.I.}$$

Where:

H = flying height above datum

h = average terrain elevation

C.I. = contour interval

- Flying tolerances

Models

- Overlap

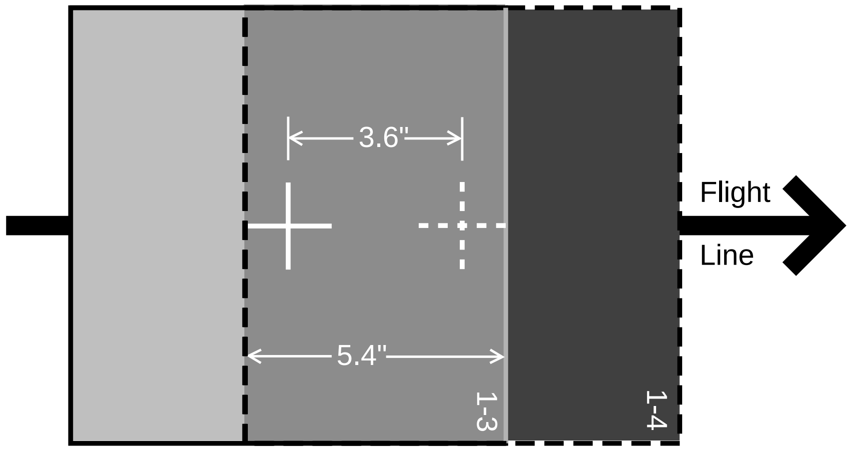


Figure 9-4. *Overlap of two successive photos.*

- Sidelap

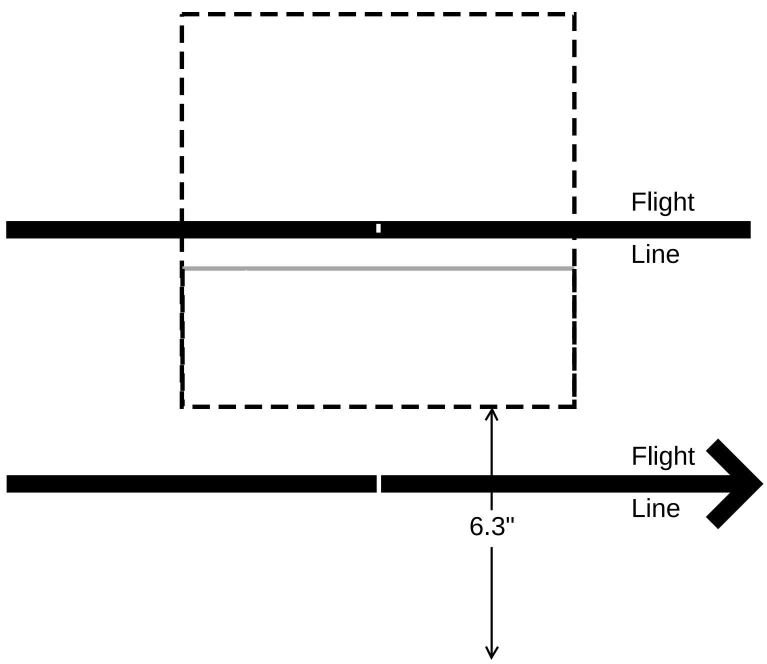


Figure 9-5. *Sidelap of two flight lines.*

- Neat model

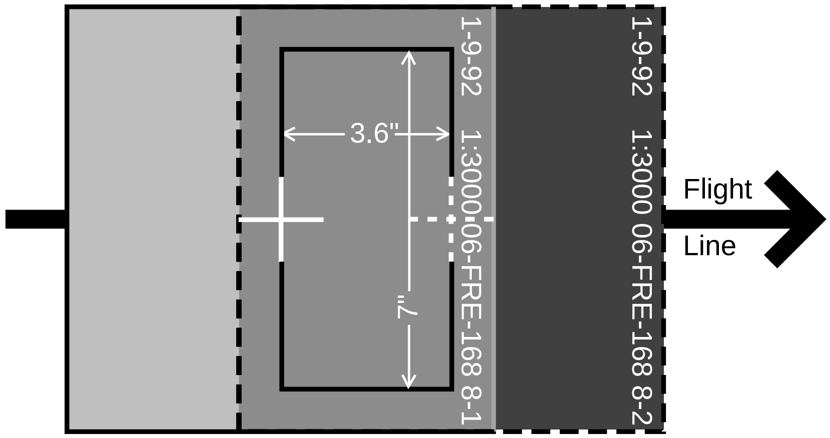


Figure 9-6. *Relationship of neat model to successive photos.*

- Flight line and models

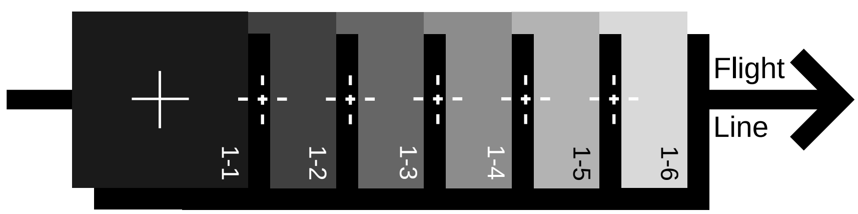


Figure 9-7. *Flight line and neat models.*

- Blocks

Photo Control

- Neat model (fully controlled)

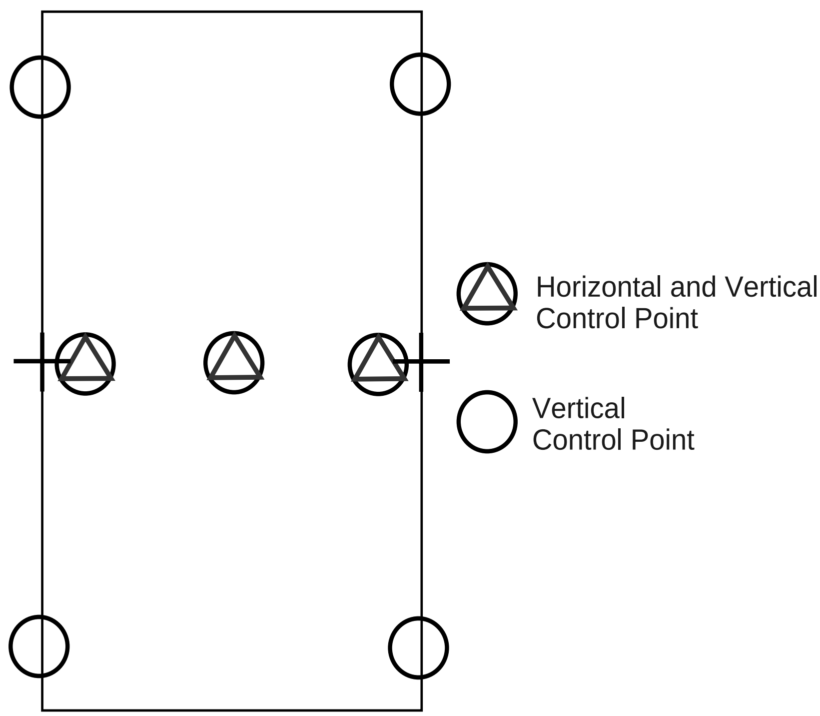


Figure 9-8. *Control for neat model.*

- Aerotriangulation

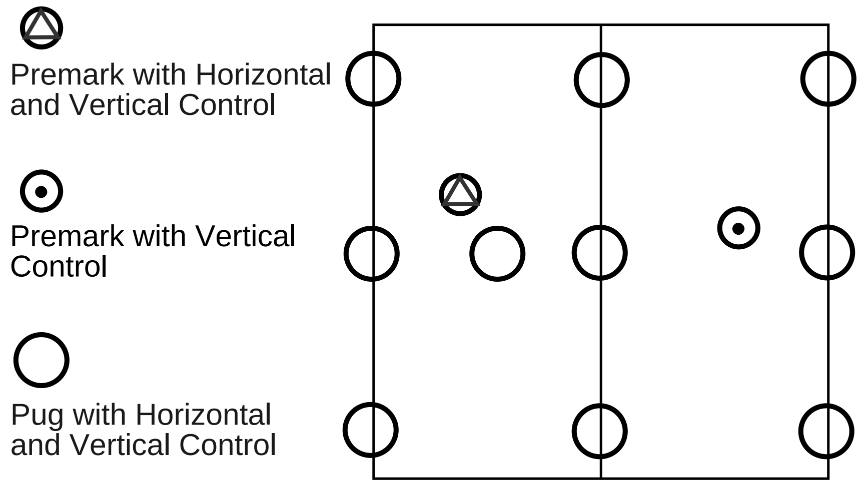


Figure 9-9. *Neat models with control.*

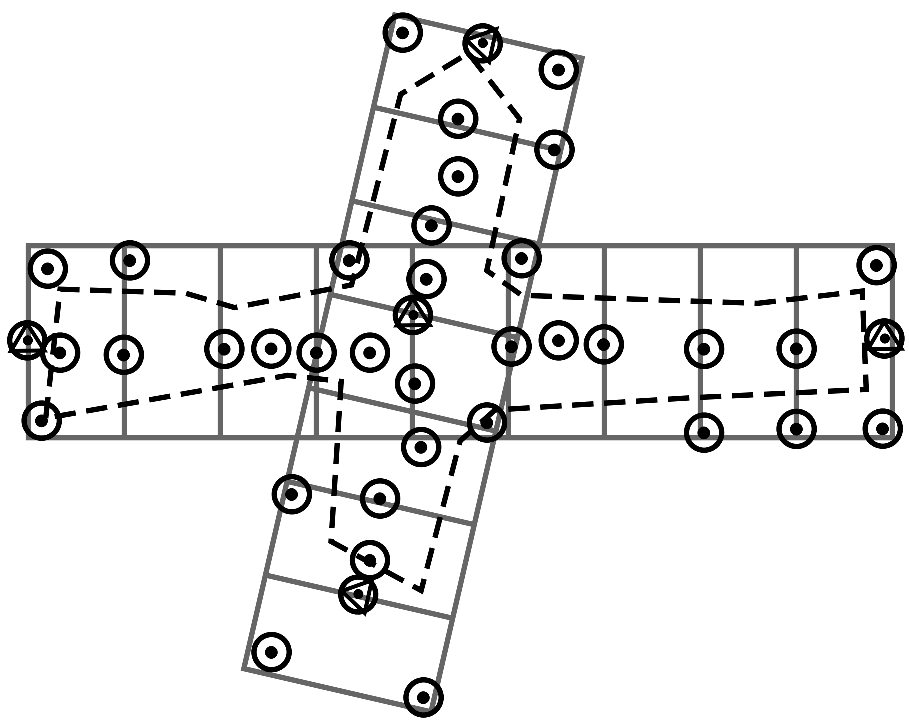
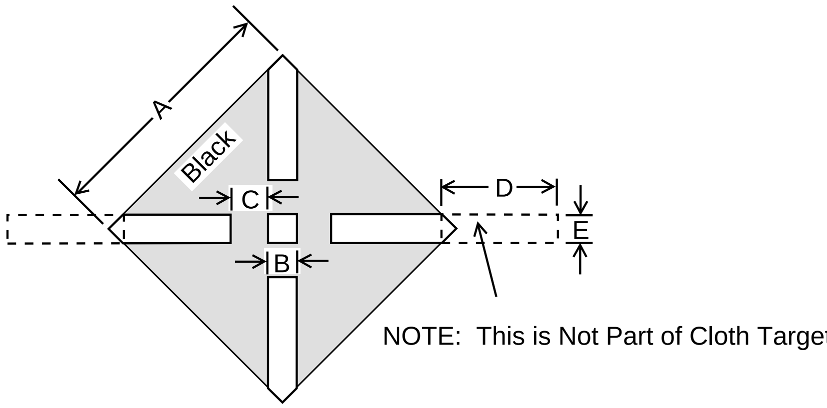


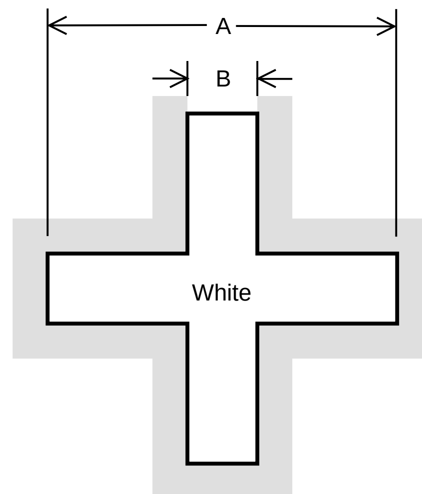
Figure 9-10. *Two flight lines with control.*

- Premarks

Quick Reference Table for Planning of Photogrammetric Mapping

Plotting Scale/C.I.	1 in = 50 ft / 2 ft	1 in = 100 ft / 5 ft	1 in = 200 ft / 10 ft	1 in = 400 ft / 20 ft	*Comments*
Photo Scale	1 in = 250 ft	1 in = 500 ft	1 in = 1000 ft	1 in = 2000 ft	Nominal. Will vary with relief.
Flying Height	1500 ft	3000 ft	6000 ft	12000 ft	*Above average ground elevation.*
Max. Range of Relief/Flight Line	300 ft	700 ft	1400 ft	2800 ft	*For mapping area – exceptions can be made if more relief outside mapping area but within photocoverage, special instructions to photographer necessary.*
Nominal Mapping Width/Flight Line	1400 ft	3000 ft	6000 ft	12000 ft	*Nominal - may be reduced 15% - 20% with maximum relief range. May be extended after aerial photography is received.*
Length of One Model	900 ft	1800 ft	3600 ft	7200 ft	*Nominal with 60% forelap, will vary with relief.*
No. Models/Mile	6	3	1.5	.75	*Approximate, for estimating purposes.*
Horizontal Control Premark Size	cloth or painted replica	cloth or painted replica with extensions	cross premark 12 ft x 1 ft	cross premark 24 ft x 2 ft	
Horizontal Control Premark Interval	see control diagram	see control diagram	planned for specific projects	planned for specific projects	*For aerotriangulation.*
Vertical Control Premark Interval	see control diagram	see control diagram	planned for specific projects	planned for specific projects	*For aerotriangulation.*
Supplemental Horizontal Control Accuracy	3rd order	3rd order	3rd order	3rd order	
Project Horizontal Control Accuracy	2nd order	2nd order	2nd order	2nd order	
Supplemental Vertical Control Accuracy	0.1 ft	0.2 ft	0.5 ft	1.0 ft	
Project Vertical Control Accuracy	2nd order	2nd order	3rd order	3rd order	





	Cloth Premark or Painted Replica		Cross	Premark
Photo Scale	1 in = 250 ft	1 in = 500 ft	1 in = 1000 ft	1 in = 2000 ft
A	45 in	45 in	12 ft	24 ft
B	4 in	4 in	1 ft	2 ft
C	7 in	7 in		
D	Not Required	24 in		
E	Not Required	6 in		

Figure 9-11. *Sizing premarks.*

National Map Accuracy Standards*

- "Photogrammetry for Highways Committee, 1968"

  Contours: 90% of elevations determined from solid line contours must be within 1/2 contour interval (C.I.) and all within 1 C.I.

  Spot elevations: 90% of spot elevations must be within 1/4 C.I. and all within 1/2 C.I.

  Planimetric Features: 90% of well defined features must be within 1/40 in and all within 1/20 in.

- "Office of Management and Budget"

  Horizontal accuracy: Maps with publication scales larger than 1:20,000 can have no more than 10% of well defined points with an error of more that 1/30 in as measured on the publication scale; for maps with a publication scale of 1:20,000 or smaller 1/50 in.

  Vertical accuracy: No more than 10% of elevations tested can be in error by more that 1/2 C.I.

Mapping

- Instruments
- Base sheets

Steps to Completion of a Photogrammetric Project

1. Determination of mapping limits
2. Select scale and contour interval
3. Plan flight
4. Plan the ground control
5. Place premarks
6. Survey premarks
7. Aerial photography
8. Survey calculations
9. Check aerial photography
10. Aerotriangulation

***Manual of Photogrammetry,* 4th Ed., American Society of Photogrammetry, 1980, page 372.

11. Base sheets
12. Map compilation
13. Map checking
14. Map corrections
15. Map acceptance

Example Problem

Problem A-3 1991 LS

Your client owns Sections 9 and 16, and the westerly 4000 ft of Sections 10 and 15, T4S, R23W, S.B.M. You have been asked to provide horizontal and vertical control for the topographic mapping that is to be used for planning purposes. Vertical photography, taken with a 6-in focal length camera on a 9-in x 9-in focal plane, is to be used. Analytical bridging is not to be considered.

The following factors control the project. **Make no assumptions.**

1. A 5-ft contour interval is required.
2. Model size is 3.6 x 7.0 in for a single flight line and 3.6 x 6.3 in for two or more adjacent flight lines.
3. The C-factor to be used for this project is 1,800 ft.
4. The map is to be compiled at a 5 to 1 ratio.
5. The average terrain elevation is 2,500 ft above sea level.
6. The minimum target size to be used for premarking the ground is not to be less than 0.001 x 0.01 in at the photo scale.
7. Per a recent Record of Survey, each section has been found to be standard dimensions.

Required:

1. Based on the above specifications, determine the following. Show all work.
   A. The minimum number of flight lines required.
   B. The required flying height above sea level.
   C. The minimum number of models required.
   D. The minimum number of photographs required.

E. The minimum number of horizontal and vertical control stations required to provide for adequate checks.

F. The negative scale.

G. The nominal map scale.

H. The minimum length and width of the target placed on the ground as a premark.

2. Give the accuracy requirements for each of the following based on requirements of the National Map Accuracy Standards:

A. Contours

B. Spot elevations

C. Planimetric features

Solution:

1. A. Minimum number of flight lines required = 1

Overall length = 10,560 ft

Overall width = 9,280 in

Model length = 5,400 ft

Model width = 10,500 ft

$$\frac{9{,}280 \text{ ft}}{10{,}500 \text{ ft}} = 0.88 = 1$$

B. Flying height above sea level = 11,500 ft

C-factor x C.I. + elevation

1800 ft x 5 ft + 2,500 ft = 11,500 ft

C. Minimum number of models required = 2

Terrain width ÷ model length = 10,560 ft ÷ 5,400 - 1.96 = 2

D. Minimum number of photographs required = 3

Photos required = models + flight lines

2 models + 1 flight line = 3

E. Minimum number of horizontal control stations required = 4

Minimum number of vertical control stations required = 6

Control for scale/model = 2 + 1 for check

Control for elev./model = 3 + 1 for check

F. Negative scale = 1,500 ft

C-factor x C.I. ÷ 6

(1800 ft x 5 ft) ÷ 6 = 1500 ft or 1:18,000

G. Nominal map scale = 1 in = 300 ft

Mapping scale = (Photo scale) (Ratio)

Photo scale = flying/focal length

Photo scale = (9,000/6) (1/5) = 300

H. Minimum length (180 in is acceptable) = 15.0 ft = 1500 x 0.01 ft

Minimum width (18 in is acceptable) = 1.5 ft = 1500 x 0.001 ft

2. A.

Accuracy	=	90%
Value	=	±1/2 C.I.
	or	
Accuracy	=	not more than 10% exceed
Value	=	±2.50 ft

B.

Accuracy	=	90%
Value	=	±1/4 C.I.
	or	
Accuracy	=	not more than 10% exceed
Value	=	±1.25 ft

C.

Accuracy	=	90% of features
Value	=	±1/30 @ map scale
	or	
Accuracy	=	not more than 10% exceed
Value	=	±10 ft

Sample Test Questions

The following questions are from Problem A-1 1988 LS

You are asked by your client to prepare a topographic map for his proposed project. He tells you that he will use this map to design a subdivision (including grading plans and street plans). He also states that he wants this map at a scale of 1 in = 40 ft with a 1-ft C.I. and spot elevations to supplement contours when the contour lines are more than 2 in apart.

Project site is 217 acres more or less, being a parcel 2700 ft north-south by 3500 ft east-west. Terrain: moderate relief.

The photo control for this mapping project will be established by field surveys.

Camera to be used for obtaining the mapping photography is equipped with a 6-in focal length lens and a 9-in x 9-in format (negative size).

The map accuracy must comply with the U.S. Map Accuracy Standards.

Required:

Circle the correct answer.

1. U.S. Map Accuracy Standards are as follows:
   - A. 90% of the contours shall be ±1/4 C.I.
   - B. 90% of the spot elevations should be within 1/2 C.I.
   - C. 80% of the spot elevations should be within 1/4 C.I.
   - D. 90% of the spot elevations should be within 1/4 C.I.
2. The C-factor for a mapping system is determined by:
   - A. Camera, aircraft, photo lab equipment, and stereo plotting equipment.
   - B. Ground control, flying height, camera, and photo lab equipment.
   - C. Flying height, weather, relief displacement, and camera.
   - D. Stereo plotter operator, camera, ground control, and airplane altitude.

3. It was determined that a mapping system with a C-factor of 1500 will be used in the above project. This system is considered:

   A. A first order system
   B. A second order system
   C. A third order system
   D. A fourth order system

4. What is the altitude from which the photography must be obtained?

   A. 4200 ft ASL (above sea level)
   B. 3000 ft AGL (above ground level)
   C. 1500 ft AGL
   D. 1500 ft ASL

5. The scale of the photography is:

   A. 1 in = 200 ft
   B. 1:3000
   C. 1 in = 100 ft
   D. 1:1500

6. The enlargement ratio from photography to final map is:

   A. 10
   B. 40
   C. 7.50
   D. 6.25

Assume for the following questions that the flight path is east-west.

7. How many photographs will be required to obtain complete stereoscopic coverage of the area?

   A. 10
   B. 8
   C. 11
   D. 7

8. How many models will be required to map the parcel?

   A. 10
   B. 8
   C. 11
   D. 7

9. How many control points will be required to fully control the mapping photography?

   A. 10
   B. 4
   C. 15
   D. 18

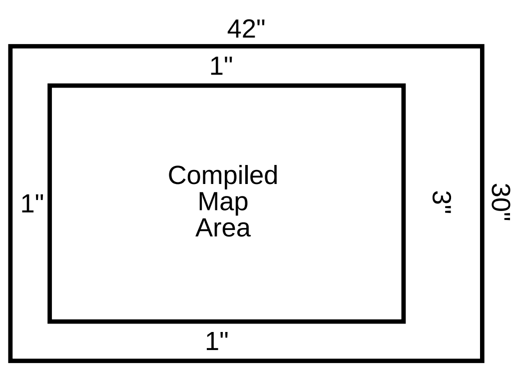
10. How many flight lines will be required?

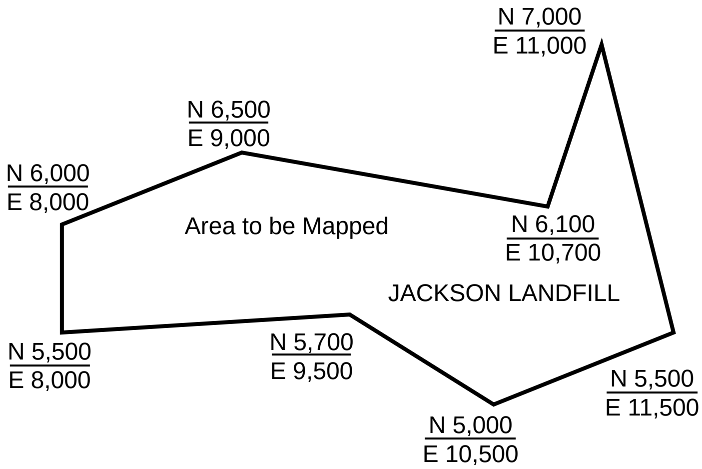
    A. 1
    B. 2
    C. 3
    D. 4

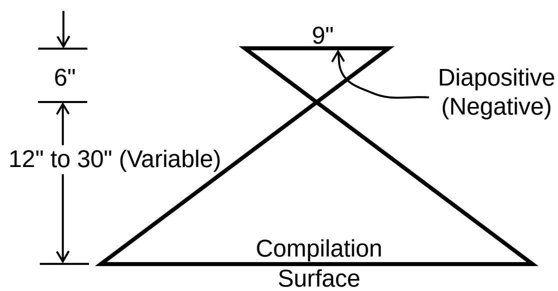
The following questions are from Problem B-5 1989 LS

Your client has requested that you provide a topographic map of the Jackson landfill by photogrammetric methods. In so doing, you are required to use the following criteria and equipment.

- The map must fit on a single mylar sheet with borders as specified in the diagram below.
- The common engineering map scale that allows the entire project to be compiled at the maximum size that will fit on the specified single sheet.
- The camera focal length is 6 in; the film format is 9 x 9 in.
- The plotter has 9-in x 9-in diapositive plate carriers and a C-factor of 2000, as shown in the diagram below.
- A forward photo overlap of 60% and a sidelap of 30% or an accepted common practice are required.
- The terrain varies from 1500 ft to 2100 ft above sea level.
- The C.I. is 1 ft.







Determine the following:

11. Usable map sheet dimensions.

12. East-west, north-south limits (length and width) of area to be mapped.

13. Maximum flying height above average terrain.

14. Flying height above sea level.

15. Photo scale.

16. Compilation scale that will fit on one map sheet (see diagram) and be drawn in one of the following common engineering scales (10, 20, 30, 40, 50, 60, 100).

17. Definition of the “neat model.”

18. Dimensions of the “neat model.”

19. Number of models required to map the given area

The following questions are from Problem A-3, 1992, LS

The following are intended to measure your knowledge of basic photogrammetry. Keep answers short, to the point, and in the correct sequence.

Required:

20. What is the scale of vertical photography if taken with a 6-in focal-length camera at an altitude of 4,500 ft above mean terrain?

21. What is the absolute **minimum** number of exposures required for a single flight line?

22. What is the minimum number of horizontal stations required to control a single model? Explain.

23. What is the **minimum** number of vertical stations required to control a single model? Explain.

24. Define a "neat model."

25. The mean elevation of the terrain being mapped is 1,600 ft above sea level (ASL). Vertical photography is taken with a 6-in focal-length camera at an ASL elevation of 4,000 ft. What is the nominal map scale utilizing an 8 to 1 plotting ratio?

Answer Key

Problem A-1, 1988 LS

1. D
2. A is the best answer. Aircraft is a questionable term for a list that should include: camera, film, photo lab, plotting equipment, and operator.
3. A
4. C
5. B
6. D
7. A
8. B

9 D

10. B

Problem B-5, 1989, LS

11. Available dimension is the sheet size less the border dimensions. Therefore:

North-south = 30 in - (2 x 1 in) = 28 in

East-west = 42 in - (1 in + 3 in) = 38 in

12. Area to be mapped is the east-west and north-south limits.

North-south = 7000 - 5000 = 2000

East-west = 11500 - 8000 = 3500

13. Flying height above average terrain must be maximun height for maximum coverage that will not exceed the C-factor of the plotter.

Maximum height = C-factor x contour interval

= 2000 ft x 1

= 2000 ft

14. Flying height above sea level is the flying height plus the elevation of average terrain above sea level.

$$\text{Avg. terrain elevation} = \frac{\text{highest elevation} + \text{lowest elevation}}{2}$$

$$= \frac{2100 \text{ ft} + 1500 \text{ ft}}{2}$$

= 1800 ft

Flying height above sea level = Flying height + elevation of average terrain

= 2000 ft + 1800 ft

= 3800 ft above sea level

15. Photo scale for maximum coverage

$$\text{Photo scale} = \frac{\text{camera focal length}}{\text{flying height}}$$

$$= \frac{6 \text{ in}}{2000 \text{ ft}} = \frac{0.5 \text{ ft}}{2000 \text{ ft}}$$

= 1:4000 or 1 in = 333.3 ft

16. Final mapping scale may vary depending on the projection distance set off in the plotter. The variation is related to photo scale divided by projection ratio; for plotters capable of ratios between 2 and 5.

$$\text{Mapping scale} = \frac{333}{2} - 166 \text{ ft to } \frac{333}{5} = 67 \text{ ft}$$

The most appropriate listed common scale is required; therefore, final mapping scale is 1 in = 100 ft.

17. A "neat model" is the maximum compiling limits of a single stereo model.

18. Dimensions of the neat model are based on the forward and sidelaps relative to the flight line. (Note: Either A or B are acceptable solutions.)

A. Common text reference

Forward = (1 - overlap) x photo scale x photo width

= (1 - .60) x 9 in x 333.3 ft = 1199 ft

Sidelap = (1 - .3) x 9 in x 333.3 ft = 2089 ft

Neat model = 1199 ft x 2089 ft 6.3 in x 3.6 in

B. Common practice

Forward = (1 - overlap) x photo scale x photo width

= (1 - .60) 9 in x 333.3 ft = 1199 ft

Sidelap = (1 - .222) 9 in x 333.3 ft = 2333 ft

Neat model = 1199 ft x 2333 ft, 7 in x 3.6 ft

19. The required number of models is based on the neat model dimensions related to the dimension of the area to be mapped. If the flight line runs east-west, then:

Width	Length	Width is adequate for a single strip
Neat model 8a or 8b	1199 ft	Models required = $\frac{3500}{1199}$ = 2.9 or 3 models
Area to map 2000 ft	3500 ft	

Problem A-3, 1992 LS

20.

$$S = \frac{f}{H\text{-}h}$$

$$1 \text{ in} = 750 \text{ ft or } \frac{1}{9{,}000}$$

$$\frac{4{,}500 \text{ ft}}{6 \text{ in}} = \frac{750 \text{ ft}}{1 \text{ in}}, \quad 1 \text{ in} = 750 \text{ ft}, \quad \frac{6 \text{ in}}{4{,}500 \text{ ft}} = \frac{1}{9{,}000}$$

21. The total number of models plus one.
    or
    A minimum of two exposures.

22. A. Two or three
    B. Two points are required to establish scale.
    or
    Two points are required to establish scale plus one additional point for check.

23. A. Three or four
    B. Any three points not in a straight line are required to establish a plane.
    or
    Any three points not in a straight line are required to establish a plane plus one additional point for check.

24. Examples of acceptable answers are:
    - The stereoscopic area between adjacent principal points and extending out sideways in both directions to the middle of the sidelap. (The mapping area of each stereopair.)
    - 3.6 x 7 in
    - 3.6 x 6.8 in

25.

$$4{,}000 - 1{,}600 = 2{,}400$$

$$\frac{2{,}400}{6} = 400$$

$$\frac{400}{8} = 50$$

Answer: 1 in = 50 ft or 1:600

References

________, *Manual of Photogrammetry*, Fourth Edition, American Society of Photogrammetry, Falls Church, VA, 1980.

Moffitt, Francis H., and Mikhail, *Photogrammetry,* Third Edition, International Textbook Co., Scranton, PA 1980.

Wolf, Paul R., *Elements of Photogrammetry*, McGraw Hill, New York, NY, 1983.

UNIT 10

THE GLOBAL POSITIONING SYSTEM

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Introduction

The Global Positioning System (GPS) has led to a revolution in the land surveying profession. The GPS technology is being employed in a variety of surveying applications while the technology is still evolving. At present there are only nationally accepted procedures/methodologies for performing static GPS surveys. Newer methodologies have been identified, e. g., pseudostatic and fast/rapid static, for which procedures are just beginning to be developed and universally accepted.

As the cost of hardware continues to decrease, surveyors will discover the benefits associated with employing the GPS technology in a variety of survey operations. Since the technology and its applications are still evolving, the land surveyor must keep abreast of these evolutionary changes.

Questions about GPS are just beginning to appear on Land Surveyor licensing examinations. This unit and the sources it cites are only a few of the available materials about GPS. Professional surveyors can expect more to become available. Methods being tested today may become the standards and specifications of tomorrow. These changes in methodology will probably be reflected on future Land Surveyor examinations.

Remember that the GPS technology is in its evolutionary phase. Surveyors must follow the changes in the use of the technology to determine how and if it can best be applied to their survey projects. GPS is not the solution to all survey problems, but can probably aid survey practices in proportion to the time put into understanding it.

Performance Expected on the Exams

Since this is a new technology there is a degree of uncertainty about the types of questions that might appear on the exams. However, knowledge of a variety of GPS issues such as those listed below should provide a sound basis for understanding the technology and for success with questions that might appear on exams in the future.

Interpret the Geometric Geodetic Accuracy Standards and Specifications for Using GPS Relative Positioning Techniques as published by the Federal Geodetic Control Subcommittee (FGCS).

Plan a small GPS project given certain criteria.

Explain the basic concepts of using a satellite-based system for land surveying and geodesy.

Name and explain the main GPS methodologies, including their uses and limitations.

Differentiate between GPS issues directed to surveying versus those directed to navigation.

Explain the overall issues which can affect the successful completion of a GPS project, including designing redundancy into the project and addition of control stations (both horizontal and vertical) to strengthen the project.

Key Terms

- FGCC standards and specifications
- Static surveying
- Kinematic surveying
- Pseudo-static surveying
- Fast or rapid static surveying
- Pseudorange measurements (primarily navigation)
- Carrier-phase measurements (primarily surveying)
- P-code measurements
- Epoch
- Point positioning
- Relative positioning
- Differential positioning (see below)
- Redundancy
- Dilution of precision (DOP, especially PDOP and GDOP)
- Loss of lock/cycle slip
- Multipath
- Antenna swap
- Selective availability (SA)
- Anti-spoofing (AS)
- Orthometric height (elevation)
- Geoid height
- Ellipsoid height

NOTE: Differential and relative positioning techniques were somewhat synonymous in earlier satellite tracking terminology. However, the term "differential" more appropriately describes the technique of determining the position of an unknown station by transmitting correction factors determined in real time at a known GPS station to the unknown station. These same correction factors are then applied to the unknown station.

Video Presentation Outline

History of Satellite Surveying

The 1960s: The Beginnings of Satellite Surveying Technologies

The 1970s: Doppler Technology

The 1980s: Evolution of Doppler to GPS

- Orbits
- Number of satellites
- Transmitting frequency
- Receiver design allowing simultaneous tracking
- Frequency standards

Technology Overview

- GPS receivers measure the time from signal generation at the satellite to signal arrival at the receiver.
- Point positioning involves station coordinate determination at a single station with a single receiver.
- Relative positioning involves GPS surveys with multiple receivers over known and unknown stations.

Positioning Methodologies

Accuracies – FGCC (now FGCS) Standards and Guidelines

Survey Categories	Order	Minimum Geometric Accuracy Standard (95 Percent Confidence Level)		
		Base Error	Line-length Dependent Error	
		e (cm)	p (ppm)	a (1:a)
Global-regional geodynamics; deformation measurements	AA	0.3	0.01	1:100,000,000
National Geodetic Reference System, "primary" networks; regional-local geodynamics; deformation measurements	A	0.5	0.1	1:10,000,000
National Geodetic Reference System, "secondary" networks; connections to the "primary" NGRS network; local geodynamics; deformation measurements; high-precision engineering surveys	B	0.8	1	1:1,000,000
National Geodetic Reference System (terrestrial based); dependent control surveys to meet mapping, land information, property, and engineering requirements	1	1.0	10	1:100,000
	2-I	2.0	20	1:50,000
	2-II	3.0	50	1:20,000
	3	5.0	100	1:10,000

Note: For ease of computation and understanding, it is assumed that the accuracy for each component of a vector base line measurement is equal to the linear accuracy standard for a single-dimensional measurement at the 95 percent confidence level. Thus, the linear one-standard deviation(s) is computed by:

$$s = \pm [\sqrt{e^2} + ((0.1d)(p))^2]/1.96.$$

Where **d** is the length of the base line in kilometers.

Table 10-1. *FGCC Accuracy Standards.*

General Survey Requirements:

- Elevation cutoff
- Tracking epoch
- Number of satellites
- Satellite configuration (dilution of precision)
- Site selection (elimination of multipath)

Static Positioning

Kinematic Positioning

Pseudo-kinematic/Pseudo-static

Project Planning

Vertical Control with GPS

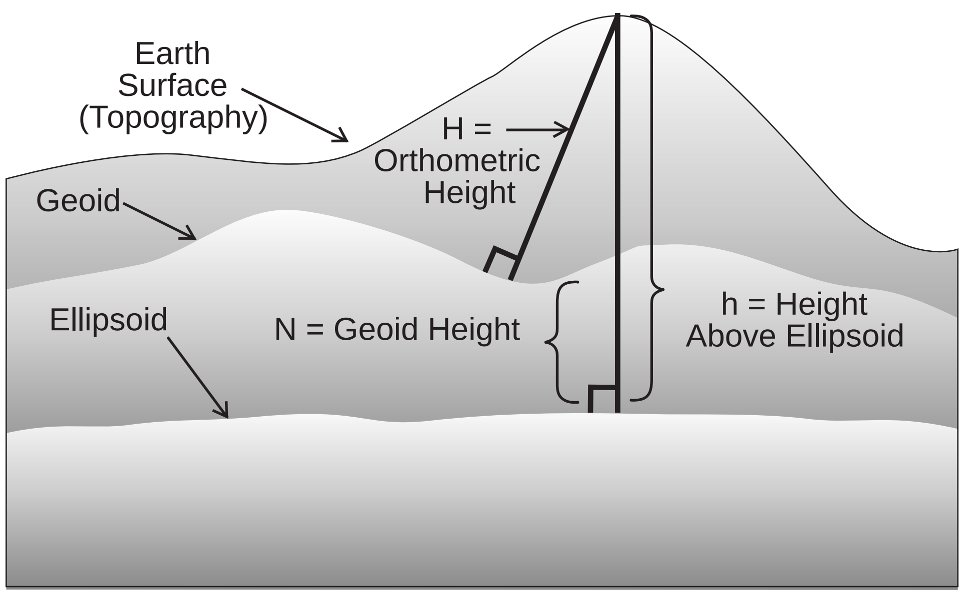


Figure 10-1. *Orthometric, ellipsoid, and geoid heights.*

GPS Provides Ellipsoid Heights

$$h = N + H$$

Where:

h = ellipsoid height
N = geoid height
H = orthometric height (elevation)

- Relative differences between GPS points can be determined with a high degree of precision.
- Absolute differences in elevation between GPS points can only be determined within 3 to 5 cm.

The California High Precision Geodetic Network (HPGN)

- The HPGN consists of about 250 stations observed to FGCS Order B accuracy (base error, 8mm + 1:1,000,000).
- Most stations are in the state highway corridors and readily accessible.
- The HPGN was constrained to about 20 California VLBI stations which are part of a worldwide network.
- The coordinates of these stations are referred to as NAD83 (1991.35).
- The pre-HPGN coordinates, NAD83 (1986), will be revised. This will probably happen in early 1993. The HPGN project helped to make the NGRS in California even more accurate.

The Latest Methodologies

- Rapid static
- GPS control of photogrammetry

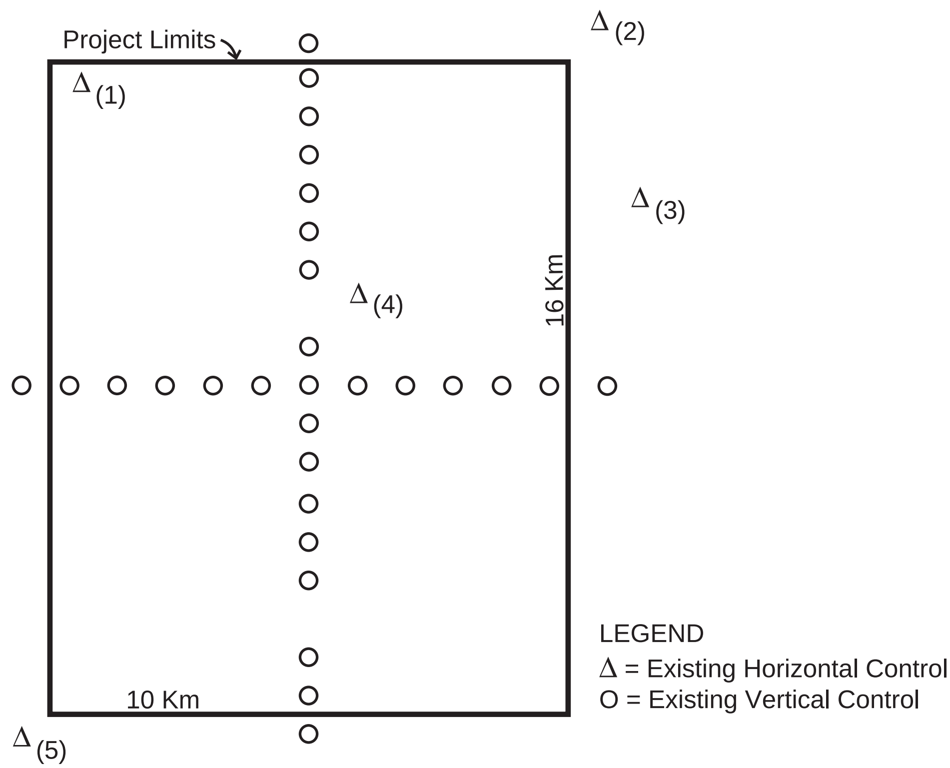
Steps to Completion of a GPS Project

1. Define the scope of the project.
2. Determine accuracy requirements, by FGCS specifications.
3. Determine project requirements, i.e., station spacing.
4. Design the project layout.
5. Research station information for existing horizontal and vertical control stations.

6. Determine suitability of existing control for GPS observations, and select those stations which are required to meet FGCS standards for the project.
7. Select sites for new project stations ensuring clear access to satellite signals and no multipath problems, set new survey monuments, and prepare station descriptions.
8. Determine number and type of receivers (single or dual frequency; P-code, etc) required to meet project specifications.
9. Plan observing schedules (accounting for satellite availability, station observation time) to ensure redundancy per FGCS specifications.
10. Conduct GPS observations, complete observation log, take station rubbing and verify final station description.
11. Download data from GPS receivers and make back-up copy.
12. Receive and process data at central processing location.
13. Review all data for completeness.
14. Perform minimal constrained adjustment.
15. Review results for problem vectors or outliers.
16. Reobserve problem vector lines, if necessary.
17. Perform constrained adjustment and review results.
18. Incorporate precise ephemeris data if appropriate.
19. Prepare final report with all pertinent sketches/maps, schedules, stations held fixed (including coordinates and elevations used), software packages used (TRIMNET, TRIMVEC, GEOLAB, GPPS, etc.), station description package, final adjustment report, and coordinates.

Sample Test Questions

1. A team of surveyors is required to design a GPS project for the city of Germantown. The project limits are about 10 km x 16 km (about 160 sq km). The project requires establishment of First Order control (FGCS) at 2 km spacing around the periphery and throughout the project limits. There is a line of levels (vertical control) running through the middle of the city in east-west and north-south directions. The vertical control bench marks are spaced about 1-1/2 km along the two lines. There are five First Order horizontal control stations in the vicinity of the city. The layout of the project limits is indicated below with existing vertical control (bench marks) and horizontal control indicated.



A. What minimum combination of horizontal control stations best fits the project requirements?

   1. 1, 2, and 4
   2. 1, 2, and 3
   3. 5, 4, and 2
   4. 1, 3, and 5

B. What is the best minimum combination of vertical control stations (bench marks), if any, required for the project?

   1. No bench marks are required as the client only requires First Order horizontal control.
   2. Four bench mark ties well dispersed throughout the project area.
   3. Four bench mark ties to any four bench marks.
   4. One bench mark tie in the center of the project.

C. What will be the approximate total number of stations (including network ties) in the project?

1. 44
2. 54
3. 61
4. 58

D. What is the error which can be expected between any two adjacent stations in the city network (at the two sigma or 95% confidence level)?

1. 1.0 cm
2. 2.2 cm
3. 3.0 cm
4. 5.0 cm

2. You are required to plan a GPS project which has a total of 40 stations. The required accuracy level for the project is First Order. The project will be accomplished using static relative positioning techniques.

   You have decided that the breakdown of the stations in the project are:

   new stations = 30
   existing horizontal control stations = 4
   existing vertical control stations = 6

   You have five suitable receivers available for the project.

   What is the minimum number of observing sessions required for the project? Show all work and appropriate FGCS specifications.

3. Are GPS heights a) orthometric heights or, b) ellipsoid heights?

4. What additional element is required to relate the orthometric height to the ellipsoid height?

5. A. How many non-trivial base lines are there in a single three station GPS session?

   B. How many trivial base lines are there in the same session?

6. What is the most likely way to avoid multipath problems?

7. Can the effects of refraction of the GPS signal through the ionosphere be removed by single frequency receivers?

8. Why is the Dilution of Precision factor important in GPS planning?

9. What is the minimum number of satellites required for most GPS applications?

10. What effect does selective availability have on the GPS signal?

Answer Key

1A. 4. According to the FGCS Standards and Specifications (pg. 15, Table 2), the location of network control should be not less than three "quadrants" relative to the center of the project. Either or both of the remaining two stations (stations 2 and 4) could be added to the network to increase the reliability and add redundancy to the network.

1B. 2. The dispersion of the bench marks throughout the project ensures a better determination of the overall accuracy of the project. According to the FGCS Standards and Specifications, bench mark ties are required for all surveys.

1C. 3. The project calls for stations around the periphery and throughout the city at 2 km spacing. Stations are at 2 km spacing at all grid intersections, therefore, 6 (at 0, 2, 4, 6, 8, and 10 km) x 9 (at 0, 2, 4, 6, 8, 10, 12, 14, and 16 km) = 54 stations at grid intersections. Thus: 54 new stations, plus 4 vertical control tie stations, plus 3 horizontal control tie stations equal a total of 61 stations.

1D. 2. The answer can be read directly from the graph on page 9 of the FGCS guidelines or by computation from the formula:

$$\sigma = \sqrt{e^2 + ((0.1)(p)(d))^2}$$

Where:

σ = maximum allowable error in cm at the 95% confidence level;
e = base error in cm;
d = distance in km between adjacent stations;
p = the minimum geometric relative position accuracy standard in parts per million at the 95% confidence level (see Table 1, page 6).

In this First Order project e = 1.0 cm, d = 2 km, and p = 10.

Substituting in the formula:

$$\sigma = \sqrt{1.0 + ((0.1)(10)(2))^2}$$
$$= 2.2 \text{ cm}$$

Thus, at the 95% confidence level, we can expect an error no greater than 2.2 cm for the stations spaced at 2 km and following First Order specifications.

2. The first step is to determine the total number of station occupations required. Table 4 on page 21 of the FGCS guidelines suggests the following:

   - 10% of all stations are occupied 3 times. Thus 4 stations (10% of 40 stations) must be occupied 3 times. This is a total of 12 occupations.

   - 100% of all vertical stations must be occupied twice. Thus 6 bench marks must be occupied twice. This is a total of 12 occupations.

   - 25% of all horizontal control stations must be occupied twice. Thus 1 horizontal station must be occupied twice. This is a total of 2 occupations.

   - 30% of all new stations must be occupied twice. Thus 9 new stations (30% of 30 new stations) must be occupied twice. This is a total of 18 occupations.

   - This leaves a total of 20 stations which are occupied once. This total is arrived at by subtracting 4 stations requiring 3 occupations, 1 horizontal station requiring 2 occupations, 6 bench marks requiring 2 occupations, and 9 new stations requiring 2 occupations, or, 40 - (4 + 1 + 6 + 9), or, 20.

     Thus, the total occupations are:

3 occupations:	12	
2 occupations:	12	(bench marks)
	2	(horizontal control)
	18	(new stations)
1 occupation:	20	
TOTAL OCCUPATIONS:	64	

   The total number of sessions is 13 (12.8) since there are 5 receivers and 64 occupations (64/5).

   REMINDER: It would be wise to consider occupying at least 1 and possibly 2 of the existing horizontal control points 3 times. 4 stations have to be occupied 3 times. Selecting 1 or 2 existing stations adds redundancy and helps strengthen the project reliability without increasing the total number of sessions.)

3. GPS heights are ellipsoid heights. GPS heights are measured to a mathematical surface, the ellipsoid, the surface of which acts as the zero reference for GPS heights.

4. The geoid (or geoidal) height is the element which relates the ellipsoid height to the orthometric height, or elevation. The relationship of the ellipsoid height to the orthometric height is given by the formula:

   $$N = h - H$$

   Where:

   N = geoid height
   h = ellipsoid height
   H = orthometric height

5. A. There are 2 non-trivial base lines.

   B. There is 1 trivial base line.

6. Multipath, the reception of a GPS signal via two or more paths, can best be eliminated by selecting sites which have no obstructions above 15 degrees from the horizon and which have no nearby (say ±50 feet) smaller obstructions such as road signs.

7. No. The removal of the ionospheric effects can be accomplished by dual frequency receivers. Single frequency receivers can be used on projects with short base lines since the effects of the ionosphere can be considered the same over small areas.

8. The Dilution of Precision (DOP) factor, a mathematical computation, provides information about how well the desired accuracy in a GPS project can be achieved. It is a measure of the geometry of the satellite constellation. The more favorable the satellite distribution in the sky, the lower will be the DOP factor. In general, surveys should not be performed with a DOP, especially PDOP or GDOP, above about 6.0 (the lower the value the better the projected accuracy).

9. Static, pseudo-static, and fast or rapid static require a minimum of 4 satellites, although 5 satellites are recommended for the latter 2 methods. Kinematic surveys generally require 5 satellites.

10. Selective Availability, implemented by the Department of Defense which maintains the GPS constellation, has the effect of reducing the accuracy

one can obtain in a GPS position. Selective Availability, or SA, falsifies the time of signals being broadcast by the GPS satellites. SA usually only affects real time GPS computations. Most GPS surveys are computed after the field surveys are completed, and the effects of SA can be eliminated.

NOTE: The references cited in the reference section provide other and more in-depth coverage of these issues.

References

__________, *Geometric Geodetic Accuracy Standards and Specifications for using GSP Relative Positioning Techniques:* Federal Geodetic Control Committee, Version 5.0: May 11, 1988, reprinted with corrections: August 1, 1989.

__________, *Geodesy for the Layman:* NOAA, 1983.

The above publications are available from:

National Geodetic Information Center
N/CG174, Rockwall Bldg., Room 24
National Geodetic Survey, NOAA
Rockville, MD 20852

__________, *GPS: A Guide for the Next Utility:* Trimble Navigation Ltd., 1989.

__________, *GPS Field Surveyors Guide: A Field Guidebook for Static Surveying:* Trimble Navigation Ltd., 1991.

__________, *GPS Field Surveyors Guide: A Field Guidebook to Dynamic Surveying:* Trimble Navigation Ltd., 1992.

Leick, Alfred, *GPS Satellite Surveying:* John Wiley & Sons, 1990.

Wells, David, editor, *Guide to GPS Positioning*, Canadian GPS Associates, 1986.

NOTE: These two books provide significantly more technical information about the GPS technology.

UNIT 11

THE CALIFORNIA COORDINATE SYSTEM

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Introduction

A 1986 change to the Public Resources Code of the State of California (§§ 8801 et seq.) created the California Coordinate System of 1983 (CCS83). The implementation is to be phased in by January 1, 1995, when state plane work must refer to it instead of the previous California Coordinate System of 1927 (CCS27). The law defined the zones for CCS27 and CCS83. It specified certain constants for each zone and defined the U.S. Survey ft.

Surveyors who are preparing for the LS examination should be aware that the new coordinate system will be phased in through the beginning of 1995. However, nothing prevents the State Board of Registration from asking questions about the older system on future examinations. Examinees need to prepare for both. While this presentation concentrates on the California Coordinate System of 1983, it is broadly applicable to the System of 1927. Calculations for the older system are not as sophisticated as now required. Therefore, surveyors who understand the System of 1983 will find it much easier to master the older system.

The North American Datum of 1927 (NAD27), developed by the U.S. Coast and Geodetic Survey, was the basis for CCS27. It used Clarke's spheroid of 1866 as its spheroid of reference. The North American Datum of 1983 (NAD83), developed by the National Geodetic Survey (NGS), underlies CCS83. NGS adjusted NAD83 during the period 1975-1986 and based it on a new ellipsoid, the Geodetic Reference System of 1980 (GRS80). To date, NGS has implemented NAD83 as a metric system.

Because the reference spheroids differ, the NAD27 latitude and longitude of a station differ from the NAD83 latitude and longitude for the same station. In fact, the differences between the two systems vary inconsistently making only approximate transforms possible. Rigorous computation of coordinates is only possible by returning to the original field observations, readjusting them and then computing positions from them.

Six Lambert conformal zones, NGS zones 0401 through 0406, comprise CCS83. Each zone is based upon a secant cone whose axis is coincident with the GRS80 axis of rotation. The secant cone intersects the surface of the ellipsoid at two standard parallels. The specification of a central meridian fixes the cone relative to the ellipsoid.

As enacted, units can be either m or U.S. Survey ft. One U.S. Survey ft equals exactly 1200/3937 m. To convert U.S. Survey ft to m, multiply by 12/39.37; to convert m to U.S. Survey ft, multiply by 39.37/12. The NGS has chosen to publish all CCS83 station data in m. State plane coordinates expressed in m or ft can be converted to the other units simply by multiplying the northing and easting by the appropriate conversion factor.

The NGS recommends using polynomial coefficients to simplify conversions between NAD83 geodetic and plane coordinates. They also allow accurate calculation of grid scale factor. Polynomial coefficients are easy to use. Therefore, they are appropriate for both manual and programmed applications. Through their use, projection tables and interpolation become unnecessary. The polynomial coefficients are used in algebraic equations that enable the use of handheld calculators. They can produce millimeter accuracy using calculators capable of carrying 10 significant digits.

The NGS developed the coefficients by polynomial curve fitting. I have translated the NGS metric coefficients into U.S. Survey ft coefficients by using the m-ft factor raised to the appropriate power. The U.S. Survey ft coefficients were not independently determined by curve fitting directly.

The California Land Surveyor's Association (CLSA) has published a book of projection tables as Special Publication No. 55/88. It is similar in form to the tables the USCGS published for NAD27. Written by Ira H. Alexander and Robert J. Alexander, it is a tool for the surveyor who has worked with CCS27 and who wishes to do work in CCS83 with little modification. Calculations for state plane coordinate systems fall into five broad classes:

1. Transformations of coordinates between ellipsoid and grid.
2. Manipulations of observations. The adjustments applied to lengths and azimuths in converting between ground, ellipsoid and grid.
3. Transformations from zone to zone.
4. Coordinate geometry with adjusted observations on the grid.
5. Calculations directly on the ellipsoid.

This presentation deals with the first four types of calculations.

Performance Expected on the Exams

Convert geodetic coordinates to plane coordinates (CCS 83).

Convert plane coordinates (CCS 83) to geodetic coordinates.

Calculate the convergence angle of a station.

Convert geodetic (astronomic azimuth) to grid azimuth (CCS 83).

Calculate scale factors.

Calculate combined scale factor and apply it to field measurements.

Key Terms

Convergence

Geodetic azimuth

Grid azimuth

NAD 27

NAD 83

U.S. Survey ft

Polynomial coefficients

Second term correction

Geodetic height

Geoid separation

Geodetic distance

Scale factor

Grid distance

Combined factor

Video Presentation Outline

The State Plane Coordinate System

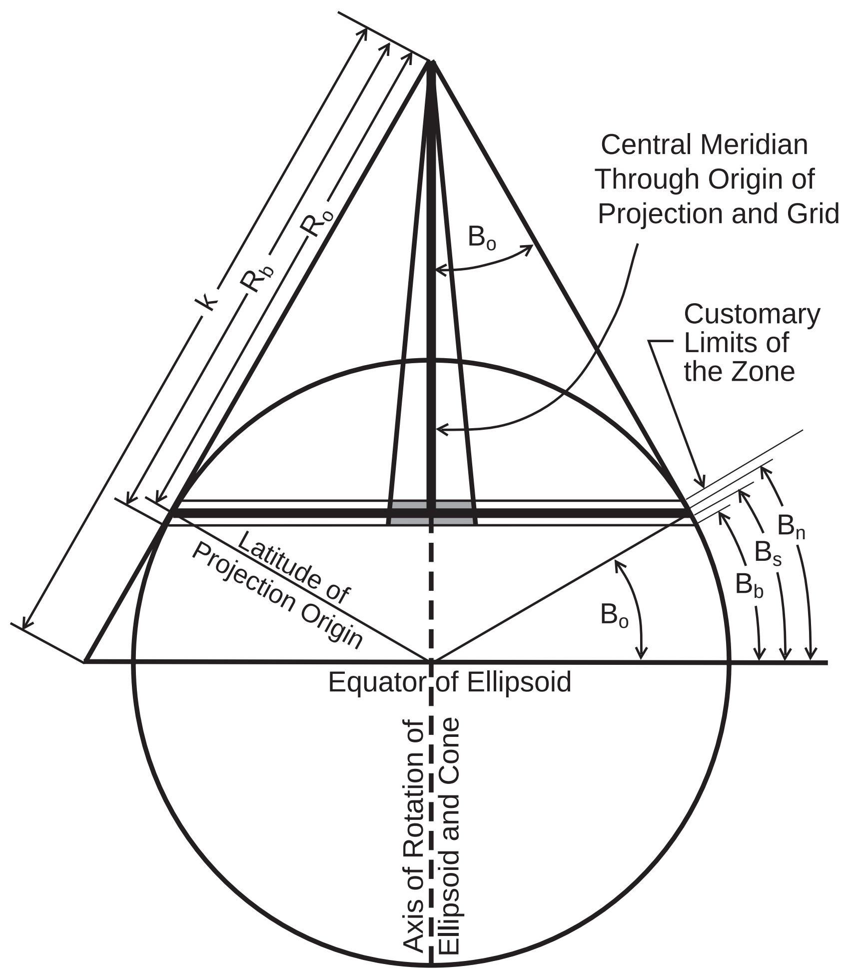


Figure 11-1. *Ellipsoid and secant cone.*

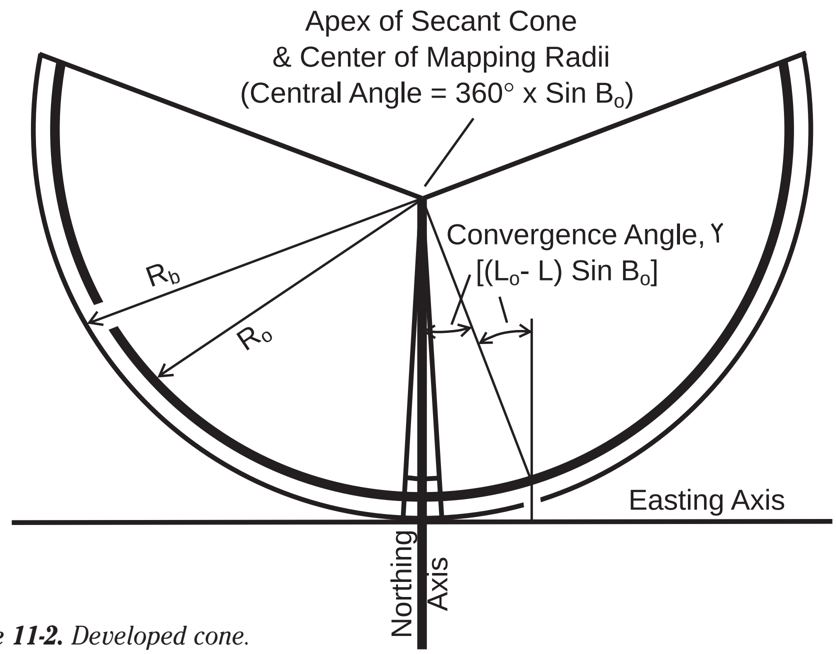


Figure 11-2. *Developed cone.*

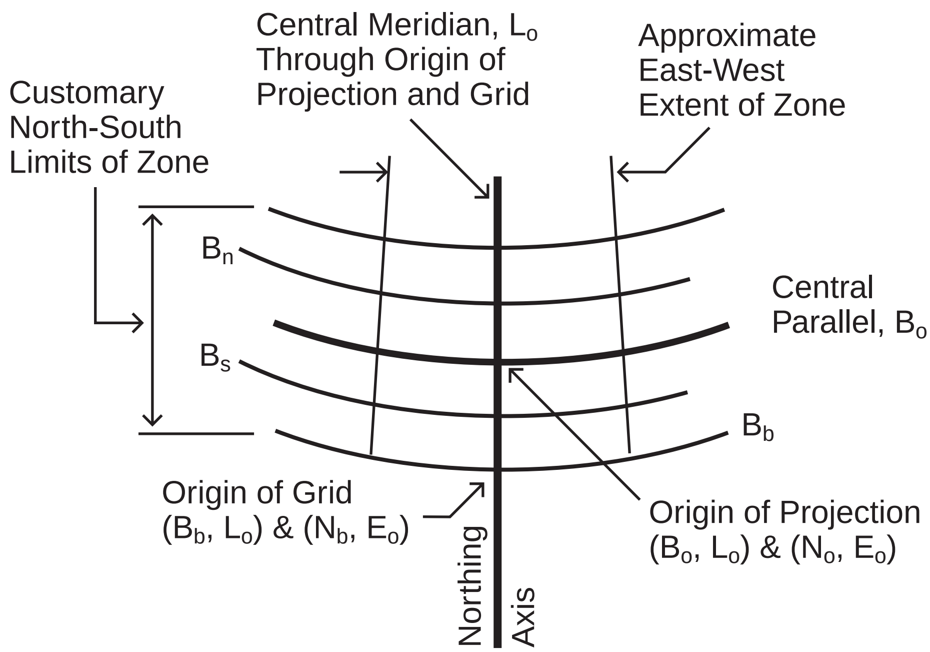


Figure 11-3. *Developed cone detail.*

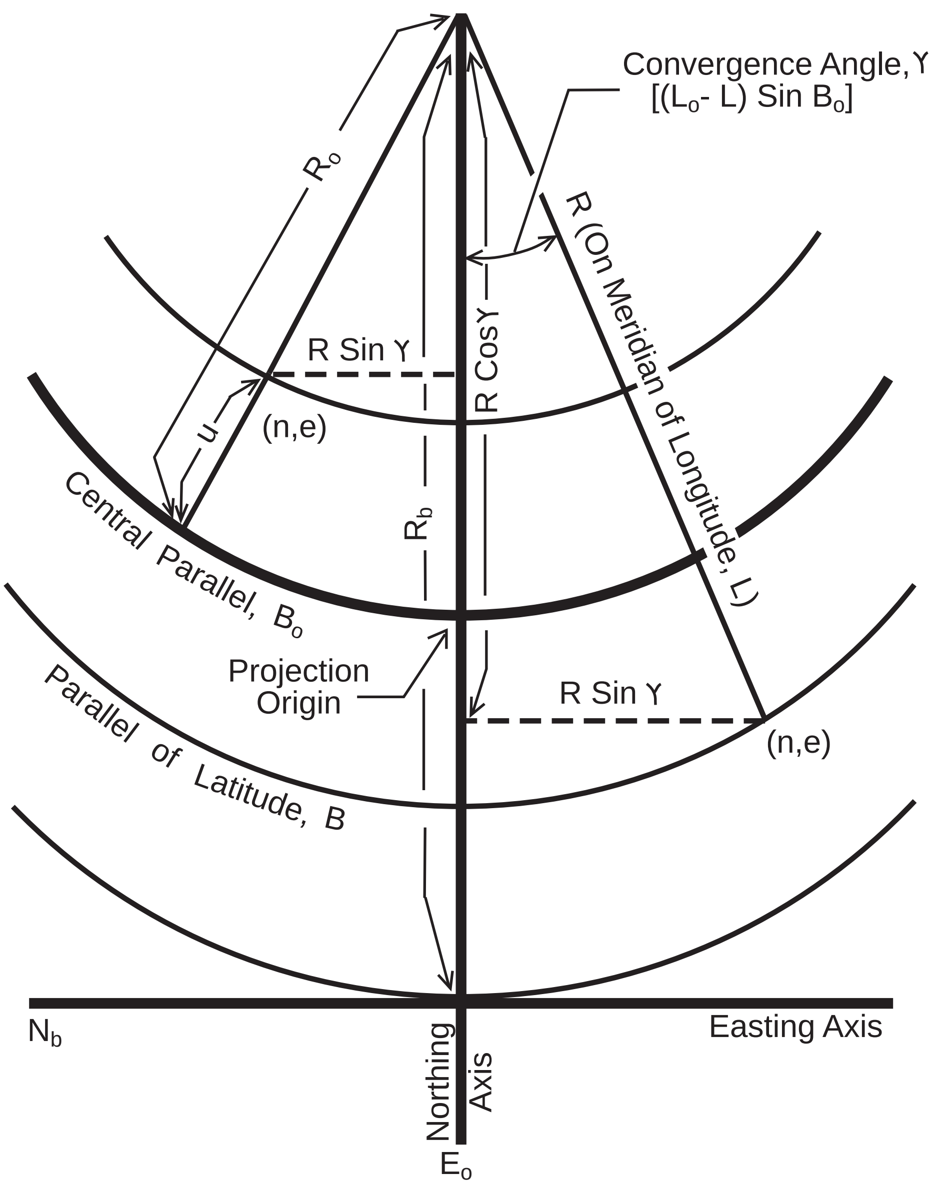


Figure 11-4. *Conversion triangle.*

Example Problems

Direct or Forward Computation: Conversion From Geodetic Latitude and Longitude to Plane Coordinates

NOTE: When working on calculators having only ten significant digits, such as a HP41, it is necessary when adding and subtracting latitudes or longitudes to truncate tens and hundreds of degrees (i.e., truncate 33.040048312° to 3.040048312°).

Direct or forward mapping equations are used to compute state plane coordinates from geodetic coordinates. L polynomial coefficients are used for direct computation. The L coefficients are used to convert the length of the meridian arc between B and B_o to the length "u" which is R_o - R. This permits the calculation of the mapping radius and subsequently the northing and easting.

Conversion of geodetic coordinates, that is latitude B, (ø), and longitude L, (λ), to plane coordinates (n, e) proceeds as follows:

Determine Radial Difference, u, if Projection Tables Will Not be Used:

$\Delta B = B - B_o$ in decimal degrees *(see note above)*

$$u = L_1\Delta B + L_2\Delta B^2 + L_3\Delta B^3 + L_4\Delta B^4$$

or for hand calculation

$$u = \Delta B[L_1 + \Delta B(L_2 + \Delta B(L_3 + L_4\Delta B))]$$

Where:

B = north latitude of station, also noted as ø
B_o = latitude of the projection origin, central parallel, a tabled constant
u = radial distance from station to the central parallel, R_o - R
L_1, L_2, L_3, L_4 = polynomial coefficients for direct computation, tabled with the zone constants

Determine Mapping Radius, R:

The mapping radius may be determined in two ways:

1. By the formula:

$$R = R_o - u$$

Where:

R = mapping radius of station
R_o = mapping radius of the projection origin
u = radial distance from station to the central parallel, R_o - R

2. If projection tables are provided, the mapping radius also may be interpolated by entering the tables with the argument of latitude against mapping radius.

Determine Plane Convergence, γ:

$$\gamma = (L_o - L) \sin B_o$$ *(see note, page 7)*

Where:

γ = convergence angle. *(Carry all significant digits for this calculation.)*
L = west longitude of station, also noted as λ
L_o = longitude of central meridian, longitude of projection and gridorigin, a tabled constant
$\sin B_o$ = a tabled constant, sine of the latitude of the projection origin

Determine Northing and Easting:

For polynomial solutions:

$$n = N_o + u + [(R \sin\gamma) \quad \tan \frac{\gamma}{2}$$

For polynomial solution or projection tables:

$$n = R_b + N_b - R \cos\gamma$$

$$e = E_o + R \sin\gamma$$

Where:

γ = convergence angle
e = easting of station
E_o = easting of projection and grid origin, 6561666.667 ft or 2000000.0000 m in all zones
n = northing of station
N_o = northing of the projection origin, a tabled constant
R = mapping radius of station
u = radial distance from station to the central parallel, $R_o - R$
R_b = mapping radius of the grid base, a tabled constant
N_b = northing of the grid base, 1,640,416.667 ft or 500000.0000 m in all zones

Inverse Computation: Conversion of Plane Coordinate Position to Geodetic Latitude and Longitude

Inverse mapping equations are used to compute geodetic coordinates from state plane coordinates. The geodetic coordinates, latitude and longitude, are on the ellipsoid of reference.

The G coefficients are used for inverse conversion. The G coefficients are used to convert the northing and easting of the station to the length "u" which is R_o - R. This permits the calculation of the length of the meridian arc between B and B_o. Adding that length to B_o, the latitude is obtained. Longitude is calculated with conventional formulas. Be careful to use the correct algebraic sign for each value in the formulas.

The applicable formulas to be solved in the order given are:

Determine Plane Convergence, γ:

NOTE: The value of the convergence angle must be carried to all available digits. Register or stack mathematics is recommended.

$$\gamma = \arctan \frac{e - E_o}{R_b - n + N_b}$$

Where:

- γ = convergence angle at the station, carry to all available digits
- e = easting of station
- E_o = easting of projection and grid origin, 6561666.667 ft or 2000000.0000 m for all zones
- R_b = mapping radius of the grid base, a tabled constant
- n = northing of station
- N_b = northing of the grid base, 1640416.667 ft or 500000.0000 m for all zones

Determine Longitude, L:

$$L = L_o - \frac{\gamma}{\sin B_o} \quad \textit{(see note above)}$$

Where:

- γ = convergence angle at the station
- sin B_o = a tabled constant, sine of the latitude of the projection origin
- L = west longitude of station, also noted as λ
- L_o = central meridian, longitude of projection and grid origin, a tabled constant

Determine Radial Difference, u:

$$u = n - N_o - [(e - E_o) \tan \frac{\gamma}{2}]$$

Where:

γ = convergence angle at the station
e = easting of station
E_o = easting of projection and grid origin, 6561666.667 ft or 2000000.0000 m for all zones
n = northing of station
N_o = northing of the projection origin
u = radial distance from station to the central parallel, $R_o - R$

Determine Latitude, B:

Where:
$B = B_o + G_1u + G_2u^2 + G_3u^3 + G_4u^4$ *(see note on page 7)*
$B = B_o + u [G_1 + u (G_2 + u (G_3 + G_4u))]$ for handheld calculation *(see note on page 7)*
B = north latitude of station, also noted as ϕ
B_o = latitude of the projection origin, central parallel
u = radial distance from station to the central parallel, $R_o - R$
G_1, G_2, G_3, G_4 = polynomial coefficients for inverse conversion, tabled with the zone constants

Only if Projection Tables are to be Used:

Determine mapping radius for interpolation arguing mapping radius against latitude by the following formula:

$$R = \sqrt{(e - E_o)^2 + (R_b - n + N_b)^2}$$

Where:
R = mapping radius of station
e = easting of station
E_o = easting of projection and grid origin, 6561666.667 ft or 2000000.0000 m for all zones
R_b = mapping radius of the grid base, a tabled constant
n = northing of station
N_b = northing of the grid base, 1640416.667 ft or 500000.0000 m in all zones

Determining Plane Convergence (Mapping) Angle and Geodetic Azimuth to Grid Azimuth

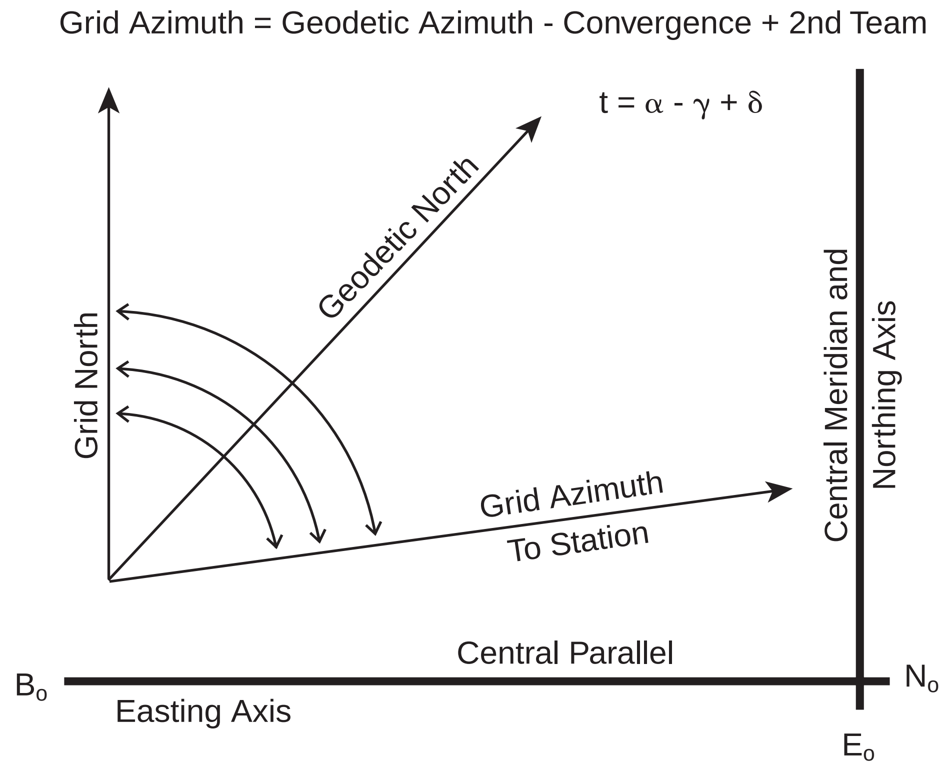


Figure 11-5. *Convergence (mapping angle).*

For CCS83 (NAD83) both geodetic and grid azimuths are reckoned from north. NGS surveys for NAD83 are reckoned from north rather than from south, which had been used for NAD27. Inverses between stations having state plane coordinates give grid azimuth and may be used directly for calculations on the grid. Plane convergence varies with longitude; therefore, the station for which convergence was determined must be specified.

Determine the Plane Convergence, γ:

The plane convergence may be determined in two ways:

1. Where the longitude is not known without calculation, the plane convergence may be calculated from the constants of the projection and the plane coordinates:

   $$\gamma = \arctan \frac{e - E_o}{R_{b-n} + N_b}$$

   Where:

   γ = plane convergence angle
   e = easting of station
   E_o = easting of projection origin, 6561666.667 ft or 2000000.0000 m for all zones
   R_b = mapping radius of the grid base, a tabled constant
   n = northing of station
   N_b = northing of the grid base, 1640416.667 ft or 500000.0000 m for all zones

2. The plane convergence also may be calculated using the difference in longitude between the central meridian and the point of question and a tabled constant. Be careful to use the correct algebraic signs.

   $$\gamma = \sin B_o (L_o - L)$$

   Where:

   γ = plane convergence
   $\sin B_o$ = sine of the latitude of the projection origin, which is also the ratio between γ and longitude in decimals of a degree, a tabled constant
   L_o = longitude of central meridian, a tabled constant
   L = west longitude of station of the desired plane convergence

The Second Term Correction, δ:

The second term correction is usually minute and can be neglected for most courses under five miles long. This correction is also noted as “t - T.” It is the difference between the grid azimuth, “t,” and the projected geodetic azimuth, “T.” It increases directly with the change in eastings of the line and with the distance of the occupied station from the central parallel. It is neglected in this presentation.

Determine Grid, t, or Geodetic Azimuth, α:

$$t = \alpha - \gamma + \delta$$

Where:

t = grid azimuth, the clockwise angle at a station between the grid meridian (grid north) and the grid line to the observed object. All grid meridians are straight and parallel.
α = geodetic azimuth, the clockwise angle between the geodetic meridian (geodetic north) and the geodetic line to the object observed.
g = plane convergence, mapping angle, the major component of the difference between geodetic azimuth and grid azimuth.
δ = arc to chord correction, known as second term or t - T, a correction applied to long lines of precise surveys to compensate for distortion of straight lines when projected onto a grid. This correction is usually minute and can be neglected for most courses under five miles long. Use 0° in those cases.

Reducing Measured Distance to Geodetic Length and then to Grid Length

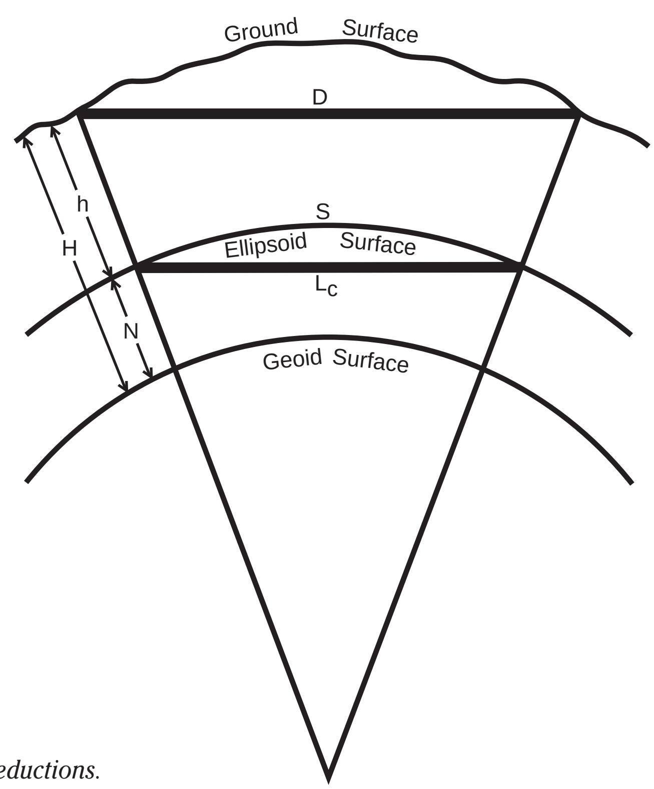


Figure 11-6. *Distance reductions.*

Reduce Measured Length to Ellipsoidal Chord Length:

Surveyors performing high precision work with NAD83 must consider the difference in elevation between the GRS80 ellipsoid and the geoid. Elevation based upon the ellipsoid is geodetic height, h. Elevation based upon the geoid (mean sea-level) is noted as H. The difference between the two surfaces is the geoid separation or height, N.

Determine Approximate Radius of Curvature of the Ellipsoid, R_α:

An approximate radius of the ellipsoid for each zone is the geometric mean radius of curvature at the projection origin. It is close enough for all but the most precise work. It can be obtained by the following formula:

$$R_\alpha \pm = \frac{r_o}{k_o}$$

Where:

$R_\alpha \pm$ = geometric mean radius of curvature of the ellipsoid at the projection origin

r_o = geometric mean radius of the ellipsoid at the projection origin, scaled to the grid

k_o = grid scale factor of the central parallel

Determine Ellipsoidal Reduction Factor, r_e, Also Known as Elevation Factor:

$$r_e = \frac{R_\alpha}{(R_\alpha + N + H)}$$

Where:

r_e = ellipsoidal reduction factor

R_α = radius of curvature of the ellipsoid in the azimuth from equation above

N = geoid separation, which is a negative value within the contiguous 48 states

H = elevation based on mean sea level to which the measured line was reduced to horizontal. The elevation to be used is usually one of the following for measured lengths:

1. Triangulation - average elevation of base line
2. EDM - elevation to which slope length was reduced
3. Taped - average elevation of the line

Each one meter error in N or H contributes 0.16 ppm of error to the distance.

Determine Ellipsoidal Chord Length, L_c:

$$L_c = r_e D$$

Where:

L_c = ellipsoidal chord length
r_e = ellipsoidal reduction factor
D = ground level, horizontal measured distance

Determine Correction of Ellipsoidal Chord Length to Geodetic Length

Geodetic lengths are ellipsoidal arc lengths. When precise geodetic lengths, s, are desired, a correction from ellipsoidal chord length, L_c, to the geodetic length on the ellipsoid surface may be applied to lines generally greater than 5 mi long. Shorter lengths and lines measured in segments are essentially arcs and need not be corrected. The correction to be applied is:

Determine Chord Correction for Geodetic Length:

L_c equals nearly 9 mi before c equals 0.01 ft and over 19 mi before equaling 0.10 ft.

$$c = \frac{L_c^3}{24R_\alpha^2}$$

Where:

c = correction, in same units as used for L_c and R_a
L_c = ellipsoidal chord length
R_a = radius of curvature of the ellipsoid in the azimuth from equation for approximating radius of curvature of ellipsoid above

Determine Geodetic Length, s:

$$s = L_c + c$$

Where:

c = correction, in same units as used for L_c and R_a
L_c = ellipsoidal chord length
s = geodetic length

Project Geodetic Length (or Ellipsoidal Chord Length) to Obtain Grid Length

Grid length, $L_{(grid)}$, is obtained by multiplying the geodetic length by a grid scale factor. Grid scale factor is an expression of the amount of distortion imposed on the length of a line on the ellipsoid as it is projected onto the grid cone. It is represented by the letter "k." It is the ratio of the length on the grid to the length on the ellipsoid. Scale factor is dependent upon latitude. It is less than unity between the standard parallels and greater than unity outside them. Scale factors can be calculated for points or lines. For this presentation, point scale factors are used exclusively. If greater accuracy is needed for a long line, the point scale factor of the mid-point or the mean of the point scale factors at each end may be used.

In a Lambert zone, if the north-south extent is not great, sufficient accuracy often may be obtained by using an average scale factor determined for the average latitude of the survey.

Determine Point Scale Factor, k:

A method which is precise enough for any work performed uses polynomial coefficients. It does not require projection tables and is easy to do using a handheld calculator with storage registers. It also readily lends itself to programmed applications. The coefficients for each zone are tabled at the end of this paper. Approximate point scale factor, k, may be interpolated from projection tables.

if γ and plane coordinates are known:

$$u = n - N_o - [(e - E_o) \tan \frac{\gamma}{2}]$$

when plane coordinates are known:

$$u = R_o - \sqrt{(R_b + N_b - n)^2 + (e - E_o)^2}$$

when geodetic latitude and longitude are known:

$$u = L_1\Delta B + L_2\Delta B^2 + L_3\Delta B^3 + L_4\Delta B^4$$

rearranged for handheld calculators:

$$u = \Delta B[L_1 + \Delta B(L_2 + \Delta B(L_3 + L_4\Delta B))]$$

Where:

u = radial distance on the projection from the central parallel to the station, $R_o - R$
N_o = northing of projection origin
n = northing of station
e = easting of station
E_o = easting of central meridian, 6561666.667 ft or 2000000.0000 m for all zones (also noted as C)
γ = convergence angle at the station
R_o = mapping radius through the projection origin, a tabled constant
R_b = mapping radius through grid base, a tabled constant
N_b = northing of grid base, a tabled constant equaling 1640416.667 ft or 500000.0000 m
L_1, L_2, L_3, L_4 = polynomial coefficients tabled with the zone constants
B = latitude of station (also noted as ø)
B_o = the latitude of the projection origin, a tabled constant for each zone (also noted as $ø_o$)
$\Delta B = B - B_o$

$$k = F_1 + F_2u^2 + F_3u^3$$

Where:

k = point scale factor
F_1, F_2, F_3 = polynomial coefficients tabled with the zone constants
u = radial distance on the projection

Determine Grid Length, $L_{(grid)}$:

$$L_{(grid)} = s\,k$$

Where:

$L_{(grid)}$ = length on grid
s = geodetic length
k = point scale factor of midpoint of line or as appropriate

Combined Factor

If no correction from ellipsoidal chord length to geodetic length is warranted and latitude and elevation differences are not great, the measured lengths may be multiplied with a combined factor, cf, to obtain grid lengths, $L_{(grid)}$.

Determine Combined Factor, cf:

$$cf = r_e k$$

Where:
- cf = combined factor
- r_e = ellipsoidal reduction factor
- k = point scale factor calculated for mid-latitude of the survey

Determine Grid Length, $L_{(grid)}$:

$$L_{(grid)} = cf\,D$$

Where:
- $L_{(grid)}$ = length on grid
- cf = combined factor
- D = ground level, horizontal measured distance

Conversion of Grid Length to Ground Length

Grid length, $L_{(grid)}$, may be converted to ground length, D, by reversing the above procedures. For measurements of similar elevation, latitude and short length, a combined correction factor may be divided into the grid length.

Area and State Plane Coordinates

Areas derived from state plane coordinates must be corrected to yield ground-level areas when they are desired.

$$A = \frac{A_{(grid)}}{cf^2}$$

Where:
- A = land area at ground level
- $A_{(grid)}$ = area of figure on the grid
- cf = combined factor

Conversion of Coordinates from One Zone to Another

To convert plane coordinates in the overlap of zones from one zone to the other of the CCS83 system, convert the plane coordinates from the original zone using the constants for that zone to GRS80 geodetic latitude and longitude. Then using the constants for the new zone, convert the geodetic latitude and longitude to plane coordinates in the new zone.

Metric–Foot Equivalency

The U.S. Survey ft, the linear unit of the State coordinate system, is defined by the equivalence: 1 international meter = 39.37 inches, exactly.

A coordinate system in ft may be converted to a coordinate system in m by multiplying the coordinate values by a scale factor of 0.304800609601. An exact conversion can be accomplished by first multiplying by 12 and then dividing by 39.37.

A coordinate system in m may be converted to a system in ft by multiplying the coordinate values by a scale factor of 3.28083333333. An exact conversion can be accomplished by first multiplying by 39.37 and then dividing by 12.

Sample Test Questions

1. Convert the position of station “San Ysidro Levee 1975” from its NAD83 geodetic coordinates of latitude 32° 32' 36.33328" N, longitude 117° 02' 24.17391" W to its CCS83 Zone 6 metric plane coordinates.
2. What are the CCS83 Zone V plane coordinates for a station at latitude 34° 08' 13.1201" N, longitude 118° 19' 32.9502" W? Use projection tables for your solution.
3. Convert the position of station “San Ysidro Levee 1975” from its CCS83 Zone 6 plane coordinates of 542065.352m N, 1925786.624 m E to its equivalent NAD83 geodetic coordinates.
4. In CCS83 Zone V, a station has plane coordinates 1872390.80 ft N, 6463072.10 ft E. What are its NAD83 geodetic coordinates? Use projection tables for your solution.
5. A line has been determined to have a geodetic azimuth of 135° 00' 00" in CCS83 Zone VI. The station is at longitude 117° 25' W. What is its grid azimuth?

6. The geodetic azimuth, *a*, is desired between two stations on the CCS83 Zone VI grid. The azimuth from Station #1 is desired. Given: Station #1 coordinates are: 1660578.090 ft N, 6570078.800 ft E. Station #2 coordinates are: 1653507.022 ft N, 6577149.868 ft E.
7. A survey party occupied station San Javier 1919 which has NGS NAD83 published CCS83 Zone 6 coordinates of 539034.888 m northing, 1977009.714 m easting. San Javier also has a published elevation of 1219.58 m and a geoid height of -33.57 m. All field measured distances were reduced to the elevation of station San Javier 1919. The field distance to a foresighted station is 13000.00 ft. What is the grid distance to the foresighted station? What is the combined factor?
8. In CCS83 Zone VI, a length of 6000.000 ft is measured at 5200 ft elevation. The mid-point of the line is at latitude 32° 36' 20" N. What is the grid length? What is the combined factor?
9. A line in CCS83 Zone VI has a grid length of 200000.000 ft and its mid-point is at latitude 32° 31' N. What is its ground length at 3800 ft elevation?

Answer Key

1. Direct forward computation.

Determine radial difference:

$\Delta B = B - B_o$
$\Delta B = 32.543425911° - 33.333922945°$ (truncate 30° from B and B_o for HP41s)
$\Delta B = -0.790497034°$

$u = \Delta B\,[L_1 + \Delta B\,(L_2 + \Delta B\,(L_3 + L_4\Delta B))]$
$u = \Delta B\,[L1 + \Delta B\,(L_2 + \Delta B\,(L_3 + (0.016171)\,(-0.790497034)))]$
$u = \Delta B\,[L_1 + \Delta B\,(L_2 + (-0.790497034)\,(5.65087 - 0.012783))]$
$u = \Delta B\,[L_1 + (-0.790497034)\,(8.94188 - 4.45689)]$
$u = (-0.790497034)\,(110905.3274 - 3.5454)$
$u = -87667.5297$ m

Determine mapping radius:

$R = R_o - u$
$R = 9706640.076 - (-87667.530)$
$R = 9794307.606$ m

Determine plane convergence:

$\gamma = (L_o - L)\sin B_o$

γ = (116° 15' - 117° 02' 24.17391") (0.5495175758) truncate 110° from L_o and L for HP41s

γ = -0° 26' 02.923552" (carry all significant digits)

Determine northing and easting:

$n = N_o + u + [(R \sin\gamma) \tan\frac{\gamma}{2}]$

$n = N_o + u + [(9794307.606)(-0.00757719469)(-0.00378865173)]$

n = 629451.7134 - 87667.5297 + 281.1686

n = 542065.352 m (exactly as published by NGS)

$e = E_o + R\sin\gamma$

e = 2000000.0000 + [(9794307.606) (-0.00757719469)]

e = 1925786.624 m (exactly as published by NGS)

2. Direct forward computation (using projection tables).

Determine mapping radius:		R = 30418256.88
for 34° 08' from Projection Table:	101.08844 x 13.1201 =	-1326.29
less diff/1" lat. from column 5:		R = 30416930.59
then for 34° 08' 13.1201" N:		

Determine plane convergence:

$\gamma = (L_o - L)\sin B_o$

γ = (118° 00' - 118° 19' 32.9502") (0.570011896174) truncate 110° from L_o and L for HP41s

γ = -0.18572099101°

γ = -0° 11' 08.5956" (carry to all significant digits)

Determine northing and easting:

$n = R_b + N_b - R\cos\gamma$

n = 30648744.93 + 1640416.67 - 30416930.59 cos(-0.18572099101°)

n = 1872390.80 ft N

$e = E_o + R\sin\gamma$

e = 6561666.67 + 30416930.59 sin (-0.18572099101°)

e = 6463072.10 ft E

3. Inverse computation.

Determine plane convergence:

$\gamma = \arctan [\frac{e - E_o}{R_b - n + N_b}]$

$\gamma = \arctan [(1925786.624 \text{ m} - 2000000.000 \text{ m}) /$
$(9836091.790 \text{ m} - 542065.352 \text{ m} + 500000.000 \text{ m})]$

$\gamma = \arctan (-74213.376/9794026.438)$

$\gamma = \arctan (-0.007577412259)$

$\gamma = -0° 26' 02.923559''$ (carry all significant digits)

Determine longitude:

$L = L_o - (\frac{\gamma}{\sin B_o})$

$L = 116.25° - (-0.4341454331/0.5495175758)$ truncate 110° from L for HP41s

$L = 117.040048312°$

$L = 117° 02' 24.17392''$ W (versus 117° 02' 24.17391" published by NGS)

Determine radial difference:

$u = n - N_o - [(e - E_o) \tan \frac{\gamma}{2}]$

$u = n - N_o - [(1925786.624\text{m} - 2000000.0000 \text{ m}) (-0.00378865173)]$

$u = 542065.352 \text{ m} - 629451.7134 \text{ m} - 281.1686 \text{ m}$

$u = -87667.530 \text{ m}$

Determine latitude:

$B = B_o + u [G_1 + u (G_2 + u (G_3 + G_4 u))]$

$B = B_o + u [G_1 + u (G_2 + u (G_3 + (-8.2753 \times 10^{-28}) (-87667.530 \text{ m})))]$

$B = B_o + u [G_1 + u (G_2 + (-87667.530 \text{ m}) (-3.73318 \times 10^{-20} + 7.25475 \times 10^{-23}))]$

$B = B_o + u [G_1 + (-87667.530 \text{ m}) (-6.55499 \times 10^{-15} + 3.266426 \times 10^{-15})]$

$B = B_o + (-87667.530 \text{ m}) (9.016699372 \times 10^{-6} + 2.8830022 \times 10^{-10})$

$B = 33.333922945° - 0.7904970372°$ truncate 30° from B for HP41s

$B = 32° 32' 36.33327''$ N (versus 32° 32' 36.33328" published by NGS)

4. Inverse computation (using projecting tables).

Determine plane convergence:

$$\gamma = \arctan \left[\frac{e - E_o}{R_b - n + N_b} \right]$$

$$\gamma = \arctan \left[\frac{(6463072.10 \text{ ft} - 6561666.67 \text{ ft})}{(30648744.93 \text{ ft} + 1640416.67 \text{ ft} - 187390.80 \text{ ft})} \right]$$

γ = arctan (-98594.57 / 30416770.80)
γ = arctan (-0.00324145422)
γ = -0.18572099571°
γ = -0° 11' 08.59558" (carry seconds to at least the fourth decimal place)

Determine longitude:

$$L = L_o - \frac{\gamma}{\sin B_o}$$

L = 118° 00' - (-0.18572099571°/0.570011896174) truncate 110° from L for HP41s
L = 118° 00' - (-0.325819500°)
L = 118.325819500° W
L = 118° 19' 32.9502" W

Determine mapping radius for use with projection tables:

$$R = \sqrt{(e - E_o)^2 + (R_b - n + N_b)^2}$$

$$R = \sqrt{-98594.57^2 + 30416770.80^2}$$

R = 30416930.59 ft

Determine the latitude:
Interpolate R in Projection Table to find latitude (ø)

(34° 08')	R =	30418256.88
(unk.)	R =	30416930.59
Diff.		1326.29

using column 5 of Projection Table:

1326.29/101.08844 = 3.1201"
therefore, B = 34° 08' 13.1201" N

5. **Determine the plane convergence.**

$\gamma = \sin B_o(L_o - L)$
$\gamma = 0.549517575763\ (116° 15' - 117° 25')$
$\gamma = -0° 38' 27.97382''$

The second term will be neglected.

Calculate the grid azimuth:

$t = \alpha - \gamma + \delta$
$t = 135° 00' 00'' - (-0° 38' 27.97382'') + 0°$
$t = 135° 38' 27.97382''$

6. **Inverse from Station #1 to Station #2.**

The inversed grid azimuth is: t = 135°00'00".

Calculate the plane convergence:

$$\gamma = \arctan \left[\frac{e - E_o}{R_{b-n} + N_b} \right]$$

$$\gamma = \arctan \left[\frac{(6570078.800\ ft - 6561666.667\ ft)}{(32270577.813\ ft - 1660578.090\ ft + 1640416.667\ ft)} \right]$$

$\gamma = \arctan (0.00026083797)$
$\gamma = +0° 00' 53.8017''$

The second term correction is neglected here.

Calculate the geodetic azimuth:

$t = \alpha - \gamma + \delta$

$135° 00' 00'' = \alpha - (+0° 00' 53.8017'') + 0°)$
$\alpha = 135° 00' 00'' + 0° 00' 53.8017'' + 0°$
$\alpha = 135° 00' 53.8017''$

7. **Convert elevation, geoid height, and measured distance to the same units. Here feet were chosen.**

Determine radius of curvature of ellipsoid:

$$R_\alpha \pm = \frac{r_o}{k_o}$$

$R_\alpha \pm = 20896729.860\ ft / 0.999954142490$
$R_\alpha \pm = 20897688.176\ ft$

Determine ellipsoidal reduction factor:

$$r_e = \frac{R_\alpha}{(R_\alpha + N + H)}$$

r_e = 20897688.176 ft / (20897688.176 ft - 110.14 ft + 4001.24 ft)

r_e = 0.999813837

Determine ellipsoidal chord length:

$L_c = r_e D$

L_c = 0.999813837 x 13000.00 ft

L_c = 12997.58 ft

Determine chord correction and geodetic length:

$$c = \frac{L_c^3}{24R_\alpha^2}$$

$c = 12997.58^3 / 24\,(20897688.176^2)$

c = 0.0002 ft

Determine geodetic length:

The chord correction is negligible, therefore the geodetic length,

s = 12997.58 ft

Determine point scale factor:

$u = R_o - \sqrt{(R_b + N_b - n)^2 + (e - E_o)^2}$

u = 31845868.317 ft -

$\sqrt{(322770577.813 \text{ ft} + 1640416.667 \text{ ft} - 1768483.63)^2 + (6486239.37 \text{ ft} - 6561666.67)^2}$

$u = 31845868.317 \text{ ft} - [1033141004 \times 10^{15} + 5689277585.29]^2$

u = 31845868.317 ft - 32142599.351 ft

u = -296731.03 ft

$k = F_1 + F_2u^2 + F_3u^3$

$k = 0.999954142490 + [1.14504 \times 10^{-15} (8.8049304 \times 10^{10})] +$

$[1.18 \times 10^{-23} (-2.6126960 \times 10^{16})]$

k = 0.999954142490 + 0.000100820 - 0.000000308

k = 1.000054654

Determine grid length:

$L_{(grid)} = s\,k$

$L_{(grid)}$ = 12997.58 ft (1.000054654)

$L_{(grid)}$ = 12998.29

Determine combined factor:

$cf = r_e k = (0.999813837)\ 1.000054654 = 0.999868481$

8. Reducing measured distance.

Determine radius of curvature of ellipsoid:

$$R_a \pm = \frac{r_o}{k_o}$$

$R_a \pm = 20896729.860\ ft/0.999954142490$

$R_a \pm = 20897688.176\ ft$

Determine ellipsoidal reduction factor:

$$r_e = \frac{R_\alpha}{(R_\alpha + N + H)}$$

$r_e = 20897688.176\ ft/(20897688.176\ ft + 0 + 5200\ ft)$

$r_e = 0.9997512306$

Determine ellipsoidal chord length:

$L_c = r_e D$

$L_c = (0.9997512306)\ (6000.000\ ft)$

$L_c = 5998.507\ ft$

Determine chord correction:

$$c = \frac{L_c^3}{24R_\alpha^2}$$

$c = 5998.507^3/(24)(20897688.176^2)$

$c = 0.000021\ ft$

Determine geodetic length:

The chord correction is negligible, therefore the geodetic length,

$s = 5998.507\ ft$

Determine point scale factor:

Scale factor for 32° 36' from Project Table = 1.0000356

20/60 diff. = (0.333) (-0.0000037) = -0.0000012

k = 1.0000344

Determine grid length:

$L_{(grid)} = s\ k$

$L_{(grid)} = (5998.507\ ft)\ (1.0000344)$

$L_{(grid)} = 5998.713\ ft$

Determine combined factor:

$cf = r_e\, k$
$cf = (0.9997512306)\ (1.0000344)$
$cf = 0.9997856$

Determine grid length:

$L_{(grid)} = cf\ (D)$
$L_{(grid)} = 0.9997856 \times 6000.00$ ft
$L_{(grid)} = 5998.714$ ft

9. Convert grid length to ground length.

Determine point scale factor:

$\Delta B = B - B_o$
$\Delta B = 32.5166666667° - 33.3339229447°$
$\Delta B = -0.8172562780°$

$u = \Delta B\ [L_1 + \Delta B\ (L_2 + \Delta B\ (L_3 + (-0.04335871457)))]$
$u = \Delta B\ [L1 + \Delta B\ (L_2 + \Delta B(18.4962412854))]$
$u = \Delta B\ [L_1 + \Delta B\ (L_2 + (-15.1161693099))]$
$u = \Delta B\ [L_1 + \Delta B\ (14.2206306901)]$
$u = \Delta B\ [L_1 + (-11.6218997086)]$
$u = \Delta B\ [363850.2731]$
$u = -297358.9199$ ft

$k = F_1 + F_2 u^2 + F_3 u^3$
$k = 0.999954142490 + (1.14504 \times 10^{-15})\ (-297358.9199^2) + (1.18 \times 10^{-23})\ (-297358.9199^3)$
$k = 1.00005507933$

Determine geodetic length:

$L_{(grid)} = s\ k$
200000.000 ft $= s \times 1.00005507933$
$s = 200000.000$ ft$/1.00005507933$
$s = 199988.985$ ft

Determine radius of curvature of the ellipsoid:

$$R_a \pm = \frac{r_o}{k_o}$$

$R_a \pm = 20896729.860$ ft $/ 0.999954142490$
$R_a \pm = 20897688.176$ ft

Determine chord correction:

$$c = \frac{L_c^3}{24R_a^2}$$

$c = 199988.985^3/[(24)\ (20897688.176^2)]$

use geodetic length as approximate ellipsoidal chord

$c = 0.763$ ft

Determine ellipsoidal chord length:

$s = L_c + c$

$L_c + 0.763$ ft $= 199988.985$ ft

$L_c = 199988.222$ ft

Determine ellipsoidal reduction factor:

$$r_e = \frac{R_a}{(R_a + N + H)}$$

$r_e = 20897688.176$ ft/(20897688.176 ft + 0 + 3800)

$r_e = 0.99981819476$

Determine level ground length:

$L_c = r_e D$

199988.222 ft = 0.99981819476 x D

D = 199988.222 ft/0.99981819476

D = 200024.587

Appendix 1

State of California Public Resources Code
Division 8. Surveying and Mapping

Chapter 1. California Coordinate System

Section 8801. California Coordinate System Defined

(a) The system of plane coordinates which has been established by the United States Coast and Geodetic Survey for defining and stating the positions or locations of points on the surface of the earth within the State of California is based on the North American Datum of 1927 and is identified as the "California Coordinate System." After January 1, 1987, this system shall be known as the "California Coordinate System of 1927."

(b) The system of plane coordinates which has been established by the National Geodetic Survey for defining and stating the positions or locations of points on the surface of the earth within the State of California and which is based on the North American Datum of 1983 shall be known as the "California Coordinate System of 1983."

(c) As used in this chapter:
(1) "NAD27" means the North American Datum of 1927.
(2) "CCS27" means the California Coordinate System of 1927.
(3) "NAD83" means the North American Datum of 1983.
(4) "CCS83" means the California Coordinate System of 1983.
(5) "USC&GS" means the United States Coast and Geodetic Survey.
(6) "NGS" means the National Geodetic Survey.
(7) "FGCC" means the Federal Geodetic Control Committee.

(d) The use of the term "State Plane Coordinates" refers only to CCS27 and CCS83 coordinates.

Section 8802. Delineation of Zones

For CCS27 the state is divided into seven zones. For CCS83, the state is divided into six zones. Zone 7 of CCS27, which encompasses Los Angeles County, is eliminated and the area is included in Zone 5.

Each zone of CCS27 is a Lambert conformal conic projection based on Clarke's Spheroid of 1866, which is the basis of NAD27. The points of control of zones one to six, inclusive, bear the coordinates: Northing (y) = 000.00 feet and Easting (x) = 2,000,000 feet. The point of control of Zone 7 bears the coordinates: Northing (y) = 4,160,926.74 feet and Easting (x) = 4,186,692.58 feet.

Each zone of CCS83 is a Lambert conformal conic projection based on the Geodetic Reference System of 1980, which is the basis of NAD83. The point of control of each of the six zones bear the coordinates: Northing (y) = 500,000 meters and Easting (x) = 2,000,000 meters.

The area included in the following counties constitutes Zone 1 of CCS27 and CCS83: Del Norte, Humboldt, Lassen, Modoc, Plumas, Shasta, Siskiyou, Tehama, and Trinity.

The area included in the following counties constitutes Zone 2 of CCS27 and CCS83: Alpine, Amador, Butte, Colusa, El Dorado, Glenn, Lake, Mendocino, Napa, Nevada, Placer, Sacramento, Sierra, Solano, Sonoma, Sutter, Yolo, Yuba.

The area included in the following counties constitutes Zone 3 of CCS27 and CCS83: Alameda, Calaveras, Contra Costa, Madera, Marin, Mariposa, Merced, Mono, San Francisco, San Joaquin, San Mateo, Santa Clara, Santa Cruz, Stanislaus, and Tuolumne.

The area included in the following counties constitutes Zone 4 of CCS27 and CCS83: Fresno, Inyo, Kings, Monterey, San Benito, and Tulare.

The area included in the following counties and Channel Islands constitutes Zone 5 of CCS27: Kern, San Bernardino, San Luis Obispo, Santa Barbara (excepting Santa Barbara Island), and Ventura (excepting San Nicholas Island) and the Channel Islands of Santa Cruz, Santa Rosa, San Miguel, and Anacapa.

The area included in the following counties and Channel Islands constitutes Zone 5 of CCS83: Kern, Los Angeles (excepting San Clemente and Santa Catalina Islands), San Bernardino, San Luis Obispo, Santa Barbara (excepting Santa Barbara Island), and Ventura excepting San Nicholas Island) and the Channel Islands of Santa Cruz, Santa Rosa, San Miguel, and Anacapa.

The area included in the following counties and Channel Islands constitutes Zone 6 of CCS27 and CCS83: Imperial, Orange, Riverside, and San Diego and the Channel Islands of San Clemente, Santa Catalina, Santa Barbara, and San Nicholas.

The area included in Los Angeles County constitutes Zone 7 of CCS27.

Section 8803. Definition of Zone 1

Zone 1 coordinates shall be named, and, on any map on which they are used, they shall be designated as “CCS27, Zone 1 or CCS83, Zone 1.”

On their respective spheroids of reference: (1) the standard parallels of CCS27, Zone 1 and CCS83, Zone 1 are at north latitudes 40 degrees 00 minutes and 41 degrees 40 minutes, along which parallels the scale shall be exact; and (2) the point of control of coordinates is at the intersection of the zone’s central meridian, which is at 122 degrees 00 minutes west longitude, with the parallel 39 degrees 20 minutes north latitude.

Section 8804. Definition of Zone 2

Zone 2 coordinates shall be named, and, on any map on which they are used, they shall be designated as “CCS27, Zone 2 or CCS83, Zone 2.”

On their respective spheroids of reference: (1) the standard parallels of CCS27, Zone 2 and CCS83, Zone 2 are at north latitudes 38 degrees 20 minutes and 39 degrees 50 minutes, along which parallels the scale shall be exact; and (2) the point of control of coordinates is at the intersection of the zone’s central meridian, which is at 122 degrees 00 minutes west longitude, with the parallel 37 degrees 40 minutes north latitude.

Section 8805. Definition of Zone 3

Zone 3 coordinates shall be named, and, on any map on which they are used, they shall be designated as “CCS27, Zone 3 or CCS83, Zone 3.”

On their respective spheroids of reference: (1) the standard parallels of CCS27, Zone 3 and CCS83, Zone 3 are at north latitudes 37 degrees 04 minutes and 38 degrees 26 minutes, along which parallels the scale shall be exact; and (2) the point of control of coordinates is at the intersection of the zone’s central meridian, which is at 120 degrees 30 minutes west longitude, with the parallel 36 degrees 30 minutes north latitude.

Section 8806. Definition of Zone 4

Zone 4 coordinates shall be named, and, on any map on which they are used, they shall be designated as “CCS27, Zone 4 or CCS83, Zone 4.”

On their respective spheroids of reference: (1) the standard parallels of CCS27, Zone 4 and CCS83, Zone 4 are at north latitudes 36 degrees 00 minutes and 37 degrees 15 minutes, along which parallels the scale shall be exact; and (2) the point of control of coordinates is at the intersection of the zone’s central meridian, which is at 119 degrees 00 minutes west longitude, with the parallel 35 degrees 20 minutes north latitude.

Section 8807. Definition of Zone 5

Zone 5 coordinates shall be named, and, on any map on which they are used, they shall be designated as "CCS27, Zone 5 or CCS83, Zone 5."

On their respective spheroids of reference: (1) the standard parallels of CCS27, Zone 5 and CCS83, Zone 5 are at north latitudes 34 degrees 02 minutes and 35 degrees 28 minutes, along which parallels the scale shall be exact; and (2) the point of control of coordinates is at the intersection of the zone's central meridian, which is at 118 degrees 00 minutes west longitude, with the parallel 33 degrees 30 minutes north latitude.

Section 8808. Definition of Zone 6

Zone 6 coordinates shall be named, and, on any map on which they are used, they shall be designated as "CCS27, Zone 6 or CCS83, Zone 6."

On their respective spheroids of reference: (1) the standard parallels of CCS27, Zone 6 and CCS83, Zone 6 are at north latitudes 32 degrees 47 minutes and 33 degrees 53 minutes, along which parallels the scale shall be exact; and (2) the point of control of coordinates is at the intersection of the zone's central meridian, which is at 116 degrees 15 minutes west longitude, with the parallel 32 degrees 10 minutes north latitude.

Section 8809. Definition of Zone 7

Zone 7 coordinates shall be named, and, on any map on which they are used, they shall be designated as "CCS27, Zone 7."

On its respective spheroid of reference: (1) the standard parallels of CCS27, Zone 7 are at north latitudes 33 degrees 52 minutes and 34 degrees 25 minutes, along which parallels the scale shall be exact; and (2) the point of control of coordinates is at the intersection of the zone's central meridian, which is at 118 degrees 20 minutes west longitude, with the parallel 34 degrees 08 minutes north latitude.

Section 8810. Definition of U.S. Survey Foot and Coordinates

The plane coordinates of a point on the earth's surface, to be used in expressing the position or location of the point in the appropriate zone of CCS27 or CCS83, shall consist of two distances, expressed in feet and decimals of a foot or meters and decimals of a meter. When the values are expressed in feet, the "U.S. Survey foot," (one foot = 1200/3937 meters) shall be used as the standard foot for CCS27 and CCS83. One of these distances, to be known as the "East x-coordinate," shall give the distance east of the Y axis; the other, to be known as the "North y-coordinate," shall give the distance north of the X axis. The Y axis of any zone shall be parallel with the central meridian of that zone. The X axis of any zone shall be at right angles to the central meridian of that zone.

Section 8811. Sources of Plane Coordinates

The state plane coordinates of a point in any zone shall be based upon the plane coordinates of published horizontal control stations or derived from published horizontal control stations of the USC&GS and the NGS or their successors.

Section 8813. Requires Accuracy of Stations and Data Required

The CCS27 and CCS83 shall be based on monumented first- and second-order stations which have been published by USC&GS and NGS or their successors. The geodetic positions of CCS27 and CCS83 stations which are used to increase the density of control and which purport to be of first- or second-order accuracy shall have been surveyed in conformity with first- or second-order survey standards and specifications in effect at the time of the survey as defined by the Federal Geodetic Control Committee. Any survey or map which is to be based on state plane coordinates shall show established field-measured connections to at least two stations of corresponding accuracy or better whose credentials are based upon published stations of USC&GS or NGS or their successors. If an FGCC order of accuracy is claimed for a survey or a map, it shall be justified by additional written data that shows equipment, procedures, closures, adjustments, and a control diagram.

Section 8814. Use of Coordinates and Constructive Notice

State plane coordinates may be used for property identification on any map, survey, conveyance, or other instrument which delineates or affects the title to real property or which delineates, describes, or refers to the property, or any part thereof. However, to constitute, when recorded, constructive notice thereof under the recording laws, the delineating, describing, or referring to the property, or part thereof, shall also refer to data appearing of record in any office, the records of which constitute constructive notice under the recording laws. That record data shall be sufficient to identify the property without recourse to those coordinates, and in case of conflict between them, the references to that recorded data shall be controlling for the purpose of determining constructive notice under the recording laws.

Section 8815. Identification of California Coordinate System

The use of the term “California Coordinate System” on any map or document or any field notes shall be suffixed either with “27” (shown as “CCS27”) for coordinates based on NAD27 or with “83” (shown as “CCS83”) for coordinates based on NAD83.

Section 8816. Use Optional

The use of the State Plane Coordinates by any person, corporation, or governmental agency engaged in land surveying or mapping is optional.

Section 8817. CCS83 Use After January 1, 1995

Prior to January 1, 1995, use of State Plane Coordinates for new projects may be based either on CCS27 or CCS83. On or after January 1, 1995, when State Plane Coordinates are used on new surveys and new mapping projects, the use shall be limited to CCS83.

Section 8818. Land Titles Referred to CCS27

This chapter does not impair or invalidate land titles, legal descriptions, or jurisdictional or land boundaries and, further, this chapter does not impair or invalidate references to, or the use of, CCS27 coordinates, except as provided in Section 8817.

Section 8819. Use of New Technologies

This chapter does not prohibit the use of new geodetic surveying technologies for which FGCC specifications have not yet been published, except that if first- or second-order accuracy is claimed for any of the resulting monumented stations, the state plane coordinates shall conform to FGCC accuracy standards.

Appendix 2

Symbols and Notations

α = geodetic azimuth, the clockwise angle between geodetic north and the geodetic line to the object observed

γ = the plane convergence angle, the major component of the difference between geodetic azimuth and grid azimuth, also sometimes called mapping angle

δ = arc to chord correction, also known as second term or "t - T" (A correction applied to long lines of precise surveys to compensate for distortion of straight lines when projected onto the grid. This correction is usually minute and can be neglected for most courses under five miles long.)

a = 6378137 m (exact) or 20925604.4742 ft, the equatorial radius of the GRS80 ellipsoid

b = 6356752.314140347 m = 20855444.8840 ft = the semiminor axis of the GRS80 ellipsoid

B = north geodetic latitude of a station, also noted as ø

B_b = north geodetic latitude of the parallel passing through grid origin, a tabled constant

B_n = north geodetic latitude of the northerly standard parallel where the cone intersects the ellipsoid (Line of exact scale)

B_o = the latitude of the central parallel passing through the projection origin, a tabled constant for each zone, also noted as $\_{o}$

B_s = north geodetic latitude of the southerly standard parallel where the cone intersects the ellipsoid (Line of exact scale)

cf = combined factor for simultaneously applying average ellipsoidal reduction and scale factors

D = ground level, horizontal measured distance

E_o = easting of projection origin and central meridian, 6561666.667 ft or 2000000.0000 m for all zones

e^2 = 0.006694380022903416 = the square of the first eccentricity of the GRS80 ellipsoid

e ft^2 = 0.006739496775481622 = the square of the second eccentricity of the GRS80 ellipsoid

F_1, F_2, F_3 = polynomial coefficients tabled with the zone constants

G_1, G_2, G_3, G_4 = polynomial coefficients for inverse conversion, tabled with the zone constants

h = geodetic height, elevation using the ellipsoid for its datum. Related to MSL datum by the formula, $h = N + H$

H = elevation using the geoid for its datum, this is approximately elevation based on mean sea level

k = point grid scale factor

K = mapping radius on the cone at the equatorial plane of the ellipsoid

k_o = grid scale factor of the central parallel, B_o, a tabled constant

L = west geodetic longitude of station, also noted as λ

L_c = ellipsoidal chord length

$L_{(grid)}$ = grid length, distance between two points on the grid plane

L_o = longitude of the central meridian passing through the projection and grid origin, a tabled constant, also noted as λ_o

L_s = measured slope length

L_1, L_2, L_3, L_4 = polynomial coefficients for direct computation, tabled with the zone constants

M_o = radius of curvature of the ellipsoid in the meridian at the projection origin, scaled to the grid

N = geoid separation or height, the distance at a station from the geoid to the ellipsoid, it is negative within the contiguous 48 states

N_b = northing of grid base, a tabled constant equalling 1640416.667 ft or 500000.0000 m for all zones

N_o = northing of the projection origin, a tabled constant

p = $1/f = \text{flattening}^{-1} = 298.2572221008827$ for GRS80

R = mapping radius through a station

R_a = radius of curvature of the ellipsoid in the azimuth

R_b = mapping radius through the grid base, a tabled constant

r_e = ellipsoidal reduction factor, also known as elevation factor

r_o = geometric mean radius of the ellipsoid at the projection origin, scaled to the grid

R_o = mapping radius through the projection origin, a tabled constant

s = geodetic length, the ellipsoidal arc length of a line

$\sin B_o$ = sine of the latitude of the projection origin, which is also the ratio between γ and longitude in decimals of a degree, a tabled constant

t = grid azimuth, the clockwise angle at a station between the grid meridian (grid north) and the grid line to the observed object (All grid meridians are straight and parallel. Grid azimuths are related to geodetic azimuths by the formula, $t = a - \gamma + \delta$>≥

T = projected geodetic azimuth (Azimuth of a straight line of the ellipsoid when projected onto the grid is slightly curved.)

u = radial distance on the projection from the station to the central parallel, $R_o - R$

Constants for the Geodetic Reference System of 1980 GRS80 Ellipsoid

a = 6378137 m (exact) = 20925604.4742 ft = the equatorial radius of the ellipsoid

b = 6356752.314140347 m = 20855444.8840 ft = the semiminor axis

p = 1/f = 298.2572221008827 = flattening^{-1}

e^2 = 0.006694380022903416 = the square of the first eccentricity

e ft^2 = 0.006739496775481622 = the square of the second eccentricity

Appendix 3

North American Datum 1983 (NAD83) – California Coordinate System 1983 (CCS83)

CALIFORNIA ZONE 1, CA01, ZONE# 0401

Meters			US Survey Feet		
B_s	=	40° 00' N	B_s	=	40° 00' N
B_n	=	41° 40' N	B_n	=	41° 40' N
B_b	=	39° 20' N	B_b	=	39° 20' N
L_o	=	122° 00' W	L_o	=	122° 00' W
N_b	=	500000.0000 m	N_b	=	1640416.667 ft
E_o	=	2000000.0000 m	E_o	=	6561666.667 ft
B_o	=	40.8351061249° N	B_o	=	40.8351061249° N
$SinB_o$	=	0.653884305400	$SinB_o$	=	0.653884305400
R_b	=	7556554.6408 m	R_b	=	24791796.351 ft
R_o	=	7389802.0597 m	R_o	=	24244708.924 ft
N_o	=	666752.5811 m	N_o	=	2187504.093 ft
K	=	12287826.3052 m	K	=	40314310.136 ft
k_o	=	0.999894636561	k_o	=	0.999894636561
M_o	=	6362067.2798 m	M_o	=	20872882.401 ft
r_o	=	6374328. m	r_o	=	20913107.780 ft
L_1	=	111039.0203	L_1	=	364300.5191
L_2	=	9.65524	L_2	=	31.6772
L_3	=	5.63491	L_3	=	18.4872
L_4	=	0.021275	L_4	=	0.069800
G_1	=	9.005843038E-06	G_1	=	2.744986448E-06
G_2	=	-7.05240E-15	G_2	=	-6.55192E-16
G_3	=	-3.70393E-20	G_3	=	-1.04884E-21
G_4	=	-1.1142E-27	G_4	=	-9.6167E-30
F_1	=	0.999894636561	F_1	=	0.999894636561
F_2	=	1.23062E-14	F_2	=	1.14329E-15
F_3	=	5.47E-22	F_3	=	1.55E-23

The customary limits of the zone are from 39° 20' N to 42° 20' N.

North American Datum 1983 (NAD83) – California Coordinate System 1983 (CCS83)

CALIFORNIA ZONE 2, CA02, ZONE# 0402

Meters			US Survey Feet		
B_s	=	38° 20' N	B_s	=	38° 20' N
B_n	=	39° 50' N	B_n	=	39° 50' N
B_b	=	37° 40' N	B_b	=	37° 40' N
L_o	=	122° 00' W	L_o	=	122° 00' W
N_b	=	500000.0000 m	N_b	=	1640416.667 ft
E_o	=	2000000.0000 m	E_o	=	6561666.667 ft
B_o	=	39.0846839219° N	B_o	=	39.0846839219° N
$SinB_o$	=	0.630468335285	$SinB_o$	=	0.630468335285
R_b	=	8019788.9307 m	R_b	=	26311590.850 ft
R_o	=	7862381.4027 m	R_o	=	25795162.985 ft
N_o	=	657407.5280 m	N_o	=	2156844.531 ft
K	=	12520351.6538 m	K	=	41077187.051 ft
k_o	=	0.999914672977	k_o	=	0.999914672977
M_o	=	6360268.3937 m	M_o	=	20866980.555 ft
r_o	=	6373169. m	r_o	=	20909305.294 ft
L_1	=	111007.6240	L_1	=	364197.5131
L_2	=	9.54628	L_2	=	31.3198
L_3	=	5.63874	L_3	=	18.4998
L_4	=	0.019988	L_4	=	0.065577
G_1	=	9.008390180E-06	G_1	=	2.745762818E-06
G_2	=	-6.97872E-15	G_2	=	-6.48347E-16
G_3	=	-3.71084E-20	G_3	=	-1.05080E-21
G_4	=	-1.0411E-27	G_4	=	-8.9858E-30
F_1	=	0.999914672977	F_1	=	0.999914672977
F_2	=	1.23106E-14	F_2	=	1.14370E-15
F_3	=	5.14E-22	F_3	=	1.46E-23

The customary limits of the zone are from 37° 40' N to 40° 30' N.

North American Datum 1983 (NAD83) – California Coordinate System 1983 (CCS83)

CALIFORNIA ZONE 3, CA03, ZONE# 0403

Meters			US Survey Feet		
B_s	=	37° 04' N	B_s	=	37° 04' N
B_n	=	38° 26' N	B_n	=	38° 26' N
B_b	=	36° 30' N	B_b	=	36° 30' N
L_o	=	120° 30' W	L_o	=	120° 30' W
N_b	=	500000.0000 m	N_b	=	1640416.667 ft
E_o	=	2000000.0000 m	E_o	=	6561666.667 ft
B_o	=	37.7510694363° N	B_o	=	37.7510694363° N
$\sin B_o$	=	0.612232038295	$\sin B_o$	=	0.612232038295
R_b	=	8385775.1723 m	R_b	=	27512330.711 ft
R_o	=	8246930.3684 m	R_o	=	27056804.050 ft
N_o	=	638844.8039 m	N_o	=	2095943.327 ft
K	=	12724574.9735 m	K	=	41747209.726 ft
k_o	=	0.999929178853	k_o	=	0.999929178853
M_o	=	6358909.6841 m	M_o	=	20862522.855 ft
r_o	=	6372292. m	r_o	=	20906428.003 ft
L_1	=	110983.9104	L_1	=	364119.7127
L_2	=	9.43943	L_2	=	30.9692
L_3	=	5.64142	L_3	=	18.5086
L_4	=	0.019048	L_4	=	0.062493
G_1	=	9.010315015E-06	G_1	=	2.746349509E-06
G_2	=	-6.90503E-15	G_2	=	-6.41501E-16
G_3	=	-3.71614E-20	G_3	=	-1.05230E-21
G_4	=	-9.8819E-28	G_4	=	-8.5291E-30
F_1	=	0.999929178853	F_1	=	0.999929178853
F_2	=	1.23137E-14	F_2	=	1.14398E-15
F_3	=	4.89E-22	F_3	=	1.38E-23

The customary limits of the zone are from 36° 30' N to 39° 00' N.

North American Datum 1983 (NAD83) – California Coordinate System 1983 (CCS83)

CALIFORNIA ZONE 4, CA04, ZONE# 0404

Meters			US Survey Feet		
B_s	=	36° 00' N	B_s	=	36° 00' N
B_n	=	37° 15' N	B_n	=	37° 15' N
B_b	=	35° 20' N	B_b	=	35° 20' N
L_o	=	119° 00' W	L_o	=	119° 00' W
N_b	=	500000.0000 m	N_b	=	1640416.667 ft
E_o	=	2000000.0000 m	E_o	=	6561666.667 ft
B_o	=	36.6258593071° N	B_o	=	36.6258593071° N
$SinB_o$	=	0.596587149880	$SinB_o$	=	0.596587149880
R_b	=	8733227.3793 m	R_b	=	28652263.494 ft
R_o	=	8589806.8935 m	R_o	=	28181724.783 ft
N_o	=	643420.4858 m	N_o	=	2110955.377 ft
K	=	12916986.0281 m	K	=	42378478.327 ft
k_o	=	0.999940761703	k_o	=	0.999940761703
M_o	=	6357772.8978 m	M_o	=	20858793.249 ft
r_o	=	6371557. m	r_o	=	20904016.591 ft
L_1	=	110964.0696	L_1	=	364054.6183
L_2	=	9.33334	L_2	=	30.6211
L_3	=	5.64410	L_3	=	18.5174
L_4	=	0.018382	L_4	=	0.060308
G_1	=	9.011926076E-06	G_1	=	2.746840562E-06
G_2	=	-6.83121E-15	G_2	=	-6.34643E-16
G_3	=	-3.72043E-20	G_3	=	-1.05351E-21
G_4	=	-9.4223E-28	G_4	=	-8.1324E-30
F_1	=	0.999940761703	F_1	=	0.999940761703
F_2	=	1.23168E-14	F_2	=	1.14427E-15
F_3	=	4.70E-22	F_3	=	1.33E-23

The customary limits of the zone are from 35° 20' N to 38° 00' N.

North American Datum 1983 (NAD83) – California Coordinate System 1983 (CCS83)

CALIFORNIA ZONE 5, CA05, ZONE# 0405

Meters			US Survey Feet		
B_s	=	34° 02' N	B_s	=	34° 02' N
B_n	=	35° 28' N	B_n	=	35° 28' N
B_b	=	33° 30' N	B_b	=	33° 30' N
L_o	=	118° 00' W	L_o	=	118° 00' W
N_b	=	500000.0000 m	N_b	=	1640416.667 ft
E_o	=	2000000.0000 m	E_o	=	6561666.667 ft
B_o	=	34.7510553142° N	B_o	=	34.7510553142° N
$SinB_o$	=	0.570011896174	$SinB_o$	=	0.570011896174
R_b	=	9341756.1389 m	R_b	=	30648744.932 ft
R_o	=	9202983.1099 m	R_o	=	30193453.753 ft
N_o	=	638773.0290 m	N_o	=	2095707.846 ft
K	=	13282624.8345 m	K	=	43578078.311 ft
k_o	=	0.999922127209	k_o	=	0.999922127209
M_o	=	6355670.9697 m	M_o	=	20851897.173 ft
r_o	=	6370113. m	r_o	=	20899279.068 ft
L_1	=	110927.3840	L_1	=	363934.2590
L_2	=	9.12439	L_2	=	29.9356
L_3	=	5.64805	L_3	=	18.5303
L_4	=	0.017445	L_4	=	0.057234
G_1	=	9.014906468E-06	G_1	=	2.747748987E-06
G_2	=	-6.68534E-15	G_2	=	-6.21091E-16
G_3	=	-3.72796E-20	G_3	=	-1.05565E-21
G_4	=	-8.6394E-28	G_4	=	-7.4567E-30
F_1	=	0.999922127209	F_1	=	0.999922127209
F_2	=	1.23221E-14	F_2	=	1.14477E-15
F_3	=	4.41E-22	F_3	=	1.25E-23

The customary limits of the zone are from 33° 30' N to 36° 20' N.

North American Datum 1983 (NAD83) – California Coordinate System 1983 (CCS83)

CALIFORNIA ZONE 6, CA06, ZONE# 0406

Meters			US Survey Feet		
B_s	=	32° 47' N	B_s	=	32° 47' N
B_n	=	33° 53' N	B_n	=	33° 53' N
B_b	=	32° 10' N	B_b	=	32° 10' N
L_o	=	116° 15' W	L_o	=	116° 15' W
N_b	=	500000.0000 m	N_b	=	1640416.667 ft
E_o	=	2000000.0000 m	E_o	=	6561666.667 ft
B_o	=	33.3339229447° N	B_o	=	33.3339229447° N
$SinB_o$	=	0.549517575763	$SinB_o$	=	0.549517575763
R_b	=	9836091.7896 m	R_b	=	32270577.813 ft
R_o	=	9706640.0762 m	R_o	=	31845868.317 ft
N_o	=	629451.7134 m	N_o	=	2065126.163 ft
K	=	13602026.7133 m	K	=	44625982.642 ft
k_o	=	0.999954142490	k_o	=	0.999954142490
M_o	=	6354407.2007 m	M_o	=	20847750.958 ft
r_o	=	6369336. m	r_o	=	20896729.860 ft
L_1	=	110905.3274	L_1	=	363861.8950
L_2	=	8.94188	L_2	=	29.3368
L_3	=	5.65087	L_3	=	18.5396
L_4	=	0.016171	L_4	=	0.053054
G_1	=	9.016699372E-06	G_1	=	2.748295465E-06
G_2	=	-6.55499E-15	G_2	=	-6.08981E-16
G_3	=	-3.73318E-20	G_3	=	-1.05713E-21
G_4	=	-8.2753E-28	G_4	=	-7.1424E-30
F_1	=	0.999954142490	F_1	=	0.999954142490
F_2	=	1.23251E-14	F_2	=	1.14504E-15
F_3	=	4.15E-22	F_3	=	1.18E-23

The customary limits of the zone are from 32° 10' N to 34° 30' N.

References

Alexander, I. A. and Alexander, R. J. (1989), "Projection Tables, California Coordinate System NAD83," California Land Surveyors Association Special Publication No. 55/88.

Burkholder, E. F. (1984), "Geometrical Parameters of the Geodetic Reference System 1980," *Surveying and Mapping*, Vol. 44, No. 4, pp. 339-340.

Fronczek, C. J. (1980), "Use of Calibration Base Lines," NOAA Technical Memorandum NOS NGS- 10, National Geodetic Information Center, NOAA, Rockville, MD 20852.

Meade, B. K. (1987), "Program for Computing Universal Mercator (UTM) Coordinates for Latitudes North or South and Longitudes East or West," *Surveying and Mapping*, Vol. 47, No. 1, pp. 37-49.

Stem, J. E. (1989), "State Plane Coordinate System of 1983," NOAA Manual NOS NGS 5, National Geodetic Information Center, NOAA, Rockville, MD 20852.

Stoughton, H. W. and Berry, R. M. (1985), "Simple Algorithms for Calculation of Scale Factors for Plane Coordinate Systems (1927 NAD and 1983 NAD)," *Surveying and Mapping*, Vol. 45, No. 3, pp. 247-259.

Vincenty, T. (1985), "Precise Determination of the Scale Factor from Lambert Conical Projection Coordinates," *Surveying and Mapping*, Vol. 45, No. 4, pp. 315-318.

Vincenty, T. (1986), "Geometric Reduction of Measured Lines," *Surveying and Mapping*, Vol. 46, No. 3, pp. 225-229.

Vincenty, T. (1986), "Use of Polynomial Coefficients in Conversions of Coordinates on the Lambert Conformal Conic Projection," *Surveying and Mapping*, Vol. 46, No. 1, pp. 15-18.

Vincenty, T. (1986), "Lambert Conformal Conic Projection: Arc-to-chord Correction," *Surveying and Mapping*, Vol. 46, No. 2, pp. 163-167.

Vincenty, T. (1986), "Application of the Chord Method to Solutions of Geodetic Lines," *Surveying and Mapping*, Vol. 46, No. 4, pp. 287-292.

State Plane Coordinate System of 1983 by James E. Stem is an excellent source of information concerning NAD83 procedures for the surveyor. It thoroughly deals with corrections to lengths and azimuths. It and other publications are available by phone. The National Geodetic Information Center number for phone orders is currently (301) 443-8631. They accept credit cards and have a listing of their publications which they will mail free of charge.

UNIT 12

U.S. PUBLIC LAND SURVEY SYSTEM

Jim McCavitt, PLS
Bureau of Land Management

Introduction

The United States Public Land Survey System (PLSS) began with the Land Ordinance of 1785. The ordinance provided for the systematic survey and monumentation of the federal lands into townships and sections. The system was largely put in place by 1805, with only minor refinements being made from time to time thereafter. The surveys in California were initiated in 1851, shortly after statehood.

The intent of the system was to survey and monument the western lands prior to patenting the lands to private citizens. The surveyed sections provided standard sized and shaped parcels of land, interrelated by reference to townships and initial points. Simple aliquot part descriptions, dividing sections into halves and fourths *ad infinitum,* were all that were necessary to describe with certainty a parcel of land.

The survey plats furnish the basic data relating to the survey and description of all areas within the particular township. All title records within the area of the former public domain are based on a government grant or patent, with the description referenced to the official plat. The plats are developed from the field notes, which are the written descriptive record of the survey. The survey plats and field notes are essential in determining the limits of PLSS parcels.

The effects of curvature and convergency on large scale surveys, such as the PLSS, can be dramatic. The PLSS has limited the effects of convergency with the use of standard parallels (or correction lines) at certain intervals. Several methods are available to correct for the effects of the curvature of lines.

The responsibility for the survey of federal lands currently rests with the Bureau of Land Management (BLM), which is the successor to the General Land Office (GLO). The *Manual of Surveying Instructions* (hereafter referred to as Manual) is issued by the BLM (and previously by the GLO) as a guide for BLM surveyors. The Manual describes how the public land surveys are made in conformance to statutory law and its judicial interpretation. The current edition of the Manual was issued in 1973. Previous editions of the Manual were issued in 1855, 1871, 1881, 1890, 1894, 1902, 1930 and 1947. The lines of a survey are established in accordance with the Manual in force on the date of the survey.

Understanding the PLSS framework of lines and corners is necessary for success on the land surveyor exams. The basic framework of PLSS lines consists of principal meridians, base lines, standard parallels, guide meridians, township lines and section lines. The corners established by the original surveys include standard, township, section, quarter-section, closing, and witness corners. It is important to know the significance of each PLSS line and corner in the scheme of the system.

Parcels of land in the PLSS, which cannot be described as aliquot parts of sections, are designated as lots or tracts. Lots are ordinarily located within a section. Tracts generally encompass an area in a township, lying in more than one section. The most common example of lots are along the north and west boundaries of a township, where the excess or deficiency in measurement was placed.

Sections ordinarily were not subdivided on the ground in the original surveys. The original survey plats show the plan and controlling measurements for the section subdivision survey. This plan may be superseded by supplemental or resurvey plats. Section subdivision surveys range from relatively simple divisions of regular sections into quarters to more complex situations involving fractional sections or arduous parenthetical distance determinations. To properly subdivide a section, it is important to be able to recognize unique situations and then apply the appropriate procedures.

As a professional land surveyor, any work you do on the PLSS will invariably involve dependently resurveying township or section lines. A dependent resurvey is first a retracement to identify original corners and other acceptable points of control. Second, it restores lost corners by proportionate measurement in accordance with the record of the original survey. The distinctions between existent corners, obliterated corners, and lost corners must be understood. An important point to remember is that the boundaries of sections based on an original survey are identical to the boundaries of the same sections based on a properly executed dependent resurvey.

To reestablish a lost corner position, it is important to know how restoration procedures are computed. Even more important, the professional surveyor must know how and when to use the various restoration procedures. The rules for the restoration of lost corners should not be applied until all original and collateral evidence has been developed and analyzed.

The PLSS topics and procedures discussed in the video, combined with this workbook, should provide a good foundation for the land surveyor examinations. Surveyors should not rely solely on this course in the PLSS studies for the exam. The pertinent portions of the 1973 Manual, such as Chapters 5 and 6, should be read and understood, along with other advised readings.

Performance Expected on the Exams

Given a survey plat, identify PLSS lines as to type.

Given a survey plat, identify the various PLSS corners and aliquot parts shown.

Explain the survey procedures necessary to subdivide a regular section or a fractional section.

Given a northerly or westerly section in a township, determine the parenthetical distances necessary to subdivide a section.

State the types of evidence that are acceptable for the restoration of an obliterated PLSS corner.

Choose the appropriate restoration procedure for a lost corner and compute the position to reestablish the corner.

Key Terms

Aliquot part	Patent
Parenthetical distance	Lot
Corner	Tract
Monument	Bearing tree
Existent corner	Standard parallel
Obliterated corner	Township
Lost corner	Section
Retracement	Meander corners
Dependent resurvey	Meander line
Standard corner	Subdivision of sections
Closing corner	Protraction
Witness corner	Proportionate measurement

Video Presentation Outline

History and Research

- Importance of Manual in effect at time of survey
- Availability of plats and notes

PLSS Datum

- Distance
- Direction
- Latitudinal lines (curved lines)

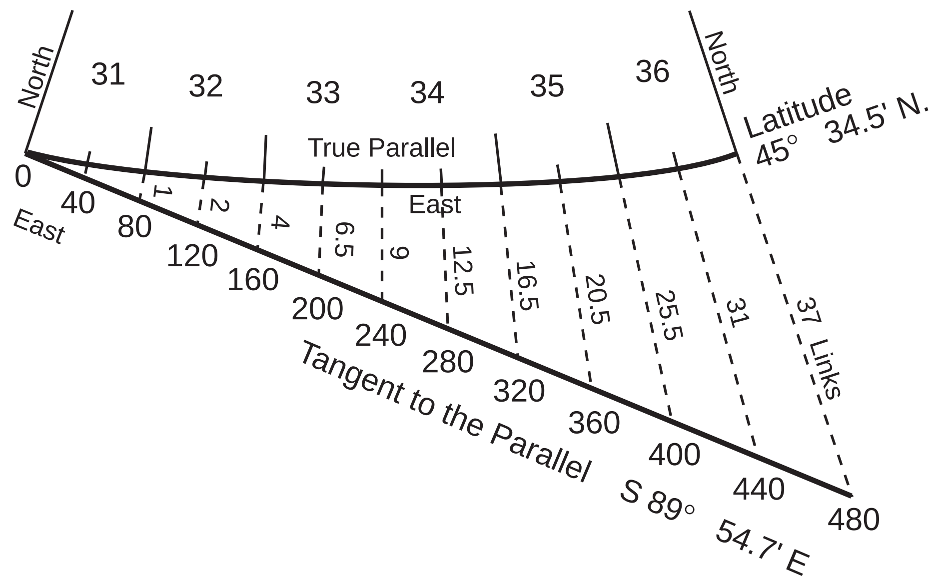


Figure 12-1. *Offsets to curved line of latitude.*

PLSS Framework

- Initial points
- Principal meridians and base lines
- Meridional lines and convergence

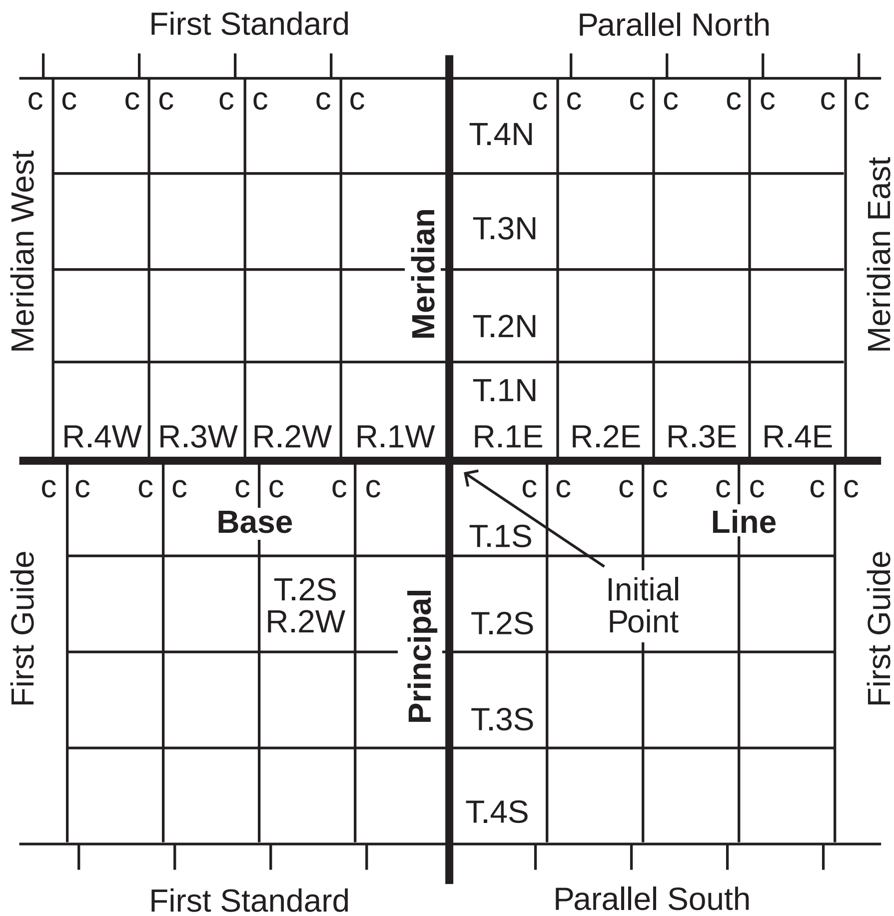


Figure 12-2. *The PLSS framework.*

- Township exteriors
- Subdivision of townships
- Differences between Manuals of 1851 and 1855
- Meander lines
- Monumentation

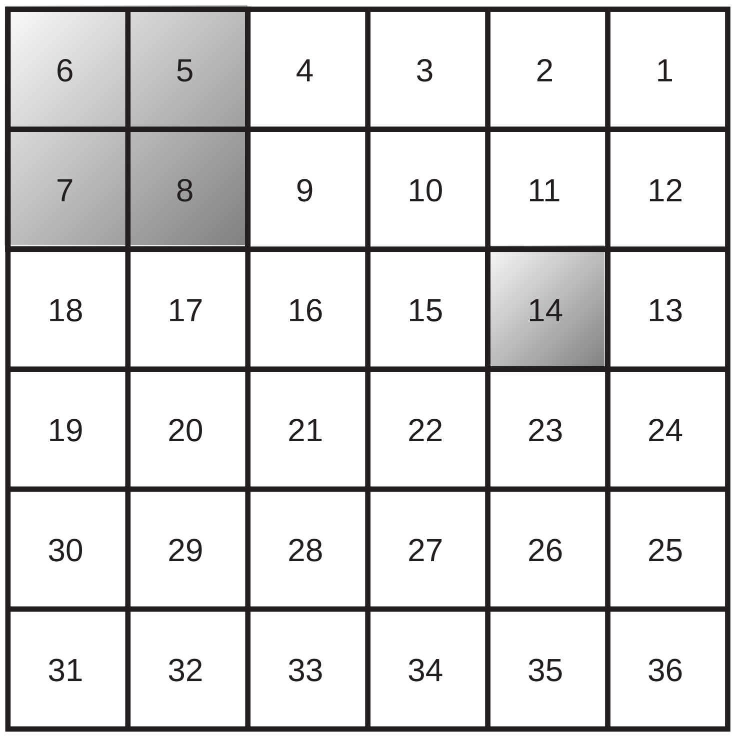


Figure 12-3. *Typical township.*

- Protracted subdivision of sections
- Aliquot parts

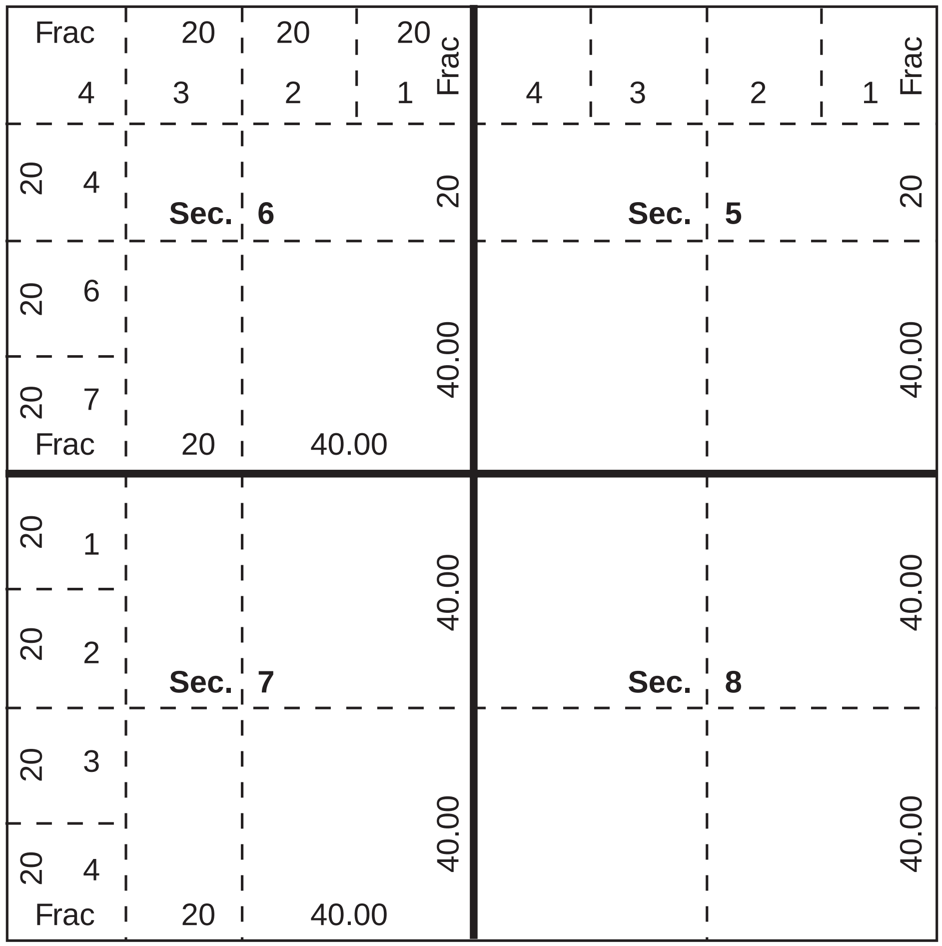


Figure 12-4. *Normal protracted subdivision of sections.*

Subdivision of Sections

- Center quarter-corner
- One sixteenth corners
- Parenthetical distances

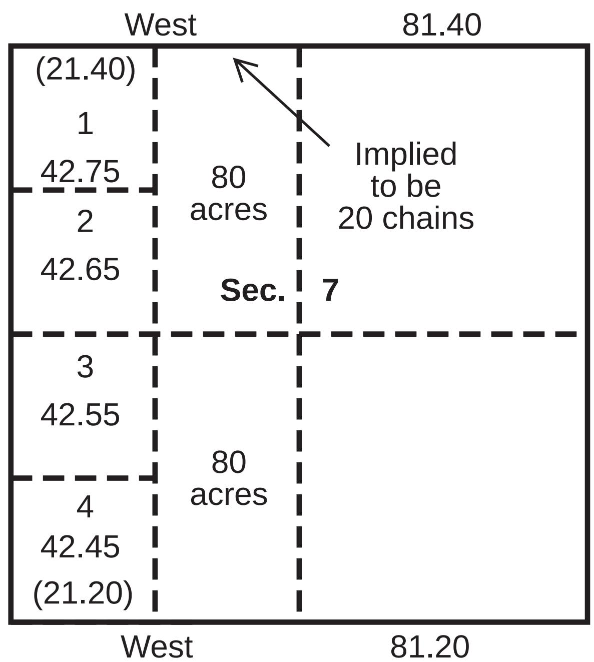


Figure 12-5. *Parenthetical distances, west tier of township.*

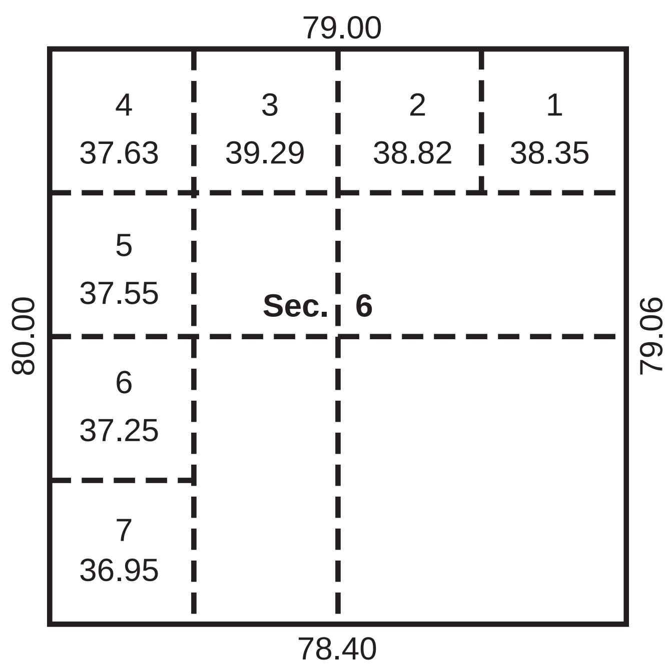


Figure 12-6. *Parenthetical distances, Section 6.*

Subdivision of Fractional Sections

- Running lines parallel to established courses
- Running lines on mean courses

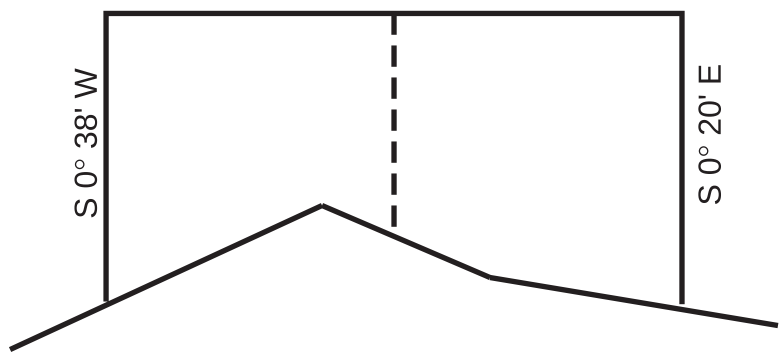


Figure 12-7. *Arithmetic mean.*

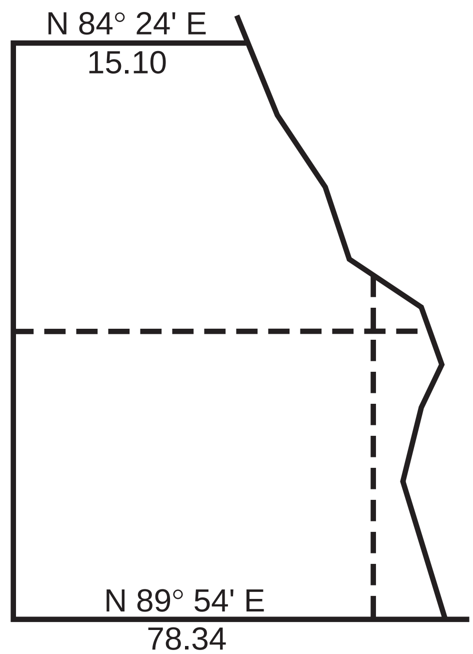


Figure 12-8. *Weighted mean.*

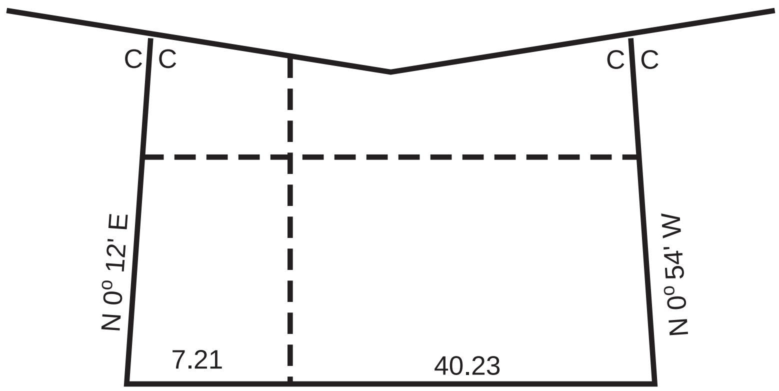


Figure 12-9. *Weighted mean of uncentered centerline.*

Dependent Resurveys

- Definitions
  - Monument
  - Corner
  - Existent corner
  - Obliterated corner
  - Lost corner
- Independent resurveys
- Topographic calls

Restoration of Lost Corners

- Proportionate measure
- Precedence of lines and corners
- Double proportion

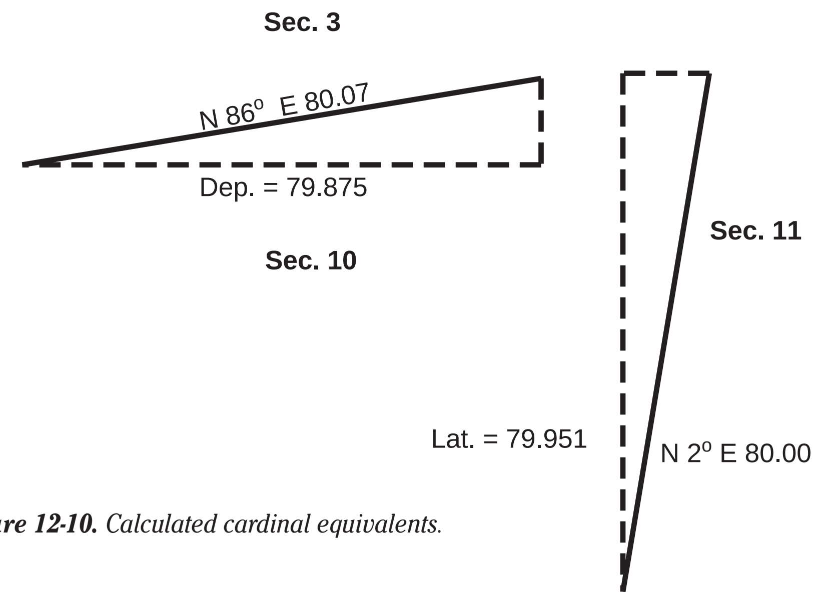


Figure 12-10. Calculated cardinal equivalents.

Calculating Coordinates

Calculate coordinates for restored corner. All dimensions are in chains, coordinates shown are per retracement.

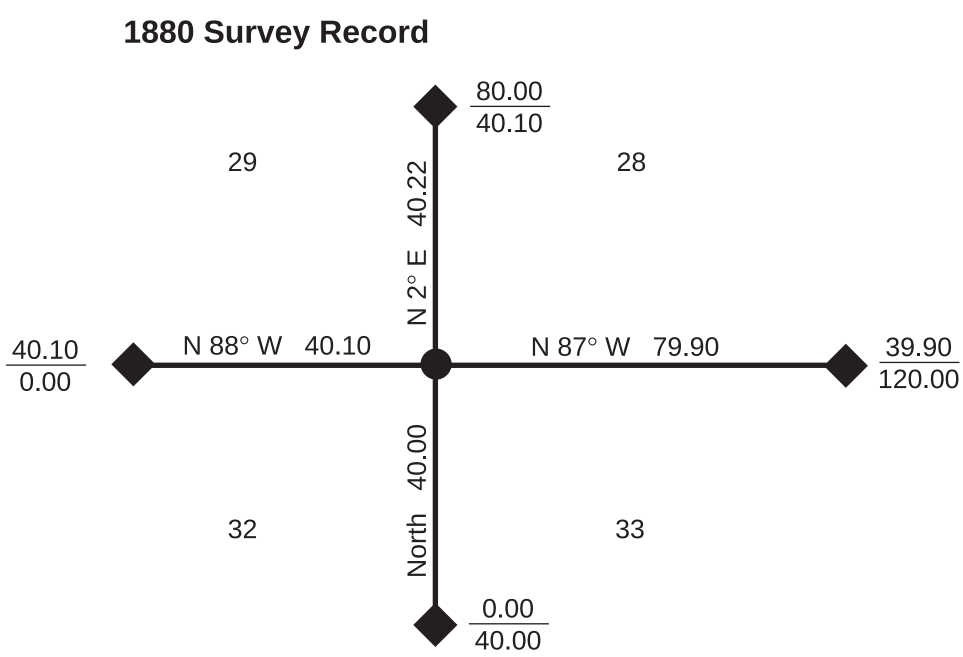


Figure 12-11. Found monuments, record, and field.

Calculate coordinates for restored corner. All dimensions are in chains.

- Single Proportion

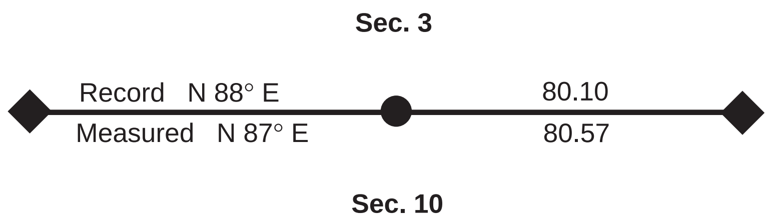


Figure 12-12. *Single proportion, record, and measured distances.*

- Proportionate measure along irregular boundaries
  3-point control
  2-point control
  1-point control
- Meander corners
- Closing corners

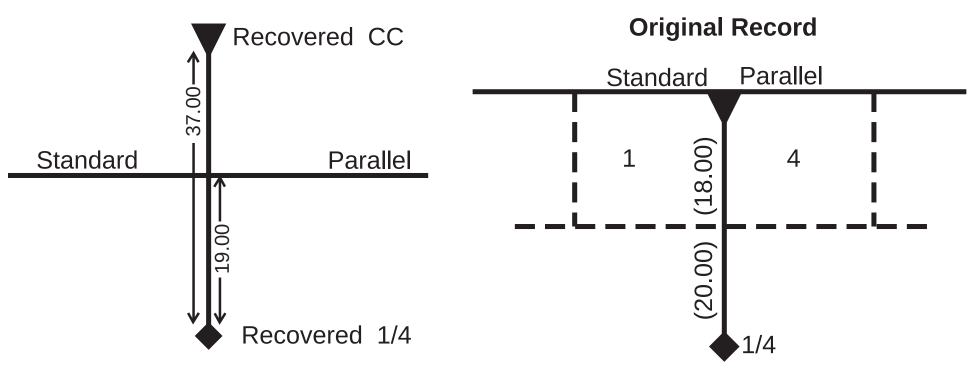


Figure 12-13. *Reset closing corner.*

- Witness corners as controlling monuments
- Line trees
- Double sets of corners

Example Problems

Problem A-5 1991 LS

You have been commissioned to survey fractional Section 8, T4S, R6W as shown on the official plat, which was approved on April 3, 1893. Your client has requested that all corners be monumented.

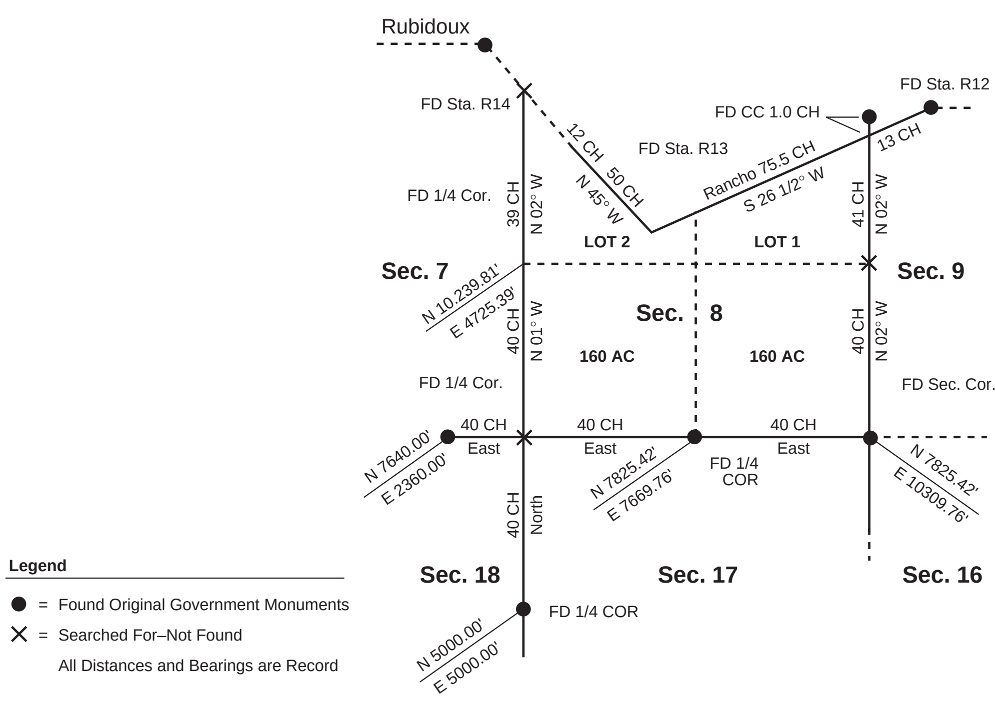
Required:

1. Identify the method and the positions and/or monuments you would hold for control to establish each of the corners denoted as "a" through "f" below. No calculations are required.
   A. Southwesterly section corner
   B. Northwesterly corner of government Lot 2
   C. North corner common to government Lots 1 and 2
   D. Northeasterly corner of government Lot 1
   E. East quarter-corner
   F. Center quarter-corner

2. Cite the governing reference that verifies the method of establishing the corners.

3. Calculate the coordinates for the southwesterly corner of Section 8. Show all work.

Plat Approved April 3, 1893



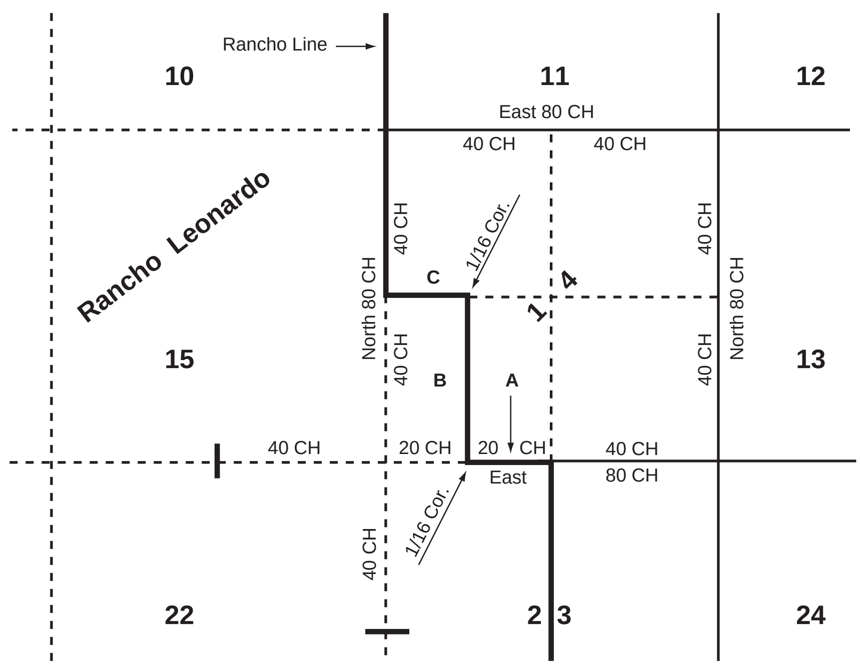
Problem A-1 1990 LS

Rancho Leonardo is shown in part on the plat dated 1860 below. The rancho follows sectional lines.

Required:

Using the coordinates indicated on the survey plat shown, calculate the bearings and distances of the rancho lines denoted as A, B, and C in Section 14. Show your answers directly on the survey plat. On the grid paper provided, identify your methods and show all your work.

Portion of 1860 Plat

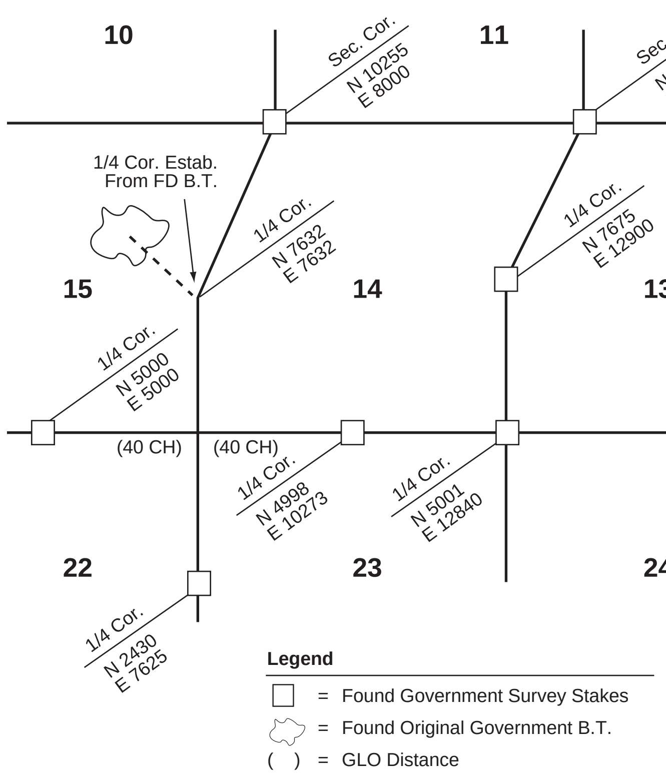


Legend

CH = Chains

Basis of Bearings Based on Solar Observation

Survey Plat



Solution:

Double proportion the southwest corner of Section 14:

Northing of proportioned position: $(\frac{40}{80})\ (7632 - 2430) + 2439 = 5031$

Easting of proportioned position: $(\frac{40}{80})\ (10273 - 5000) + 5000 = 7636.50$

Single proportion the north quarter-corner of Section 14

Northing of proportioned position: $\frac{(10300 - 10255)}{2} + 10255 = 10277.5$

Easting of proportioned position: $\frac{(13250 - 8000)}{2} + 8000 = 10625$

Establish center quarter of section by intersecting the lines between opposite quarter-corners:

N 7655.01; E 10450.15

Position of west 1/16 corner:

$$\text{Northing} = \frac{(5031 - 4998)}{2} + 4998 = 5014.5$$

$$\text{Easting} = \frac{(10273 - 7636.5)}{2} + 7636.5 = 8954.75$$

Position of center west 1/16 corner:

$$\text{Northing} = \frac{(7655.01 - 7632)}{2} + 7632 = 7643.50$$

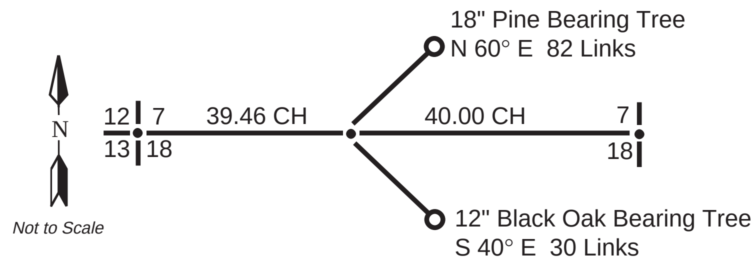
$$\text{Easting} = \frac{(10450.15 - 7632)}{2} + 7632 = 9041.08$$

Dimensions by inverse:

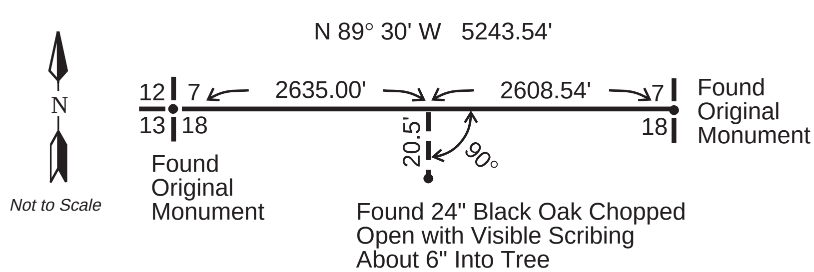
A. N 89° 16' 58" W 1318.35'
B. N 01° 52' 51" E 2630.42'
C. S 89° 31' 7" W 1409.13'

Problem B-3 1990 LS

Below is a sketch compiled from GLO plat and field notes dated July 26, 1879.



Below is a sketch showing the results of a field survey performed in January 1990.

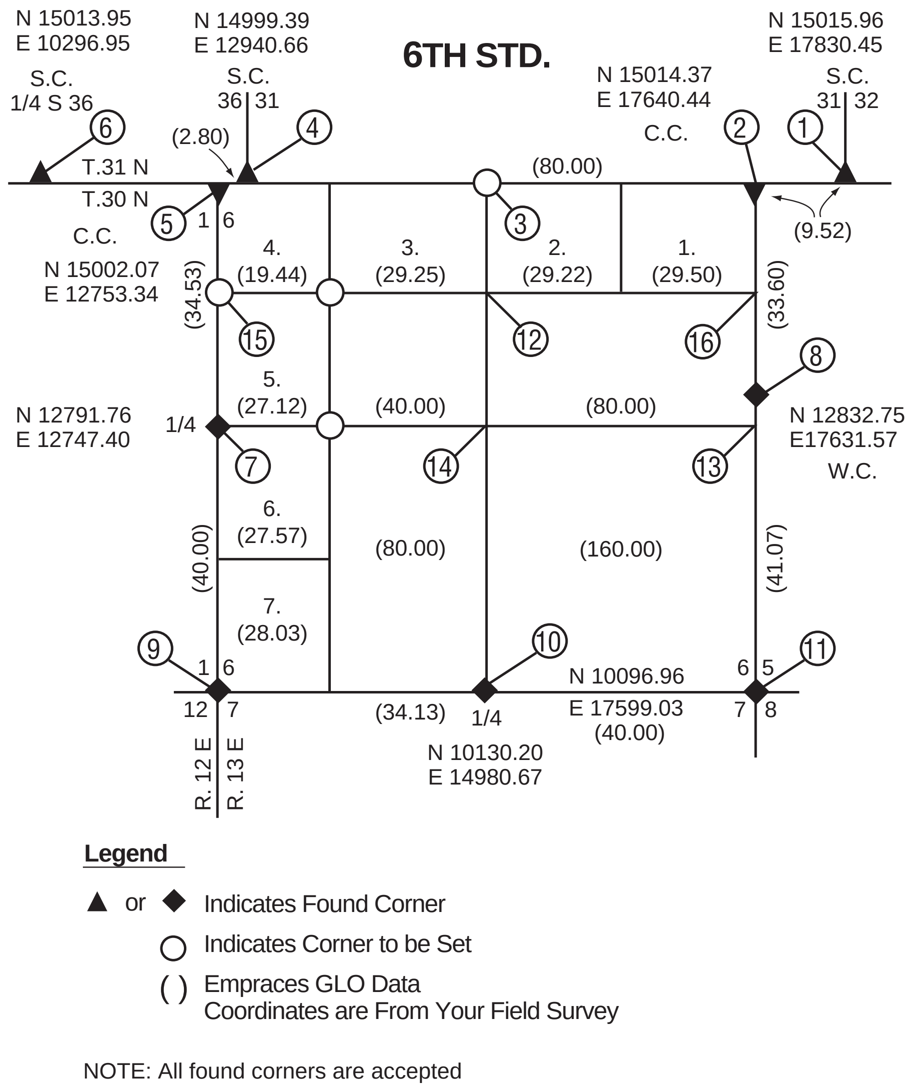


Required:

1. Describe how you would re-establish the missing quarter-corner monument position.
2. Assume that, in addition to the oak tree, you had found a blazed 18 in living pine tree without visible scribing near the location called in the notes for the 18 in pine bearing tree. How would you re-establish the missing quarter-corner monument position? Explain your answer.

Sample Test Questions

1. Problem A-4 1988 LS



Required:

Based on the previous diagram, answer the following multiple choice questions, A through I, by selecting the answer you believe to be correct from the choices given as indicated by the letters a, b, c, and d. *(Circle the correct answers.)*

A. What is the record length of the north line of Section 6, T 30 N, R 13 E?

1. 92.32 chs
2. 73.28 chs
3. 80.00 chs
4. 67.68 chs

B. How would you establish the northeast corner of Section 6, T 30 N, R 13 E?

1. Accept the found monument, as it is an original corner.
2. On line between Pt. 1 and Pt. 4, holding the distance of 9.52 chs from Pt. 1.
3. At the intersection of the line between Pt. 8 and Pt. 2 and the standard parallel.
4. On the standard parallel at the proportionate distance from Pt. 1.

C. How would you establish the north quarter-corner of Section 6, T 30 N, R 13 E?

1. Midway between Pt. 2 and Pt. 5.
2. On the standard parallel 40 chs from Pt. 2.
3. On the standard parallel and 40.42 chs from the northeast corner of Section 6.
4. On the standard parallel and 36.64 chs from the northeast corner of Section 6.

D. How would you establish the northwest corner of Section 6, T 30 N, R 13 E?

1. On line between Pt. 5 and Pt. 7 at 34.53 chs from Pt. 7.
2. On line between Pt. 4 and Pt. 6 at 2.80 chs from Pt. 4.
3. Accept the found monument, as it is an original corner.
4. None of the above.

E. The record length of the north line of Lot 4, Section 6, T 30 N, R 13 E is?

1. 20.00 chs
2. 19.44 chs
3. 17.2 chs
4. 13.28 chs

F. The record length of the west line of Lot 4, Section 6, T 30 N, R 13 E is?

1. 17.26
2. 14.53 chs
3. 20.00 chs
4. 19.44 chs

G. What are the record dimensions of Lot 7, Section 6, T 30 N, R 13 E?

1. 20 chs on all sides.
2. East and west lines 20 chs; north and south lines 17.06 chs.
3. East and west lines 20 chs; south line is 14.13 chs; north line is 13.90 chs.
4. East and west lines 20 chs; north and south lines 14.13 chs.

H. How would you establish the northwest corner of the southwest quarter of the northeast quarter of Section 6, 30 N, R 13 E?

1. Single proportion between Pt. 14 and Pt. 3.
2. At the intersection of lines 15-16 and 14-3.
3. On line between Pt. 14 and Pt. 3 and 20 chs from Pt. 3.
4. Single proportion between Pt. 15 and Pt. 16.

I. What are the ground coordinates of the east quarter-corner of Section 6, T 30 N, R 13 E as you would establish it based on measured values?

1. N 12762.13; E 17630.73
2. N 12832.75; E 17631.57
3. N 12761.47; E 17630.72
4. None of the above.

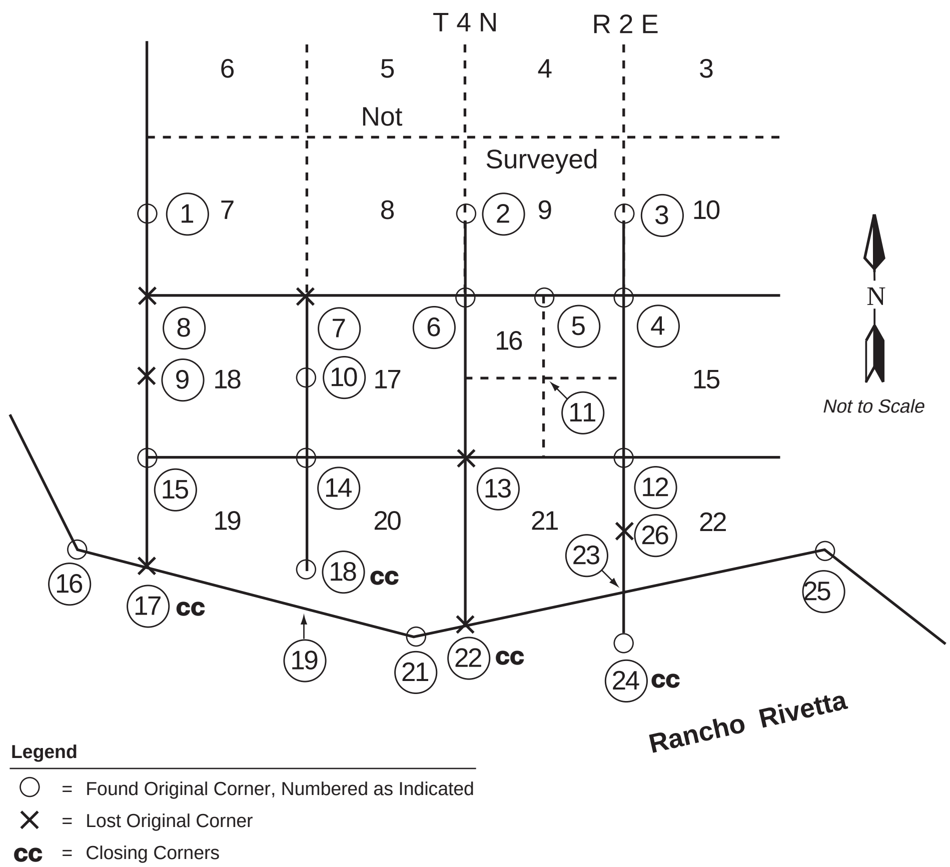
2. Problem A-3 1989 LS

The portion of a township plat shown below has been annotated to show which corners are lost and found at this date. Township 4 North, Range 2 East is bounded by regularly surveyed townships to the west, north and east, and by Rancho Rivetta to the south.

The found original corners are U.S. Government Survey monuments described in the notes of the survey. There are no topographical or accessory calls recovered; the lost original corners have been properly identified as such.

No excessive distortion was found in the record dimensions indicated in the plat and field notes for the rancho and township.

Exhibit Plat



Required:

A. Explain the procedure necessary to establish the corners in the following order: 8, 9, 17, 19, 23, 22, 26, 13, 11, 7.

B. As a licensed land surveyor, what would you be required to file to show the monumentation of:

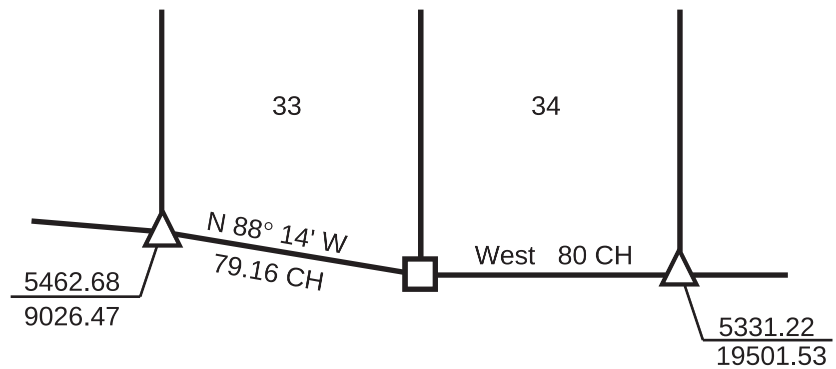
1. Corner 11
2. Corner 5

C. Describe or cite the definition of:

1. A lost corner
2. An obliterated corner

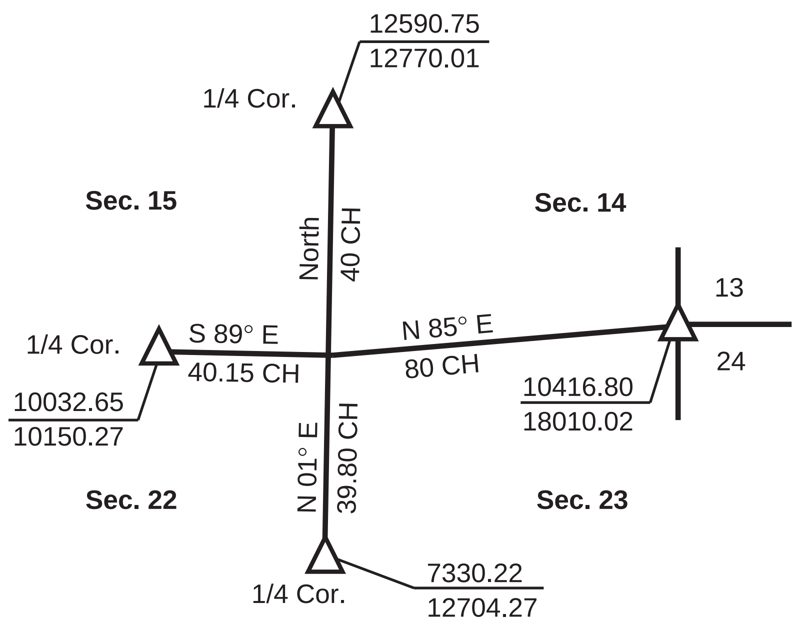
3. Your client has asked you to monument the west line of his property. The property is described as "the east one-half of Lot 2, Section 3, Township 33 North, Range 3 East, Mt. Diablo Meridian, as shown on the GLO plat dated March 5, 1888." How would you set the corners?

4. You are to set the southwest corner of Section 34 which is lost. The sketch shows the record dimensions per GLO. The south boundary of the township along Section 34 was surveyed in 1863. The completion survey of the south boundary of the township was completed in 1879 (random and true line) giving the record bearing and distance for the south line of Section 33 as shown. Coordinates shown are per your field survey. What are the coordinates of the southwest corner of Section 34?



5. If you find a record closing corner and set a new monument at its true position on the line closed upon, what, if anything, should be done to the original closing corner? What are your responsibilities according to the Land Surveyors Act?

6. Record GLO dimensions and coordinate values for field tied existent corners are shown on the sketch. The coordinates shown are for a local coordinate system based on the true meridian. What are the coordinates of the restored section corner common to Sections 14, 15, 22 and 23?



7. If the existent corners in problem 6 were tied from CCS control, how would the proportioning be carried out to ensure correct values to reestablish the corner of Sections 14, 15, 22 and 23?

Answer Key

1. A. 2.
   B. 3.
   C. 3.
   D. 4.
   E. 4.
   F. 2.
   G. 3.
   H. 2.
   I. 4.

2. A. **Corner 8** is reestablished by single proportionate measurement between corners 1 and 15 (5:30 and 5:25).

   Corner 9 is reestablished by single proportionate measurement between corners 1 and 15 (5:30).

   Corner 17 is established by single porportionate measurement between corners 16 and 21 (5:41).

   Corner 19 is established at the intersection of a line run southerly from corner 14 through corner 18, with the line between corner 16 and corner 21 (5:41).

   Corner 23 is established at the intersection of the line between corners 12 and 24, and the line between corners 21 and 25 (5:41).

   Corner 22 is established by single proportionate measurement between corners 21 and 25 (5:41).

   Corner 26 is reestablished on line at a proportionate measurement between corners 12 and 24 (5:41).

   Corner 13 is established by double proportionate measurement between corners 12 and 14, and corners 22 and 6 (5:29).

   Corner 11: The lost exterior quarter-corners of Section 16 are first reestablished by single proportioning between corners 6 and 13, 12 and 13, and 12 and 4. Corner 11 is then established by intersecting straight lines between opposite corresponding quarter-corners (3:87).

   Corner 7: The position in departure is determined by single proportioning between corners 6 and 8. (The record bearings for the north boundaries of Sections 17 and 18 may be different; if so, it may be necessary to use cardinal equivalents.) The position in latitude is determined at the record latitudinal distances from corner 10 (5:29).

   B. **Corner 11:** Record of Survey (8773A).

   Corner 5: Record of Survey or corner record (8773A).

C. "A lost corner is a point of a survey whose position cannot be determined, beyond reasonable doubt, either from traces of the original marks or from acceptable evidence or testimony that bears upon the original position, and whose location can be restored only by reference to one or more interdependent corners" (5:20).

"An obliterated corner is one at whose point there are no remaining traces of the monument or its accessories, but whose location has been perpetuated, or the point for which, may be recovered beyond reasonable doubt by the acts and testimony of the interested landowners, competent surveyors, other qualified local authorities, or witnesses, or by some acceptable record evidence" (5:19).

References are to *Professional Land Surveyors Act/Board Rules and Manual of Instructions,* 1973.

3. Federal rules would apply in this case because of the reference to the GLO plat. The west line is the line between the midpoints of the north and south lines of the lot.

4. Cardinal equivalent departure for south line Section 33:

Cos 1° 46' x 79.16 ch = 79.1224 ch

Proportional departure along south line of Secion 34:

$$\frac{80\ \text{ch}}{159.1224\ \text{ch}} = \frac{d}{10{,}475.06\ \text{ft}}$$

d = 5266.423 ft

Easting of southwest corner of Section 34:

19501.53 - 5266.42 = 14,235.11

Cardinal equivalent of record latitude:

sin 1° 46' x 79.16 ch = 2.4404 ch

Difference in latitude from field to record:

131.46 ft - 2.4404 (66) = -29.61 ft

Proportion difference in latitude for south line Section 34:

$$\frac{80\ \text{ch}}{159.16\ \text{ch}} = \frac{d}{-29.61}$$

L = - 14.88 ft

Northing of southwest corner of Section 34:

5331.22 - 14.88 = 5316.34

5. The original closing corner should be marked "AM." A Corner Record must be filed unless the new closing corner will be shown on a record map (Record of Survey, Parcel Map or Final Map).

6. Record cardinal equivalent departure from south quarter of Section 15 to lost corner:

   sin 89° (40.15) = 40.1439 ch

   Record cardinal equivalent departure from lost corner to the southeast corner of Section 14:

   sin 85° (80) = 79.6956 ch

   Departure from south quarter-corner of Section 15 to proportioned position of lost corner:

   $$\frac{40.1439 \text{ ch}}{119.8395 \text{ ch}} = \frac{d}{7859.75 \text{ ft}}$$

   d = 2632.86

   Easting of proportioned position:

   10150.27 + 2632.86 = 12783.13

   Record of cardinal equivalent latitude from west quarter-corner of Section 23 to lost corner:

   cos 01° (39.80) = 39.7939 ch

   Latitude from west quarter-corner Section 23 to proportioned position of lost corner:

   $$\frac{39.7939 \text{ ch}}{79.7939 \text{ ch}} = \frac{L}{5260.53}$$

   L = 2623.47 ft

   Northing of proportioned position:

   7330.22 + 2623.47 = 9953.69 ft

7. Before the latitudes and departures can be proportioned, the field ties and record information must be on the same basis of bearings. In this case field ties are based on the California Coordinate System and the original record information is astronomic. Cardinal equivalent (astronomic) latitudes and departures will differ from the latitudes and departures based on CCS grid north.

   One way to ensure correct values is to rotate the grid bearings to an astronomic basis and then proceed to proportion the cardinal equivalent latitudes and departures. After the proportioning is complete and bearings and distances have been computed to the reestablished corner, the bearings can be rotated back to a grid basis.

References

______, *Manual of Instructions for the Survey of the Public Lands of the United States*, Bureau of Land Management, 1973.

______, *Restoration of Lost or Obliterated Corners & Subdivision of Sections*, Bureau of Land Management, 1974.

Brown, Curtis, *Boundary Control and Legal Principles*, Third Edition, John Wiley & Sons, New York, 1988.

Robillard and Bouman, *Clark on Surveying and Boundaries*, Fifth Edition, Michie Co., Charlottesville, VA.

White, C. Albert, *A History of the Rectangular Survey System*, U.S. Government Printing Office, Washington, D.C. 20402, Stock Number 024-011-00150-6.

UNIT 13

PRINCIPLES OF BOUNDARY DETERMINATION

Robert Lee McComb, PLS
Julian, CA

Introduction

Boundary determination accounts for a major portion of the California LS exam and is being emphasized more on the LSIT. Examinees must be able to solve problems that require knowledge of the U.S. Public Land Survey System, knowledge of sequence and simultaneous conveyances, and knowledge of conflicting elements in deeds and rules governing the relative value of different pieces of evidence.

There are a number of good sources of information on boundary principles, several of which are listed in the reference section of this unit. It is a good idea to become familiar with a number of books, then choose the one that is most understandable, studying it thoroughly so that it can be used as a quick reference. Knowing one good source thoroughly is almost always better than having a passing knowledge of several.

This unit is organized around a fictional survey. The survey crew's mistakes and discoveries will serve to point out many boundary principles, but more than that, they will show the importance of research, planning, and organization in solving surveying problems.

Performance Expected on the Exams

Determine property boundaries based on the U.S. Public Land Survey System.

Establish boundaries along waterways.

Apply the rules for boundary determination in simultaneous conveyances.

Apply the rules for boundary determination between the holders of junior and senior rights.

Establish boundaries based on interpretation of legal descriptions.

Identify and resolve conflicting deed calls in legal descriptions.

Key Terms

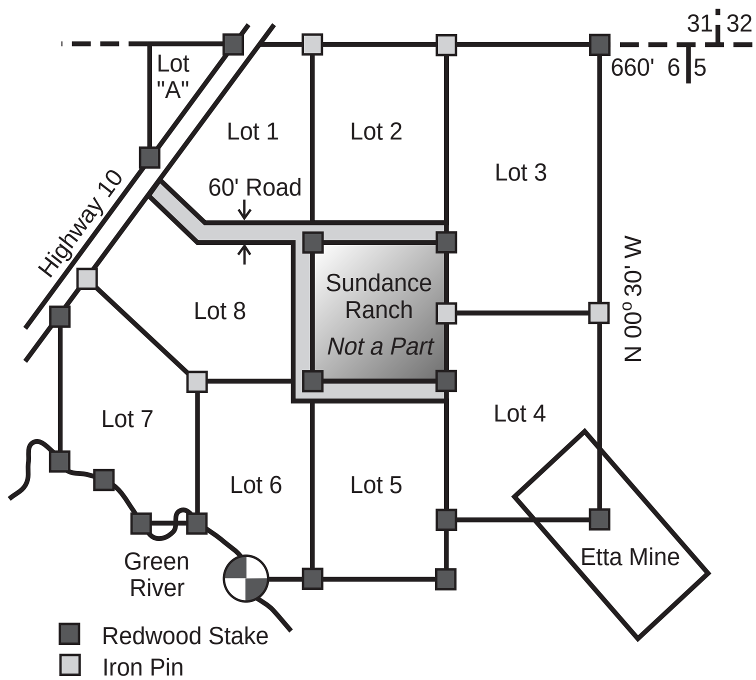
Subdivision	Proration
Senior rights	Monument
Natural monument	Record monument
Artificial monument	Accessories
Corner	Closing corner
Call	Deed
Intent of deed	Fee simple
Easement	Appurtenant
Basis of bearings	Common law
Statute law	Course
Encroachment	Exception
Reservation	Prescriptive right
Evidence	Parole evidence

Video Presentation Outline

The surveyor in our story has been hired by the Sundance Ranch Partnership to survey their land, locally known as the Sundance Ranch, together with the road that serves the property.

The following maps and descriptions will be used by the field crew as they locate the boundaries of the ranch.

Parker's Subdivision



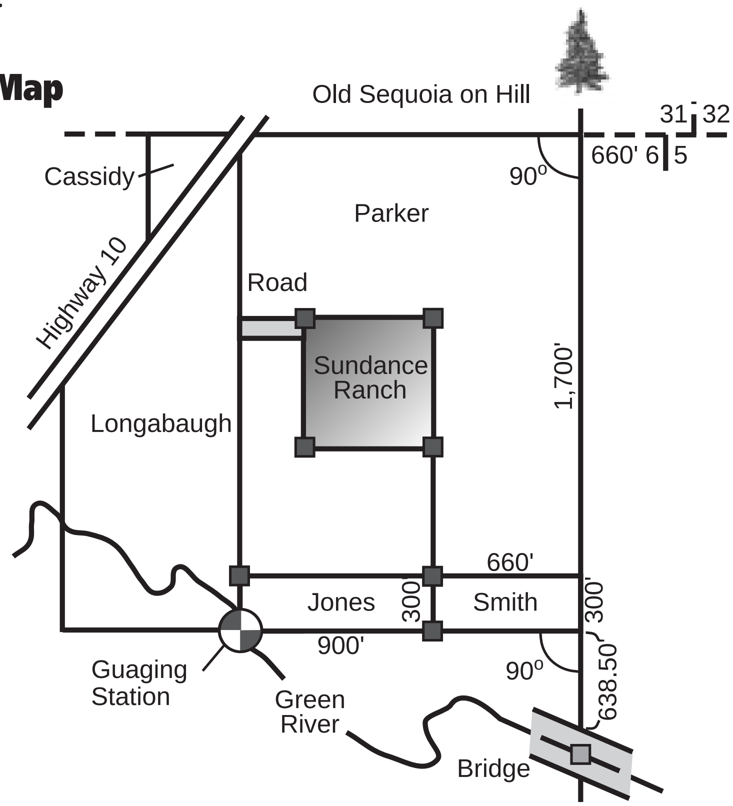
Sundance Ranch Description

The northeasterly 1,250 ft of Parker's land in Section 6, excepting the north 700 ft and the east 660 ft, containing an area of 8.45 acres.

Parker's Description

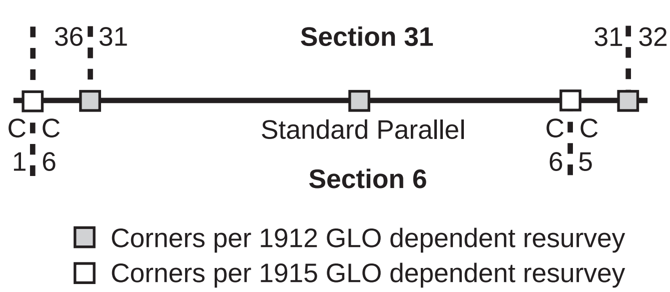
Beginning at the old sequoia on the hill north of town; thence toward the center of the bridge crossing the river to an intersection with the north line of said Section 6 and the True Point of Beginning; thence continuing toward said bridge 1,700 ft; thence south 89° 30' west 660 ft, thence south 00° 30' east 300 ft; thence south 89° 30' west 900 ft to Logan's gauging station; thence along the top of the left bank of the river north 46° 30' west 200 ft; thence south 89° 30' west to the cliff road; thence along the cliff road to the west line of the easterly 400 ft of the west half of Section 6, being the southeast corner of land conveyed to Ketchum by deed in the year 1900; thence northerly along Ketchum's east line to Highway 10; thence along said Highway 10 to the north-south centerline of Section 6; thence northerly along side north-south centerline to the north quarter-corner of Section 6; thence easterly to the True Point of Beginning.

L. Baltimore Map



Survey of a portion of Section 6 prepared by L. Baltimore, 1905.

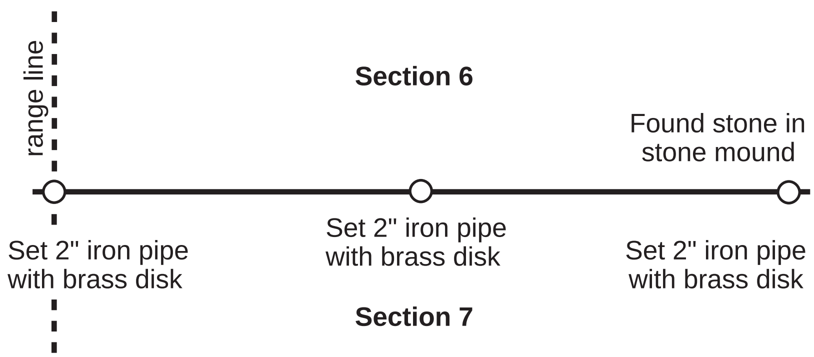
1912 and 1915 Dependent Resurveys of the Standard Parallel



Cassidy's Description

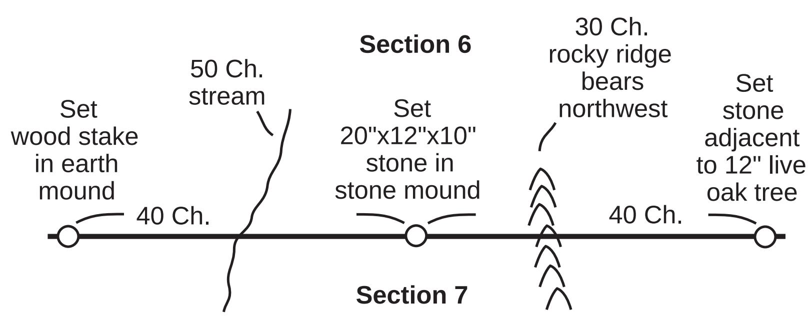
All of Section 6 except the east one-eight. Also excepting the west one-half. Also excepting the south one-half.

1915 Dependent Resurvey of South Line of Section 6



Corners per 1915 GLO dependent resurvey

1870 Original Survey of South Line of Section 6



Corners per 1870 GLO dependent resurvey topo calls from original notes

Sample Test Questions

1. Use the following chain of title and the sketch showing bearings obtained from a field survey to answer the following questions.

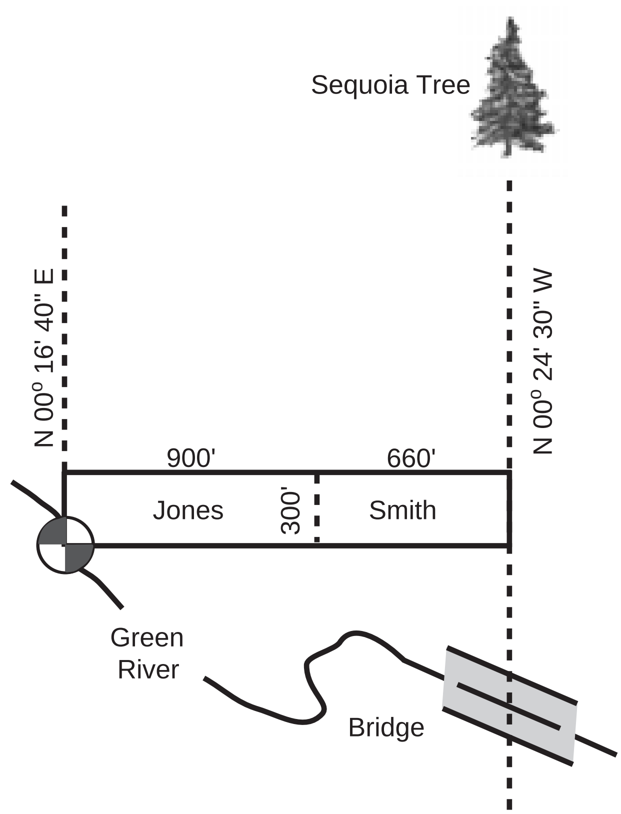
   1895 - Harriman deeds to Woodcock:
   "The southerly 300 ft of the northerly 2,000 ft of the easterly 2,220 ft of Section 6. EXCEPTING that portion lying easterly of a line drawn from the old sequoia to the center of the bridge."

   1898 - Woodcock wills to his heirs upon his death:
   To Jones: "The westerly 900 ft of Woodcock land."
   To Smith: "The easterly 660 ft of Woodcock land."

   A. Who has senior title?

   B. How should the line common to Smith and Jones be reestablished?

   C. If the conveyances were worded, "the westerly 6.20 acres" and "the easterly "4.55 acres," how would reestablishment of the common line be done?



2. From the following chain of title, how many acres did Kellerman receive in 1898?

   1895 - Harriman deeds to McCarty:
   "The westerly 820 ft of the northerly 2,000 ft of the easterly 3,040 ft of Section 6."

   1896 - McCarty deeds to Logan:
   "The south 8 acres of the westerly 820 ft of the northerly 2,000 ft of the easterly 3,040 ft of Section 6."

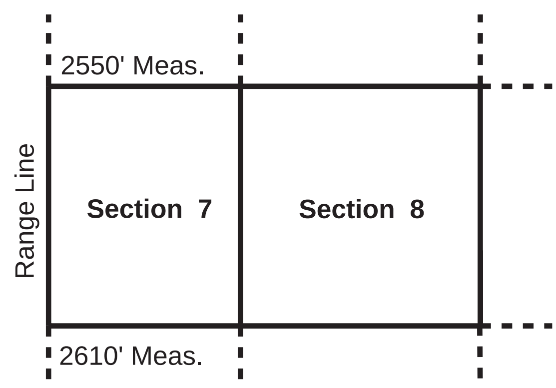
   1897 - McCarty deeds to Place:
   "The west 10 acres of the westerly 820 ft of the northerly 2,000 ft of the easterly 3,040 ft of Section 6. EXCEPTING the south 8 acres."

   1898 - McCarty deeds to Kellerman:
   "The northerly 25 acres of the westerly 820 ft of the northerly 2,000 ft of the easterly 3,040 ft of Section 6, EXCEPTING the west 10 acres and the south 8 acres."

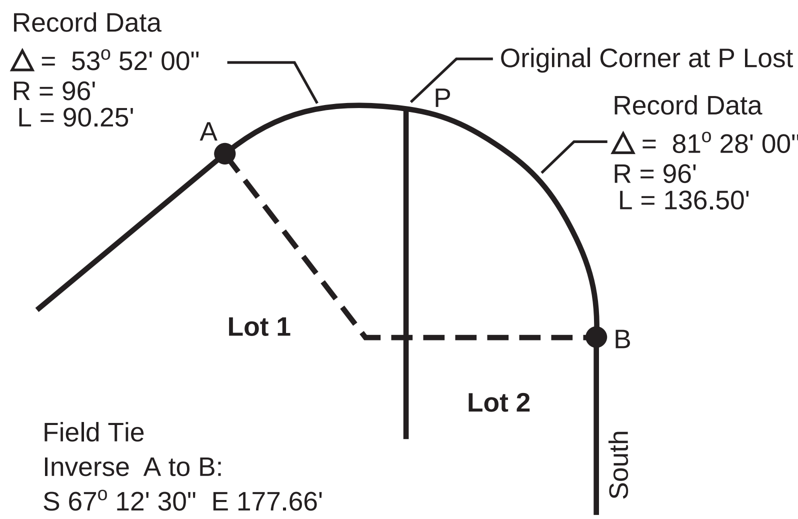
3. In the video presentation, the question was raised concerning the validity of proportioning the monuments, apparently lost, along the east line of the Sundance Ranch as part of the subdivision map.

   Should the Sundance Ranch, which is shown as "not a part" on the subdivision map (see map in Video Outline) bear a proportional excess or deficiency with other lots in the subdivision, such as Lots 2 and 5?

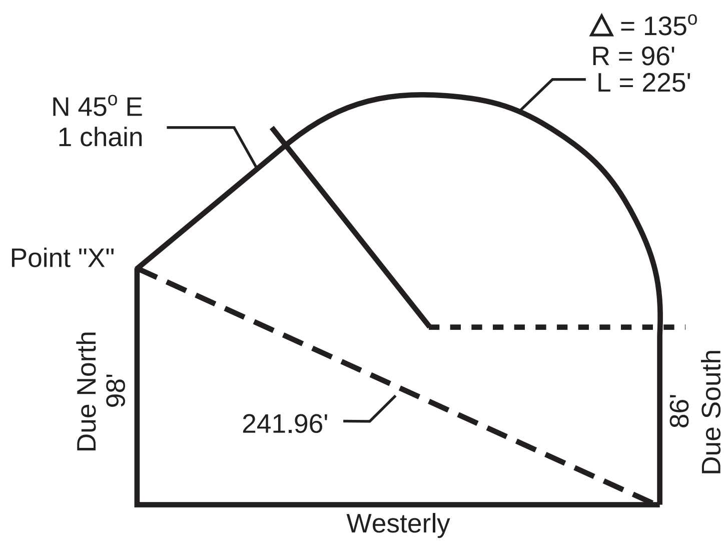
4. In Section 7 shown below, explain how it is possible for the west half of the Section to be nonexistent.



5. Lost corners, originally set from a single point along a bearing and distance, are restored by what method?
6. Based on the information shown below, what is the curve data for Lot 1? Original monuments at A and B are existent.



7. Is there enough information given in the following sketch to write a nonambiguous legal description for the figure given, provided *all* the information is used, excepting that which can be disproved? Identify the two errors in the drawing and reflect them in a correct legal description.



8. Sketch the following easements across Section 6 shown below.

   "The north 200 ft and the east 100 ft of the south 1600 ft of the north 1800 ft of Section 6."

Can you write a better legal description for the above set of easements?

9. Problem B-1 1990 LS

You have been asked to survey and monument Lot 3 of the XYZ Subdivision in Rainbow County, California shown on page 13-10. Lot A of the ABC Subdivision was conveyed in August 1933. Lot B of the ABC Subdivision was conveyed to a different party in June 1936.

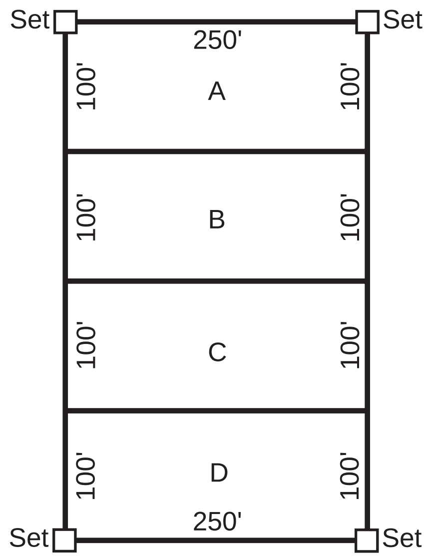
A. Given only the information shown on the plats below, indicate the lengths of the sides w, x, y, and z of Lot 3 on your field survey.

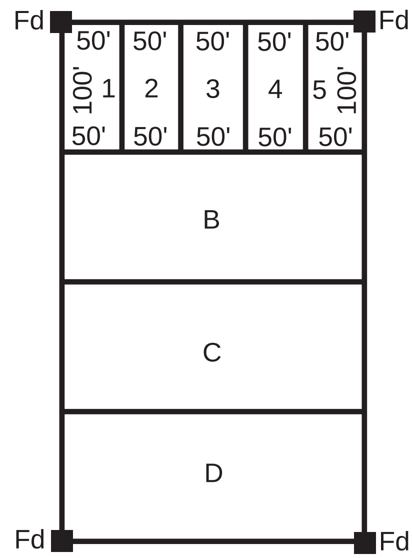
B. Using only the information given in the problem statement above and in the plats shown below, write a legal description for Lot B.

C. What type of document(s) will you be required by law to prepare?

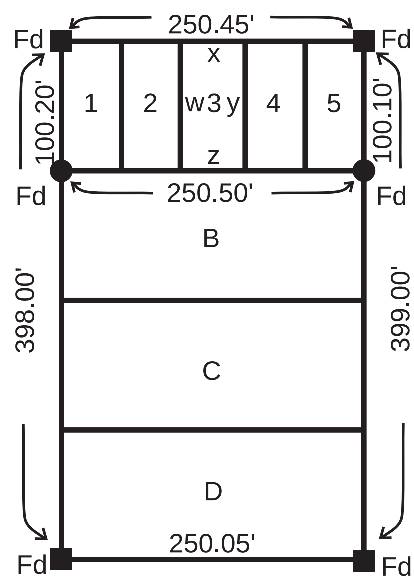
XYZ Subdivision
Filed in Book 7
of Maps at Page 73
in May 1946
(a Division of Lot A
of ABC Subdivision)

ABC Subdivision
Filed in Book 3
of Maps at Page 25
in July 1932





Your Field Survey
Prepared in
April 1990



Legend

= Set 3" x 3" Redwood Post with Nail

= Found 3" x 3" Redwood Post with Nail (Original)

= Found 1" Iron Pipe (Origin Unknown)

10. Problem A-3 1990 LS

In June 1969, Meyers conveyed to your client the west one-half of Lot 5 of Rainbow Acres. In December 1970, Meyers sold a portion of the remainder of Lot 5 to Landis with the following legal description:

> “The southerly 100.00 ft of that portion of Lot 5 of Rainbow Acres, in the County of Rainbow, in the State of California, as per map recorded April 16, 1954 in Book 3 of Maps, Page 3, in the Office of the County Recorder of said county described as follows:
>
> Beginning at the southeast corner of Lot 5; thence west along the southerly line thereof 100.00 ft to a 1-in iron pipe; thence north parallel with the easterly line of said Lot, 317.60 ft to a 1-in iron pipe on the northerly line of said lot; thence north 84° 04' 00" east along said northerly line 100.54 ft to the northeast corner thereof; thence south, 327.90 ft to the point of beginning.”

You have been asked to survey and monument your client’s parcel of land. The record values of Lot 5, as well as the results of your boundary survey, are shown in the plat on the following page.

Required:

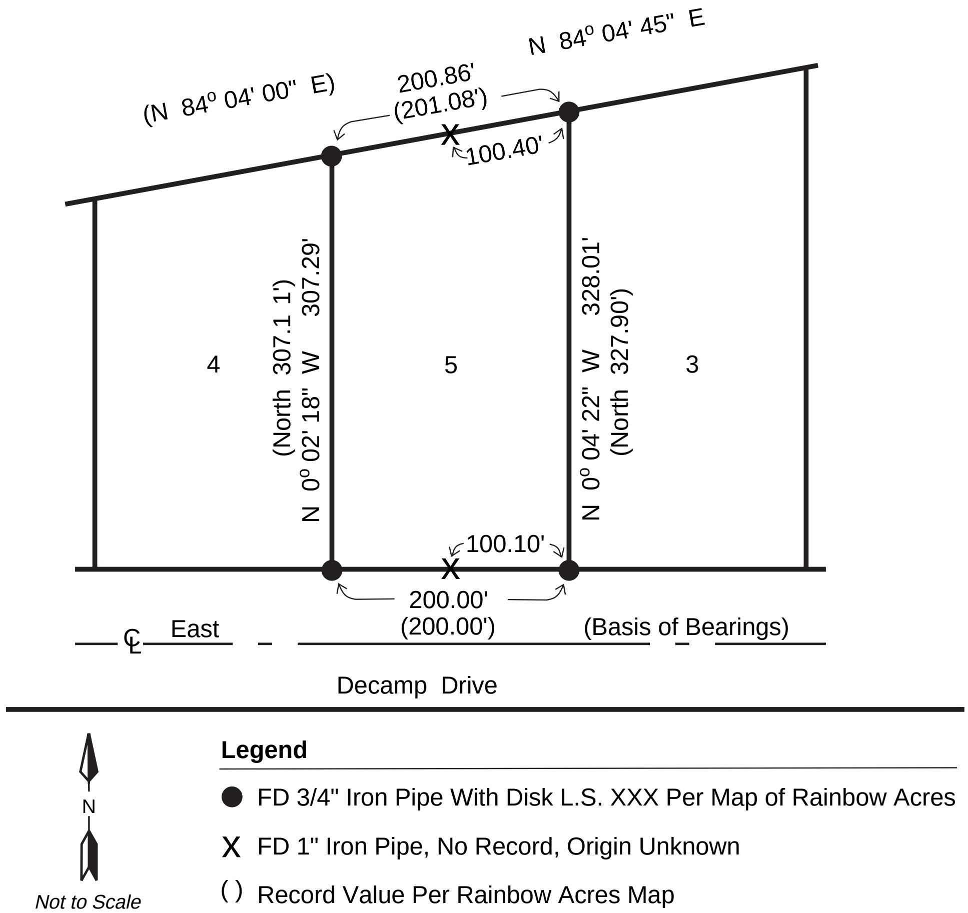
A. Describe how you would determine the boundaries of your client’s property. (Calculations are not required.)

B. Describe the effect of the Landis deed on your client’s property.

C. Prepare a legal description for the remainder of Meyers’ property. (Do not use a metes and bounds description.)

E. Is the filing of a Corner Record sufficient documentation of your survey? Explain your answer and cite references.

RAINBOW ACRES
Recorded in Book 3, page 3 of Maps, County of Rainbow



Answer Key

Two challenges faced by surveyors are to assert a confident solution to real-world problems, and to assume the liability therefor. Reliance upon texts, or popular interpretations of court cases, or the opinions generated by helpful colleagues cannot be used to indemnify one from the weighty responsibility of professional judgment. So it is with these problem solutions. The answers do not purport to be exact solutions, and are subject to other valid solutions and future legal opinions which may spring into being and suddenly apply.

1. A. Smith and Jones were willed land at the same time. Because they received simultaneous conveyances, neither Smith nor Jones enjoys seniority.

   B. The initial task is to survey Woodcock's original land as described in his 1895 deed from Harriman. Next, the footages provided in Woodcock's will of 1898 are compared to this survey and any measured excess or deficiency proportionally applied to the 900 ft and 660 ft calls in the will. The correction to be applied to record demensions is determined independently for the north and south lines of Smith and Jones and is linear, as with subdivision lots. If the excess, or deficiency, is denoted by "e," then the overall length of the line as measured would be a + b + e. The overall record length is, of course, a + b (900 + 660), so the ratio (a + b + e)/(a + b) would be applied as a multiplier, the so-called factor of proration, to the 900 ft and 660 ft lengths to be prorated along the line.

   C. Linear proportion applies when either area or footage are stated. When area is stated, it is not sufficient to merely fix the common line between Smith and Jones by a linear ratio, but another rule is necessarily added to control the direction of the common line. The rule of mean bearing may be considered here, but that rule usually applies to equal subdivisions of area. In this case, it makes sense to proportion the bearing to reflect the prescribed ratio of areas. Since the east bearing would be rotated 0° 41' 10" to become parallel to the west boundary, the proportional line will rotate 4.55/10.75 parts from the east line, i.e.:

   0° 41' 10" (4.55/10.75) + N 0° 24' 30" W = N 0° 07' 05" W.

   Another solution would be to make the common line bear astronomic north. The rationale for this is strengthened by the fact that the east and west lines, as stated in Woodcock's deed, are presumed nonparallel, and no basis of bearings is contained in the deed wording.

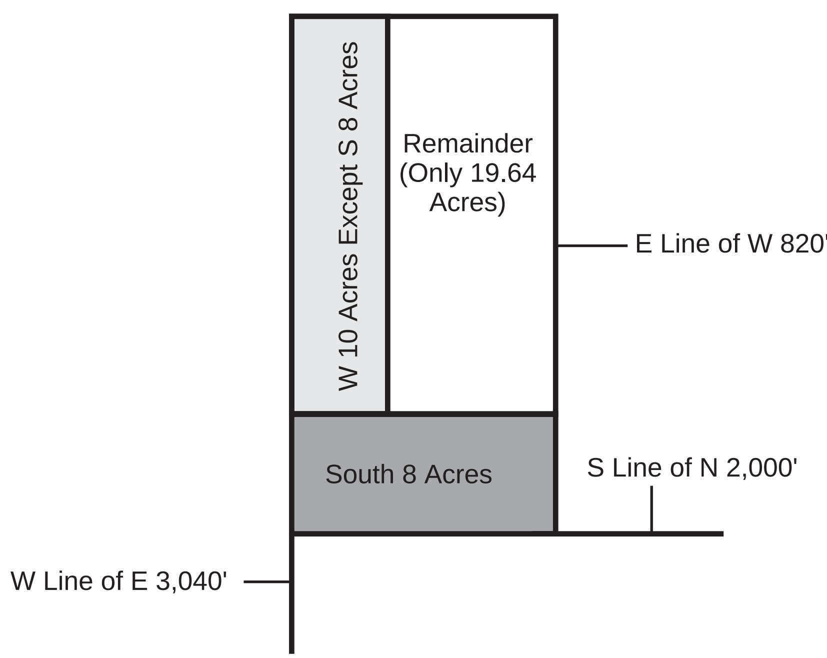
   The actual areas of Smith and Jones will be equal to their record area times the total surveyed area divided by the record total area.

2. The deeds are sequential because they occurred at different times. The earliest deed, Harriman to McCarty, in 1895, is senior to the others, the bottom of the hierarchy ending with the most junior deed, McCarty to Kellerman, in 1898.

   The 1896 deed from McCarty to Logan is straightforward. Next comes McCarty's deed to Place, the description of which raises a strong objection. In the first sentence 10 acres is clearly conveyed, but the exception in the second sentence then seems to subtract 8 acres from the grant. That would leave only 2 acres to the grantee, an unlikely intent, particularly since we have the earlier 1896 deed to Logan which establishes a clear meaning to the phrase, "the south 8 acres".

   A surveyor may not interpret those terms of a deed suggesting intent, but it is permitted for a surveyor to demonstrate that alternate meanings within the wording exist. A court might interpret the given situation in light of the principle that grants are to be interpreted in favor of the grantee in cases where two meanings are possible, which would give Place a full 10 acres.

   Following this reasoning, the 1898 deed to Kellerman was intended to grant 25 acres. Unfortunately, only 19.64 acres remained after the previous two grants. As junior in sequence, the 1898 deed receives only what is left.



3. The confusion here, if there is any, probably stems from the proper interpretation of the situation in terms of either simultaneous or sequential conveyances. The Sundance Ranch is senior to the subdivision and not a part of the subdivision. Its position should be reestablished according to its deed as a sequential conveyance before the time of the subdivision. Proration would not be expected since there are no simultaneous lines to infer proportional distribution.

4. It is possible, and fairly common, for the original government surveys to have accumulated large measurement errors. Because all of the error in measurements east and west in a township ends up in the last half mile of the north and south lines of Section 7, accumulated errors of a half mile can result in elimination of the west half of the section. If the error is in favor of the section, the west half can theoretically be at least as large as an entire section.

5. Reestablishment is by record bearing and distance, using an astronomical basis of bearings. This is true if the original field survey was based upon an astronomical reference bearing and if there is no other evidence available to retrace the original survey.

6. This curve solution follows the rules of simultaneous conveyances for subdivision lots. As subdivision lots, linear proration is used to distribute any excess or deficiency. One solution to this problem is to proportion the radius and length of curve based on the relationship between the field tie inverse between A and B and the record calculated inverse. This method preserves the original delta of the curve.

$$\frac{R}{96\text{ ft}} = \frac{177.66\text{ ft}}{177.60\text{ ft}}$$

$$R = 96.03\text{ ft}$$

$$\frac{L}{90.25\text{ ft}} = \frac{177.66\text{ ft}}{177.60\text{ ft}}$$

$$L = 90.28\text{ ft}$$

An alternative solution will proportion the frontage along the curve but preserve the original even footed radius for the curve. Start by calculating the delta for the curve from A to B from the inverse distance from A to B and a radius of 96 ft.

$$LC = 2R \sin\frac{\Delta}{2}$$

$$177.66 = 2\,(96)\sin\frac{\Delta}{2}$$

$$\Delta = 135^\circ\ 25'\ 52''$$

$$\frac{53^\circ\ 52'\ 00''}{\Delta} = \frac{135^\circ\ 20'\ 00''}{135^\circ\ 25'\ 52''}$$

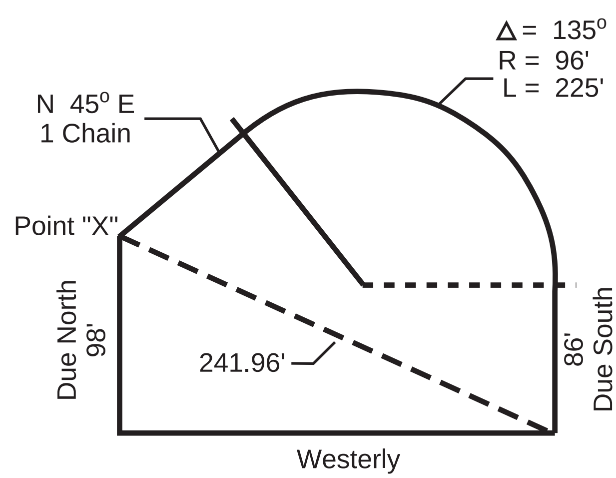
$$\Delta = 53^\circ\ 54'\ 20''$$

$$L = \frac{\Delta}{360}\ 2\,\pi\,R$$

$$L = 90.32$$

7. Let's begin this problem by a straightforward plunge into a legal description. Let's assume that everything on the drawing is correct for now. We will also impose the rule that everything on the drawing *must* be used to generate the legal description. Let's also plan to be brief and non-redundant. Later on, we'll throw out ambiguous as well as surplus information. We can begin anywhere, but the point "X" has been conveniently marked, so we'll use that.

"Beginning at Point 'X'; thence north 45° east 1 (one) chain to the beginning of a tangent 96-ft radius curve, concave southwesterly; thence along the arc of said curve through a central angle of 135° a distance of 225 ft; thence leaving said curve due south 86 ft to a point that is 241.96 ft southeasterly of the point of beginning; thence westerly to a point lying due south, 98 ft, from the point of beginning; thence to the point of beginning."



The next thing we should do is run a traverse around this boundary to see if it closes mathematically. If we're lucky, during the process we may spot errors in the information provided and hopefully correct them with certain boundary principles.

The curve data contains an arc length rounded to the nearest five ft, or so it seems based upon holding the delta and radius. Verification of the delta is inferred by the bearings in and out of the curve, and use of the radius as an even ft stems from the notion that arc lengths are usually derived from calculation from the other curve elements. The first source of error is found in the length of curve.

The next course runs due south. The use of the term "due" suggests that the basis of bearing should be taken as astronomical. The appearance of this term along this course in the boundary does not change the bearing along the first course, i.e., N 45° E, but clarifies the direction of that line as referential to true north. However, the length of this course must accommodate the next piece of information in the diagram, that of the distance of 241.96 ft from the point of beginning. Holding the bearing of south and performing a bearing-distance intersection from the end of the curve to the point of beginning yields a distance of 98.00 ft along this south course, not 86 ft as shown in the diagram. Thus, the second error in the description is located.

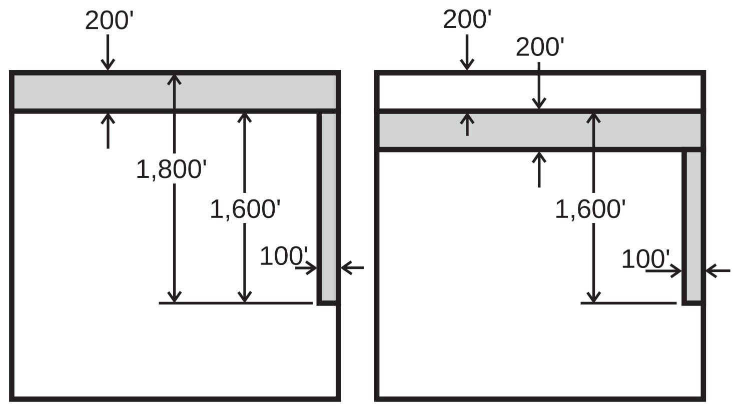
A complex legal description similar to the example at hand may be relegated to Wattles' discussion of majority probability (see *Land Survey Descriptions,* William C. Wattles) whereby all factors in a description interplay to suggest the most logical solution. If a scenario contains factors that predominate toward a given interpretation, then the predominance tends to support the interpretation.

The correct legal description in this case would then be worded:

> "Beginning at Point 'X'; thence N 45° E 1 (one) chain to the beginning of a tangent 96-ft radius curve, concave southwesterly; thence along the arc of said curve through a central angle of 135°; thence leaving said curve due south to a point that is 241.96 ft southeasterly of the point of beginning; thence westerly to a point on a line bearing due south, 98 ft, from the point of beginning; thence to the point of beginning."

The above legal description is correct according to the diagram given, with ambiguities removed. Notice that certain computed information has been omitted without affecting the meaning of the description.

8. Taken from real life, this description originated within a title company and was later reinterpreted by a private engineer who filed a map depicting Solution B. The engineer was unaware of Solution A, which, it turned out, was the original intent of the scrivener. The engineer's map was revised after great effort through the public agency and after several thousand dollars were expended to attorneys and professional consultants.



Proper grammar, punctuation, word usage are all essential to unambiguous legal descriptions. What the scrivener had in mind originally can be compressed into the following noun-preposition sentence structure:

"(The north 200 ft) *and* (the east 100 ft of the south 1600 ft) (of the north 1800 ft of Section 6)."

The last phrase in parenthesis, "the north 1800 ft . . . of Section 6" modifies both parts of the two-part, or collective, noun contained in the first two parentheses, which are connected by the conjunction, *and*. The prepositional phrase "of the south 1600 ft" is intended to modify only the second part of this two-part noun. The title company intended to impart this meaning which produces Solution A; however, the engineer chose to pick out the modifiers as follows:

"(The north 200 ft) *and* (the east 100 ft) (of the south 1600 ft of the north 1800 ft of Section 6)."

The conjunction connects a two-part noun, as before, but this time the entire series of prepositional phrases that occurs afterwards is taken as a single phrase to modify both parts of the collective noun. The meaning on the ground imparts Solution B.

Which interpretation is correct? It may not be possible to tell without additional information about the intent of the sentence which, in most cases, we would not have. In the real life situation, that additional information existed, but only in terms of an admission by the title company that they had created an ambiguous description. This example typifies the reason legal descriptions for land and easements are often separated as individual parcels like this:

Parcel 1: "The north 200 ft of Section 6."

Parcel 2: "The east 100 ft of the south 1600 ft of the north 1800 ft of Section 6."

Parcel 2 can also be written without reference to the south 1600 ft. It is perfectly all right to let easements overlap in this case.

9. A. w = 99.60 ft Hold block corners and prorate lot lines.
   x = 50.09 ft Hold block corners, compute original lot lines, and prorate new lot lines.
   y = 99.65 ft Hold block corners and prorate lot lines.
   z = 50.07 ft Prorate between found block corners.

   B. Lot B of ABC Subdivision filed in Book 3 of Maps at page 25, in Rainbow County, California.

   C. Record of Survey is required.

10. A. 1. Calculate the boundaries of the west one-half of Lot 5 by using one-half the area of Lot 5.

   2. The easterly boundary will be set on a mean bearing of the east and west lines of Lot 5.

   B. The Landis deed has no effect on the client's property since it is junior to the client's deed.

   C. The east one-half of Lot 5 of Rainbow Acres, in the County of Rainbow, in the State of California, as per map recorded April 16, 1954 in Book 3 of Maps, page 3, in the Office of the County Recorder of said county.

   Excepting the southerly 100 ft.

   D. A Record of Survey is required. Section 8762 of LS Act.

References

______, *Manual of Instruction for the Survey of the Public Lands of the United States,* U.S. Government Printing Office, 1973.

Brown, Curtis, *Evidence and Procedures for Boundary Location,* Second Edition, John Wiley and Sons, New York, 1981.

Brown, Curtis, and Robillard, and Wilson, *Boundary Control and Legal Principles*, Third Edition, John Wiley and Sons, New York, 1986.

Robillard, Walter, and Bouman, *Clark on Surveying and Boundaries,* Fifth Edition, Bobbs Merrill Co., New York, 1987.

Skelton, Ray Hamilton, *Boundaries and Adjacent Properties,* Bobbs Merrill Co., Indianapolis, 1930.

Wattles, Gurdon H., *Writing Legal Descriptions,* Gurdon H. Wattles Publications, Orange, CA, 1979.

UNIT 14

WATER BOUNDARY LOCATION

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Boundary Section
California State Lands Commission

Introduction

Property boundary location along waterways in California is neither certain nor predictable because each situation is settled on its individual merits. However, there are guidelines useful to those answering questions on land surveying examinations, based on the basic concepts of property boundary location along California's waterways. Using these will not only provide a valuable test-taking aid, but will also serve as an introductory foundation for study and application beyond the examination and into real property surveying.

Performance Expected on the Exams

Explain the effects of navigability on the title to the beds of lakes and rivers.

Explain the effects of avulsion on land title.

Explain the effects of erosion and accretion on land title.

Describe how you would locate the mean high tide line in the field.

Calculate the elevation of the mean high water line for a parcel of land given the NGVD elevation of mean lower low water of a tidal bench mark.

Describe effects of engineering works such as dams and breakwaters on land title of riparian and littoral property owners.

Key Terms

Accretion

Alluvium

Artificial conditions

Avulsion

Deed terms: to, along, with

Erosion

Last natural location

Littoral

Mean high tide line

Mean low tide line

Mean lower low tide

Meander: stream, lake, ocean

Navigability: in fact, by statute, susceptible

Ordinary high water mark

Ordinary low water mark

Public trust easement

Reliction

Riparian

Sovereign lands

Stream: center, thread

Swamplands

Tidal datums

Tidelands

Video Presentation Outline

Introduction

Boundary location along waterways is a marriage of four factors:

- The title held by the owners of land along and underneath waterways.
- Law, both common and statutory, that may be generally applicable on a state or national basis, or applicable to a specific water body, or even a specific site along a shore.
- Actual uses made of the waterway in the past, the present, or possibly in the future.
- Physical science for the effect of rainstorms, wet seasons, dry seasons, and years of ordinary rainfall.

Navigable and Non-navigable Waterways

The beds of non-navigable waterways belong to adjacent upland owners. The beds of navigable waterways belong to the State.

- Determination of navigability.
- Effect of navigability on property boundaries.
- Boundaries defined along non-navigable lakes, rivers, or streams.
- Boundaries defined along navigable lakes, rivers, streams, or oceans.

Tidal Datums and Water Boundaries

The mean high tide line is an elevation contour that can be located at any time by a surveyor using the proper information. It may or may not be a boundary line. The ordinary high water mark is the boundary by California statute. It may or may not be the same as the mean high tide line.

- Tidal datums in California.
- National Geodetic Survey Tidal Benchmark Program.
- Tidal datums as boundaries—field location.
- Comparing and reconciling legal terminology and scientific methodology.
- Apportioning by survey along non-tidal, non-navigable waterways.

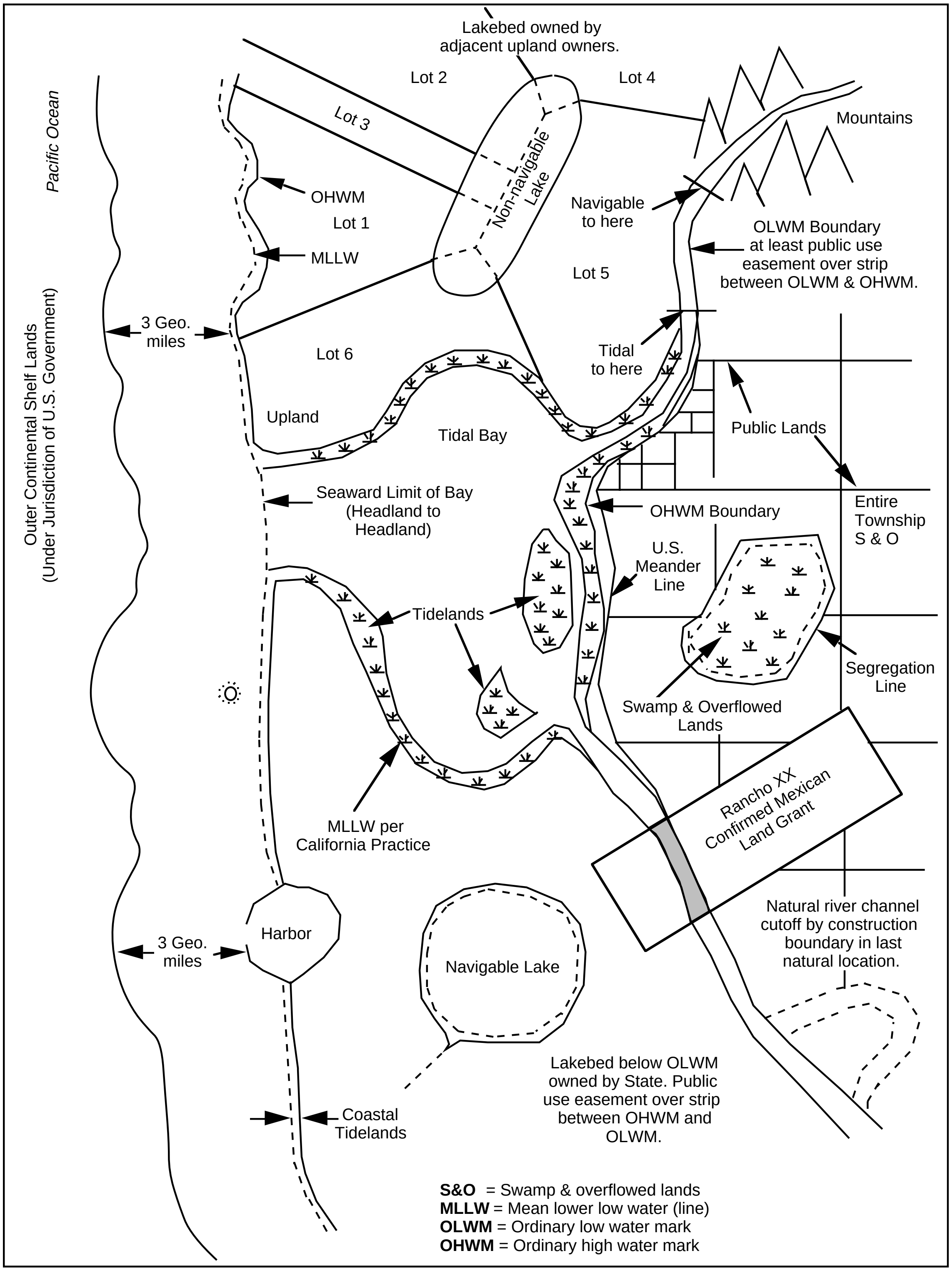


Figure 14-1. *From "Water Boundaries for Land Surveyors," permission granted by Landmark Enterprises.*

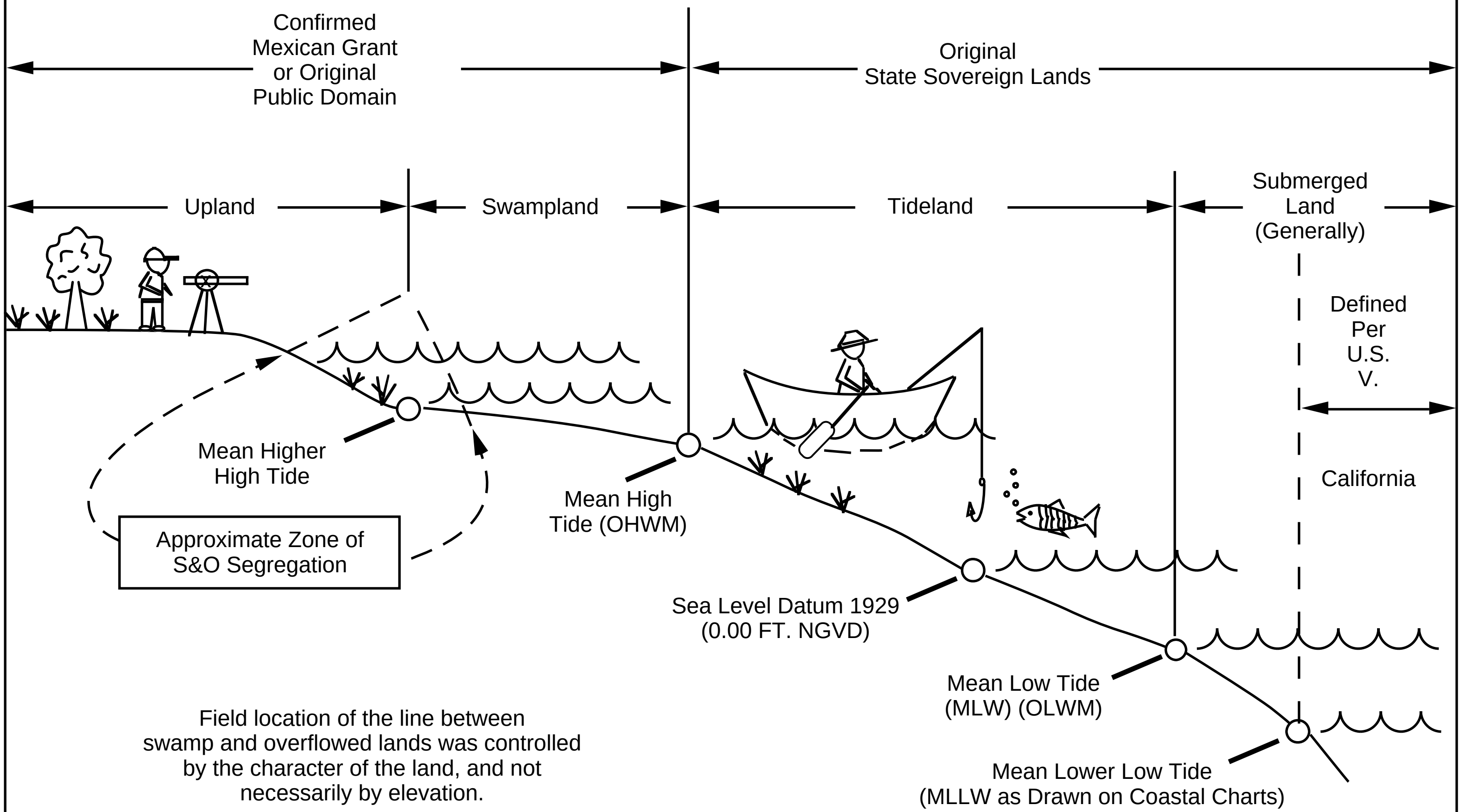
- Locating boundary lines along non-tidal lakes and rivers.
- Swamplands and tidelands in California.
- Locating boundaries along tidal waterways.

Factors Affecting Property Boundaries Along Waterways

- Accretion and erosion.
- Artificial changes generally, and in California.
- Meander lines related to water boundaries.
- Land descriptions with certain and uncertain "calls."
- Islands in streams—navigable and non-navigable.
- Senior grants of land bordering or including navigable waterways.

Sample Test Questions

1. Describe one method for apportioning the bed of a cigar-shaped lake among several owners.
2. Describe two methods of apportioning ownership of accretion along a river between adjoining owners.
3. The lands beneath a non-tidal, navigable river, belong to _________.
4. The boundary of the uplands along the Pacific Coast in California is _______________.
5. If a new island forms in the bed of a navigable river, who will be the owner?
6. In a land description, a "call" is made to a stream which is non-navigable. Title thus conveyed is to ____________.
7. Define the term reliction. Explain how it differs from erosion.
8. Explain why United States public land surveyors run meandered bodies of water.
9. Define tidal epoch and explain its relationship to property boundaries along waterways.
10. Title to lands lying beneath a navigable waterway are usually vested in the ____________.



Land Title and Tide Line Cross Section Along the Pacific Ocean

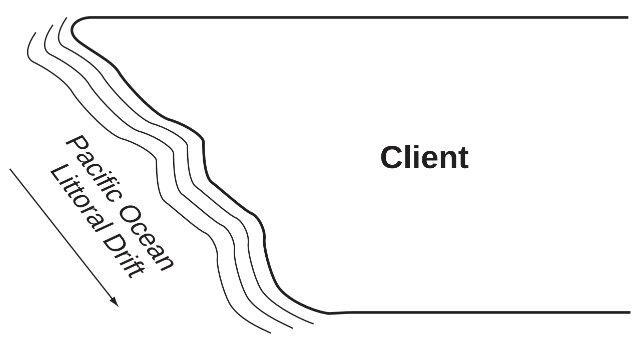
Figure 14-2. *From "Water Boundaries for Land Surveyors," permission granted by Landmark Enterprises.*

11. Describe the effect on property boundary lines when a meandered, non-navigable stream abandons its original bed by an avulsive action and forms a new bed. Who will own the new bed? The old bed?

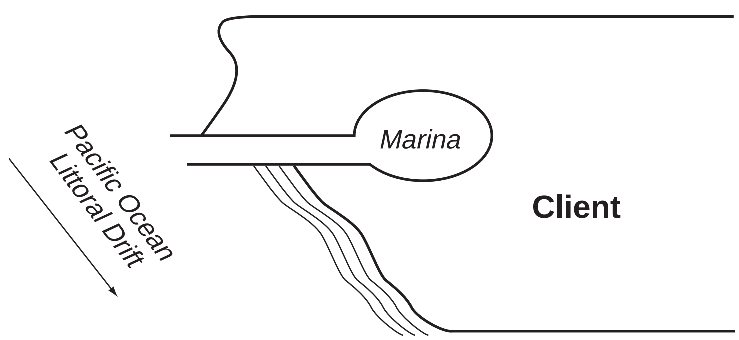
12. Distinguish between the ordinary high water mark and the mean high tide line.

13. Describe how to locate, in the field, the mean high tide line when it is defined at the survey site as 4.73 ft above mean lower low water. The equation between mean lower low water and NGVD at the site is 1.89 ft, and the location is along the California coast.

14. Suppose you were asked to survey the deed description of the owner to a piece of property along the Pacific Ocean and the call was "...to the ordinary high water mark of the Pacific Ocean, thence along the Pacific Ocean to the section line between..." The original deed was dated January 7, 1866. It is clear from your research that the shore line is in a natural condition, as shown in the sketch below. How would you survey your client's property along the ocean? What factors would you consider? Describe the steps you would take.



Would you conduct your project differently if your research uncovered, about midway between the ends of the property, breakwaters that had been constructed about 1950 to protect a man-made harbor? This alternative situation is shown below.



Answer Key

1. The “long lake method” with the “pie method” at the ends of the lake would be the most equitable in dividing the bed among riparian owners. A median line is established at the thread of the lake. Division lines are erected perpendicular from the median line to the upland property lines. The ends of the lake are treated as arcs, with property lines converging toward the ends of the median line, in the manner of cutting a pie.

2. The “Proportionate shore-line method” divides the accreted land so that each riparian owner receives the same proportionate amount of water frontage. Regardless of whether the shoreline condition is concave, convex, or straight, the length of the new shoreline is in proportion to the length of the old. This method is the most commonly used for the division of accretions.

   The “Proportionate acreage method” apportions each riparian owner an area of accreted land in relation to one’s former proportionate length of water frontage. While this method provides a more equitable division for particular cases, it lacks specified direction for the line of division in general applications.

3. The State.

4. The ordinary high water mark.

5. The ownership of an island formed in a navigable river, after the date of the admission of a state into the Union, belongs to the state.

6. Regardless of whether the “call” is specific or uncertain, the title, by law, is conveyed to the middle or thread of a non-navigable stream, except where the bed of the stream is held under another title.

7. Reliction is the gradual and imperceptible withdrawal of water resulting in the uncovering of land once submerged. Erosion is the washing away of land by water. Although erosion is usually considered an imperceptible action, the rate of erosion may be quite rapid.

8. A meander line run by a surveyor is for the purpose of platting the size and extent of a body of water and for the purpose of ascertaining the quantity of upland, not to determine the location of the bank of the body of water.

9. A tidal epoch is a periodic variation in the rise of water above sea level having a period of 19 years. The epoch is based upon theoretical considerations of an astronomical character and the continuous statistical observation of the rise of water above sea level in the United States since 1854.

   The boundary between tidelands and upland ownership is the mean high tide line, which is an average height of all high waters at the place over a tidal epoch. Because the mean high tide line is a fluctuating line, the property boundary is also a moving boundary.

10. The State.

11. Avulsion has no effect on boundaries whether State or private. The land ownership remains as it was.

12. The ordinary high water mark is a legal term as cited in the Civil Code and the Public Resources Code defining the State's boundary. It can represent the natural condition of the shoreline. However, it can also be an adjudicated line rendered by a court decision.

    The mean high tide line is a scientific term. It is the average of all high tides in an 19-year epoch of observation. It is an intersecting line of water and land on any given date. It may or may not be the boundary between the uplands and the State, depending on whether the shoreline is natural or artificial.

13. The topographic contour method is an acceptable procedure to locate the mean high tide line.

    First, from the known data, the elevation of the mean high tide line must be determined. Mean lower low water = 0.00 ft. The elevation of the mean high tide line in this example is 2.84 ft NGVD (4.73 - 1.89 = 2.84 ft).

    From a known bench mark elevation, based on sea level datum, the topographic contour line of 2.84 ft is located on the shore by leveling and conventional horizontal surveying procedures. The rod person would place the rod at the mean high tide line elevation indicated by the level person. The horizontal bearing and distance of each point on the contour would be fixed by a theodolite operator from a known cadastral point. As each mean high tide line elevation is established along the subject property, a stake or lath is placed, thus providing a visual means to see or photograph the mean high tide line. The survey should be timed when the tide is low to avoid any hazardous surf.

14. Because the coast line of the Pacific Ocean is subject to erosion and accretion, it is probable the described 1866 ordinary high water mark would not be in the same location today and the property boundary would move with that line.

    The ordinary high water mark may or may not be synonymous with the mean high tide line in reference to tidal waters. To survey the ordinary high water mark, the topographic contour method is an acceptable procedure. From technical data published by the National Oceanic and Atmospheric Administration, the elevation of the mean high tide line (ordinary high water mark) would be determined from a local tide station or interpolated between the two nearest stations. From a known bench mark elevation, based on sea level datum, the calculated topographic contour line is located on the shore by leveling and conventional horizontal surveying procedures. As each mean high tide line station is established along the subject property, it is fixed by bearing and distance from a known cadastral point. A stake or lath is placed at each elevation point on the shore, thus providing a visual means to see or photograph the mean high tide line. The survey should be timed when the tide is low to avoid any hazardous surf.

    In California, an artificial influence, such as the construction of breakwaters, has no effect upon littoral boundaries between the State and upland owner. The boundary will remain as it existed, at the last natural location of the shoreline. Determining the last natural shoreline is dependent upon historical research to locate evidence of the shoreline before the artificial condition was introduced. Historic charts, maps, surveys, and historic tide station data are very informative when searching for the best available evidence of the location of the historic water lines.

References

_______, *Tidal Benchmark Information-California,* Boundary Section, California State Lands Commission, Sacramento, CA, 1991.

Brown, Curtis, *Boundary Control and Legal Principles*, Third Edition, John Wiley & Sons, New York, 1988.

Cole, George, *Water Boundaries*, Second Edition, Landmark Enterprises, Rancho Cordova, CA, 1991.

Minnick, Roy, *Glossaries for Surveyors*, Second Edition, collected by, Landmark Enterprises, Rancho Cordova, CA, 1989.

Minnick, Roy, *Laws for the California Land Surveyor*, 1991, Landmark Enterprises, Rancho Cordova, CA, 1991.

Minnick, Roy, and Russell Brinker, Editors, *The Surveying Handbook*, Van Nostrand Reinhold, New York, 1991.

Minnick, Roy, *Water Boundaries for Land Surveyors*, Landmark Enterprises, Rancho Cordova, CA, 1990.

Robillard and Bouman, *Clark on Surveying and Boundaries*, 5th Edition, Michie Co., Charlottesville, VA.

UNIT 15

LEGAL DESCRIPTIONS

R. W. "Russ" Forsberg, PLS

Introduction

Legal descriptions are word pictures that accurately describe parcels of land. The California Land Surveyors Act designates the writing of legal descriptions as part of the practice of land surveying. Drafting legal descriptions requires knowledge of the rules of evidence and proper use of legal terms, ability to write clearly and concisely, understanding of the effects of the description writing process on land title, and the use of a rigorous mathematical checking procedure.

Because description writing requires the use of so many basic land surveying skills, it has been heavily emphasized on recent exams. Typically, about 60 percent of the points on an LS exam will deal with descriptions and boundary determination.

Examinees must know what is required of a good description, the parts of a description, types of descriptions, and a method for attacking description problems. Proper planning of the task of description writing will save time, eliminate errors, and ensure that the description can have only one interpretation.

Performance Expected on the Exams

Write surveyable metes and bounds, strip, “of,” and line descriptions.

Write description preambles and lot, block, and sectionalized land descriptions with information presented in surveyable, logical sequence.

Given the results of a field survey and project requirements, choose the type of description that will give the least ambiguous location for the land being described.

Write curve descriptions using acceptable calls and phrases.

Utilize calls in descriptions that clarify and emphasize the intent of the parties to the description.

Key Terms

Adjacent	Adjoining
Aliquot	Along
Basis of bearings	Bearing
Call	Caption
Coincident	Commence
Contiguous	Continuation
Course	Conveyed
Described	Double call
Due	Each
Either	Excepting
Free line	Half
Lot	More or less
Northerly	Parallel
Point of Beginning (POB)	Point of commencement
Point of termination	Preamble
Prolongation	Reserving
Said	Sufficiency
Thence	Title line
To	Tract
True north	True Point of Beginning
Pedigree	

Video Presentation Outline

Characteristics of a Legal Description

- Can be interpreted in only one way
- Clear and concise
- Surveyable
- Legal
- Insurable

The Description Writer

- Qualifications
- Frame of mind
- Vocabulary
- Clauses

The Parts of the Description

- Preamble
- Body
  - Mathematical closure
  - Courses
  - Free line/title lines
  - Point of beginning
  - Basis of bearing
  - Second basis of bearing
- Clauses

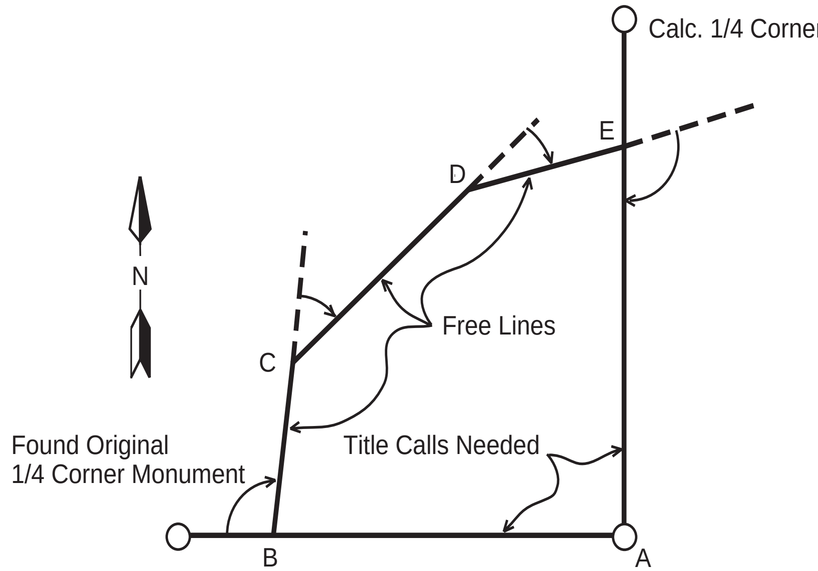


Figure 15-1. *Free lines/title lines, Point of Beginning.*

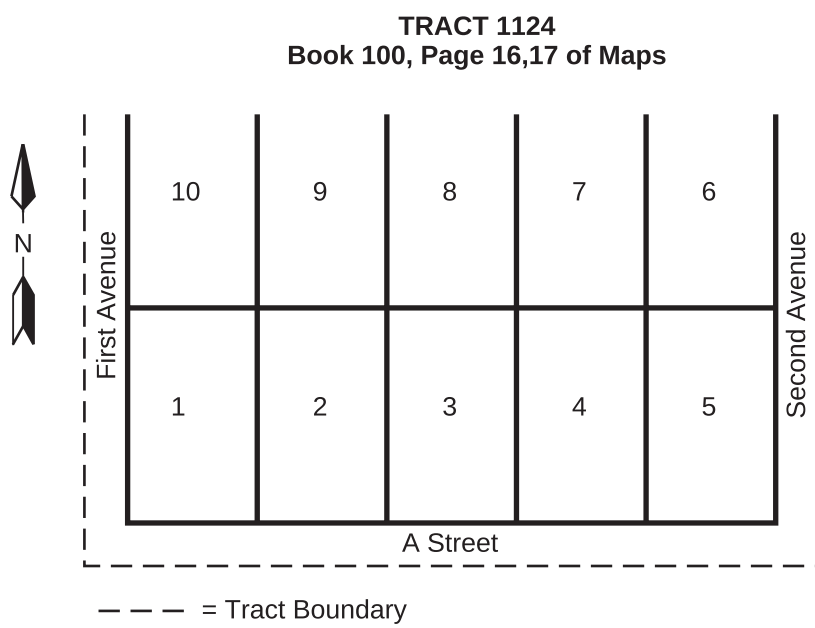


Figure 15-2. *Tract 1124.*

(NOTE: In the video, this drawing is used to illustrate different types of descriptions.)

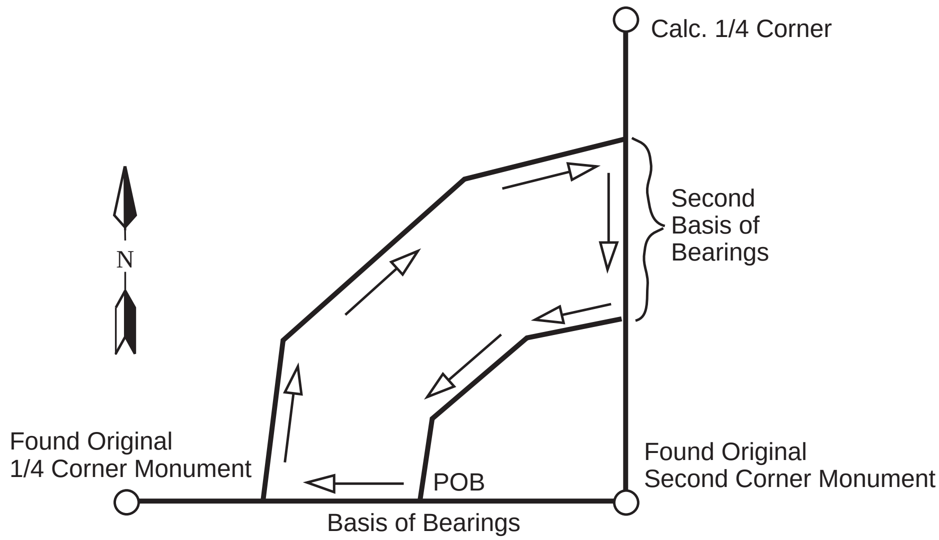


Figure 15-3. *Title calls can change basis of bearings.*

Types of Descriptions

- Sectionalized land
- Map reference
- Metes and bounds
- Strip
- “Of”
- Line
- Exception
- Inclusive

Planning the Task

A good legal description is *legal, insurable,* and *surveyable.* The best way to meet these criteria is to plan ahead.

Before beginning to write, ask the following questions about the parcel to be described:

1. What type of description is best for this parcel?
2. If the parcel to be described is part of a project requiring descriptions of parcels from different ownerships, what type of description should be used?
3. What is the best location for the Point of Beginning?
4. Where should the basis of bearings be established?
5. Clockwise or counterclockwise?
6. What calls should be made?
7. What clauses should be used?

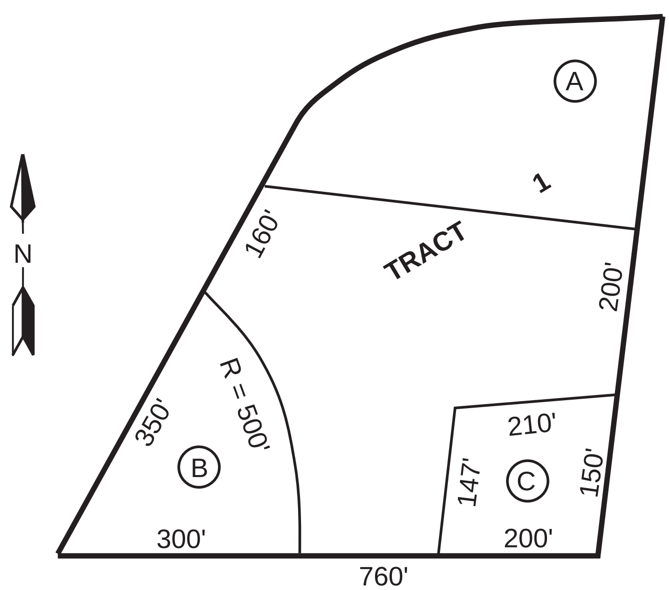
Example Problems

Problem C-4 1977 LS

A parcel of land identified as “Tract One of Mann’s Subdivision” per Map Book 3, page 17, Records of _____ County, California, is shown below. Bearing and distances not shown are not to be assumed or calculated. Disregard possible requirements for parcel map.

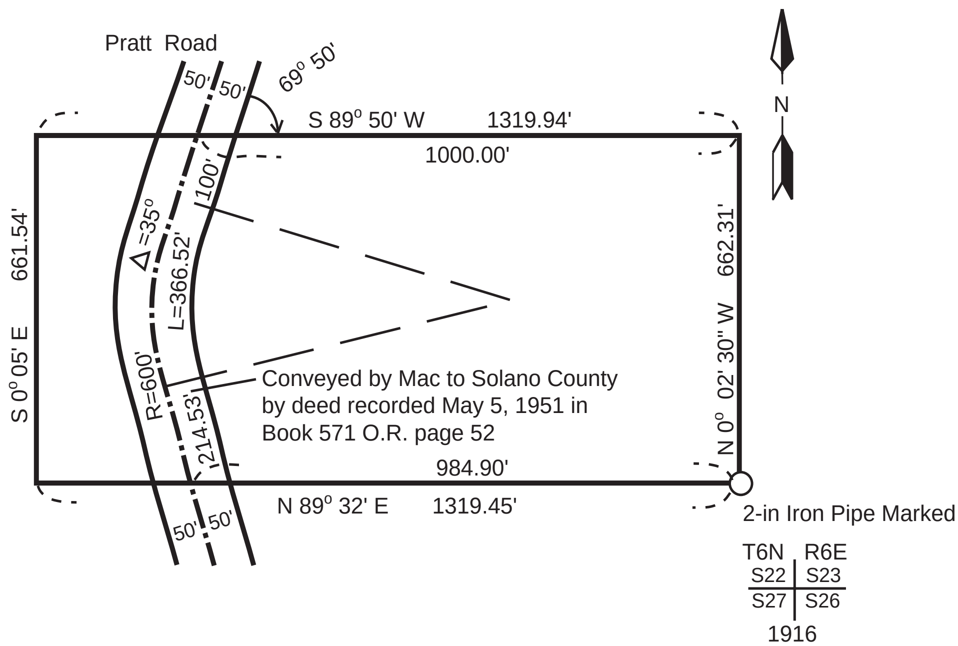
Required:

1. Write a metes and bounds description sufficient to convey the parcel identified as “A.”
2. Write a metes and bounds description sufficient to convey the parcel identified as “B.” Disregard references to official records.
3. Write a metes and bounds description sufficient to convey the parcel identified as “C.” Disregard references to official records.



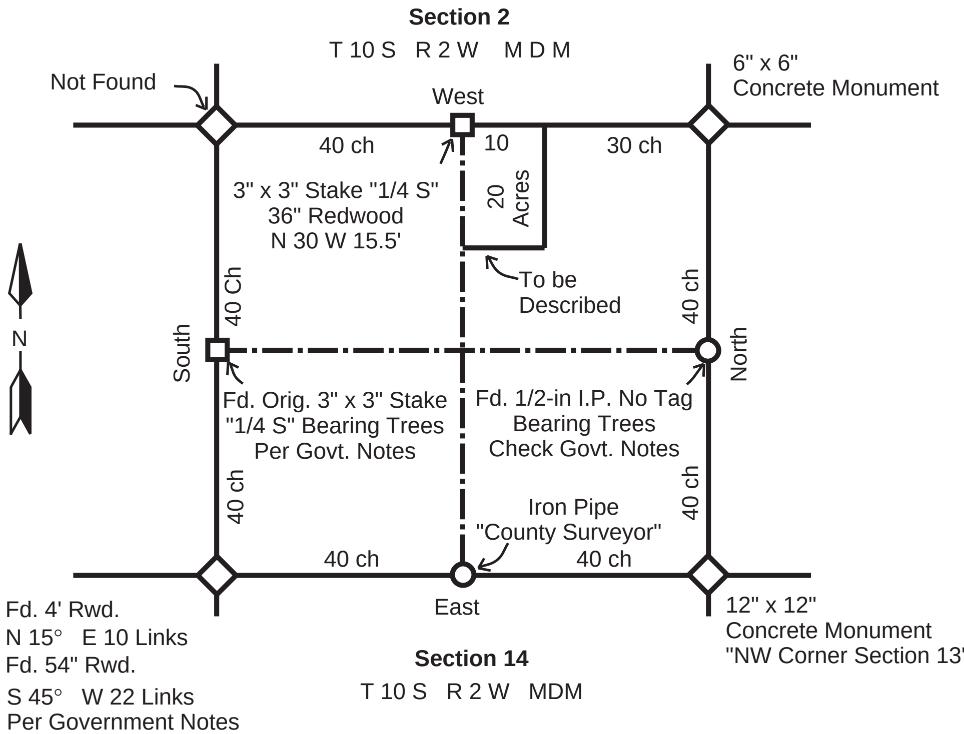
Problem C-1 1978 LS

Calculate the dimensions and write a metes and bounds description for that portion of the S 1/2 of the SE 1/4 of the SE 1/4 of Section 22 which lies east of Pratt Road. (Area not required.)



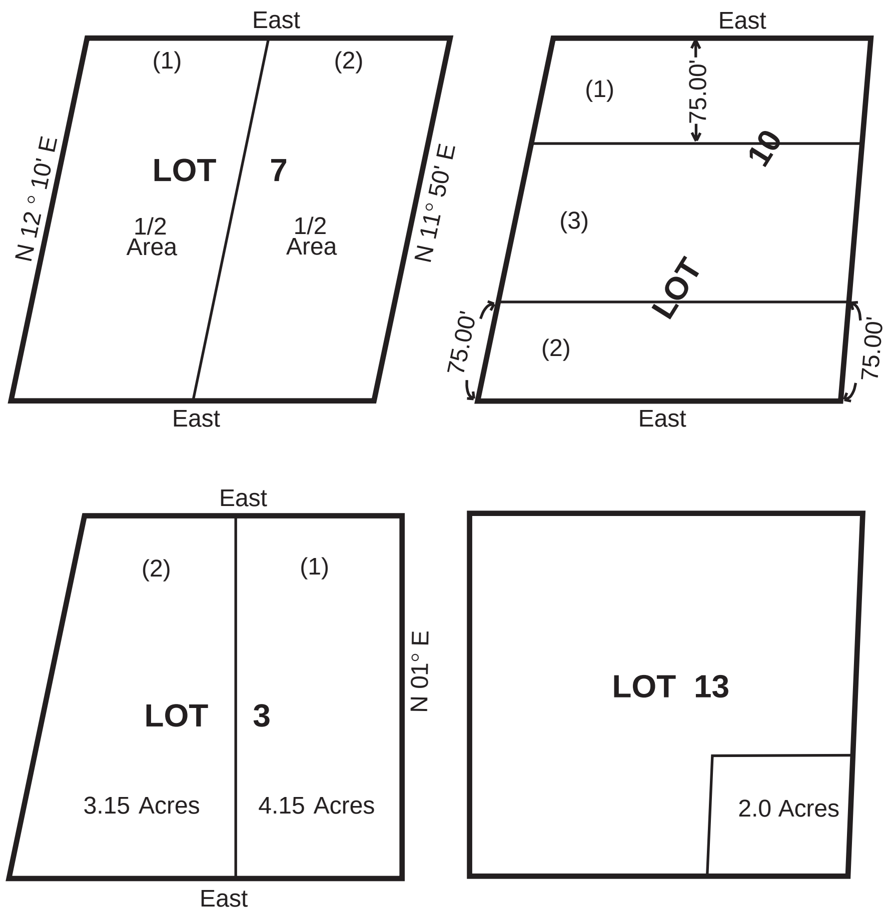
Problem D-5 1984 LS

Write a legal description for the 20-acre parcel shown in County of Santa Cruz, State of California. All dimensions shown are record and measured. Bearings are based on Polaris observation.



Problem D-6 1984 LS

Prepare legal descriptions for all portions of each lot in the order shown, i.e., (1) First, (2) Second, etc. The legal descriptions must be sufficient to properly convey each parcel and there must be no ambiguities created. The parcels will be conveyed in the order shown.



Problem A-3 1986 LS

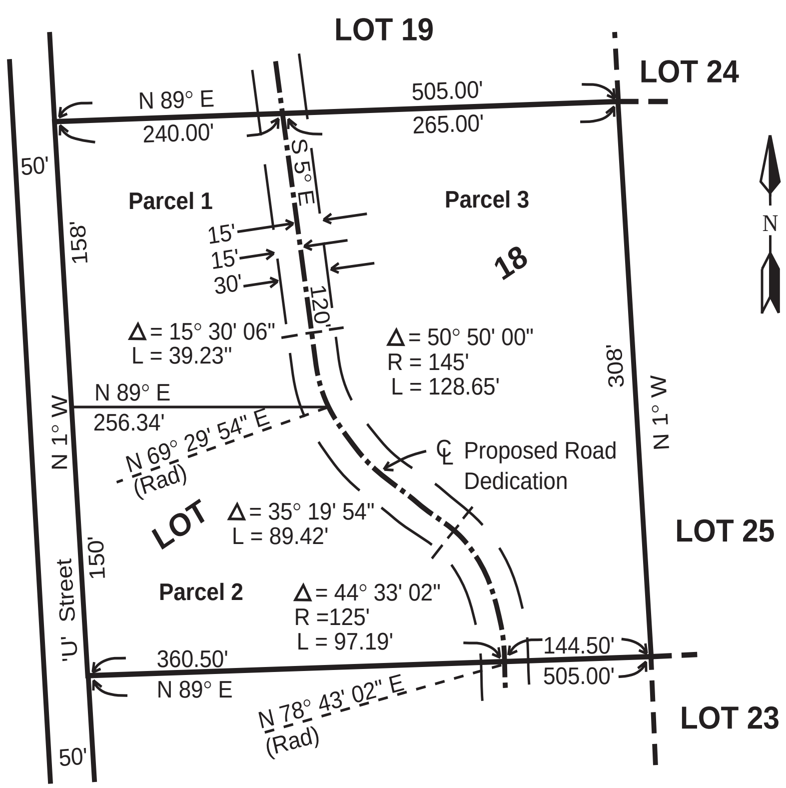
The sketch below shows Tentative Parcel Map Number 12687, based on record data, proposing the subdivision of Lot 18, Tract 400, into three parcels. The tentative map has been approved along with a waiver of final parcel map. (No parcel map need be filed.)

In order to complete the subdivision process, the county requires that a Certificate of Compliance be prepared and recorded and that the 30-ft road easement be dedicated to the county by a separately recorded document. The bearings and dimensions shown are record or calculated from record per Tract 400 and are not the result of a survey.

Required:

1. Prepare a legal description for the road easement. Include the caption.
2. Prepare legal descriptions for proposed Parcels 1, 2, and 3 in such a way that if a survey reveals different bearings and dimensions from those shown on Tract 400, no gaps or overlaps will result. Include captions. Each description must stand alone. If reference is made to the road easement, the entire description need not be rewritten, just referred to.

   No calculations are required.

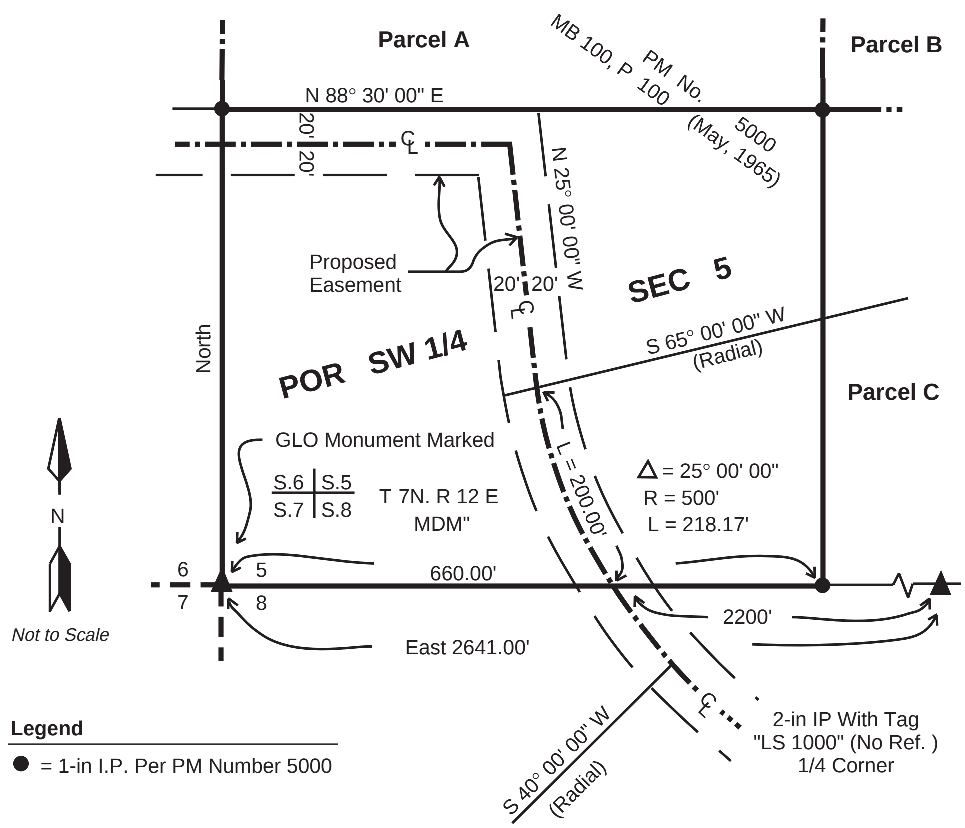


Problem B-1 1991 LS

A client owns a portion of the southwest 1/4 of Section 5, T7N, R12E, M.D.M., and has provided the plat shown below. The surveyor has been asked to prepare a legal description for a proposed 40-ft-wide easement across the client's property.

Required:

1. Prepare a strip legal description, including caption (preamble), for the proposed easement.
   - Begin your description at the southwest corner of Section 5.
   - Use only the information provided on the plat. Do not make assumptions.



Sample Test Questions

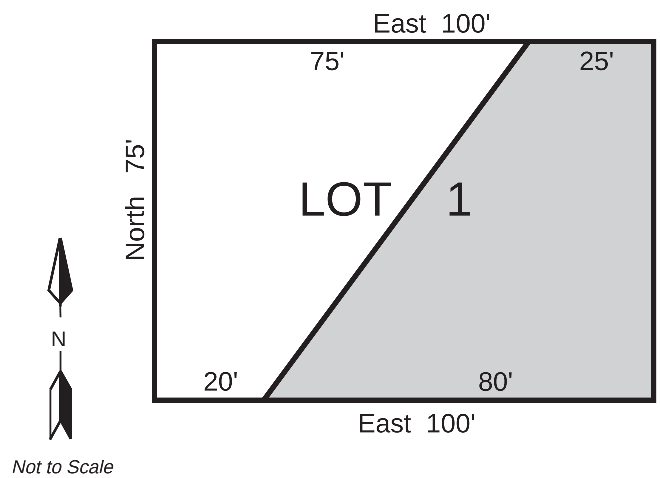
1. Problem A-5 1992 LS

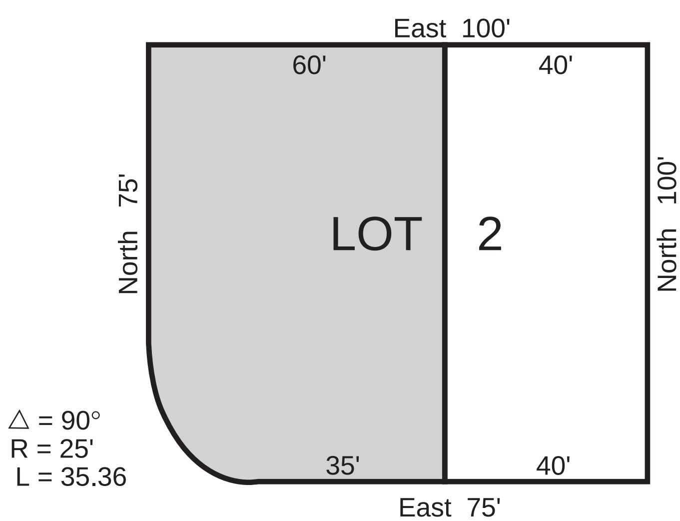
You have been asked to prepare legal descriptions for the shaded portions of the lots below. Treat each shaded portion as the first division of each lot.

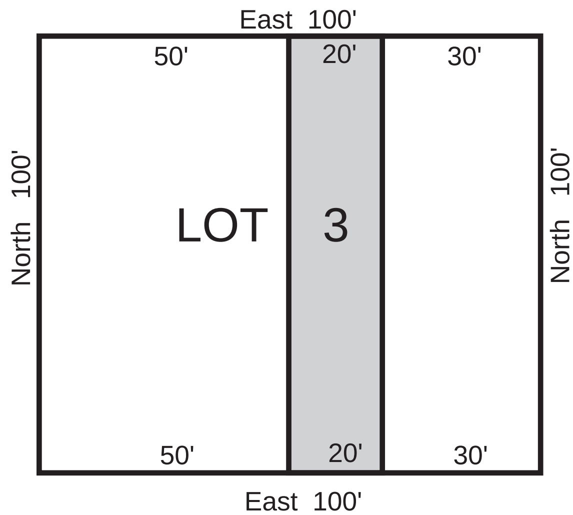
Required:

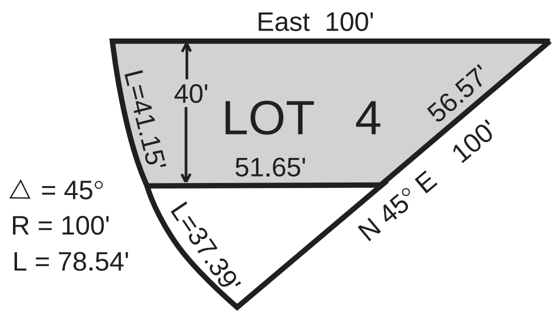
Use only the information provided to solve the following. Make no assumptions. **Do not write a caption for your description. No calculations are required.**

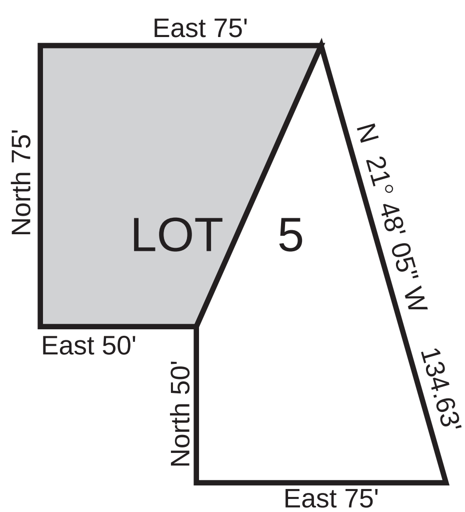
Prepare legal descriptions for the shaded portions of Lots 1 through 5. The shaded areas of the lots depict the client's intent. Prepare your legal descriptions to ensure this regardless of subsequent re-establishments of lots.






East 100'
50'
20'
30'
North 100'
LOT 3
North 100'
50'
20'
30'
East 100'


East 100'
L=41.15'
40'
LOT 4
56.57'
100'
51.65'
N 45° E
△ = 45°
R = 100'
L = 78.54'
L=37.39'


East 75'
North 75'
LOT 5
N 21° 48' 05" W
134.63'
East 50'
North 50'
East 75'

2. Problem A-1 1991 LS

You have surveyed a pipeline route as shown on the plat on the following page. Utilizing a property line tie that you made on the northerly line of Sycamore Street, record map values, and the Tarantino deed, you calculated pipe lengths on the Tarantino property. You have been asked to prepare an easement description for the two pipelines as they relate to the Tarantino property.

The Tarantinos bought the northerly 150.00 ft of the easterly 200.00 ft of Lot 3 of Block 14 as shown on a deed recorded on August 9, 1943, in Book 27, Page 83 of Official Records.

The easement for pipeline “A” is 30.00 ft wide. The pipeline is 10.00 ft southeasterly and easterly of the northwesterly and westerly line of the easement.

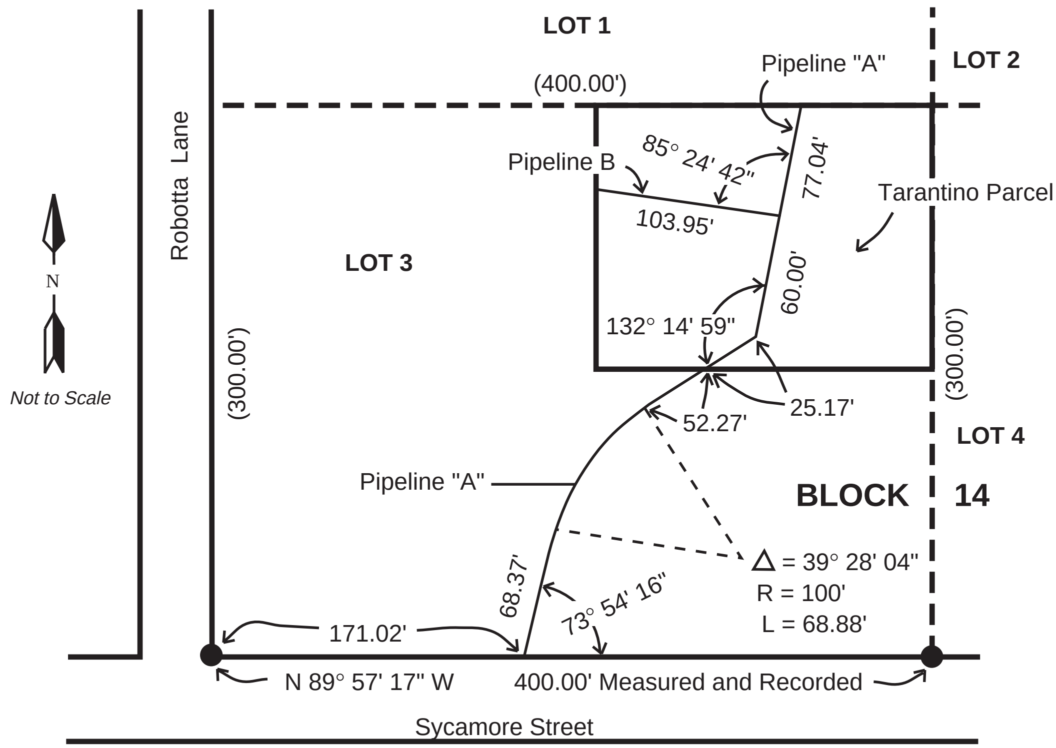
Pipeline “B” is in the center of a 20.00-ft-wide easement.

Required:

A. Calculate the bearings necessary to write the description for the two pipeline easements.

B. Utilizing the surveyed information, prepare a complete legal description entitled “Exhibit A” for the two pipeline easements on the Tarantino parcel. The legal description will be attached to a grant deed that reads, in part, that Tarantino grants the easements for pipeline purposes as described in Exhibit A.
   - Begin the description at the southwest corner of Lot 3.
   - The TRUE POINT OF BEGINNING must be on the southerly line of the Tarantino parcel.
   - No further surveying or property line calculations are required.

A PORTION OF MAP 3619
Book 36, Page 54 of Maps,
County of Testing Showing Pipelines



Legend

● = FD 2-in I.P. with Disk LS 10,000 Per Map 3619

( ) = Record Value Per Map 3619

3. Problem B-4 1988 LS

Mr. Giraf owns Parcels 1 through 3 inclusive of Parcel Map No. 12,000 in the County of Calaveras, State of California as shown on the map recorded in Book 14 of Parcel Maps at page 35. He is selling Parcel 1 to Mr. Moosehead.

He needs to retain a well easement, a pipeline easement and an access easement.,

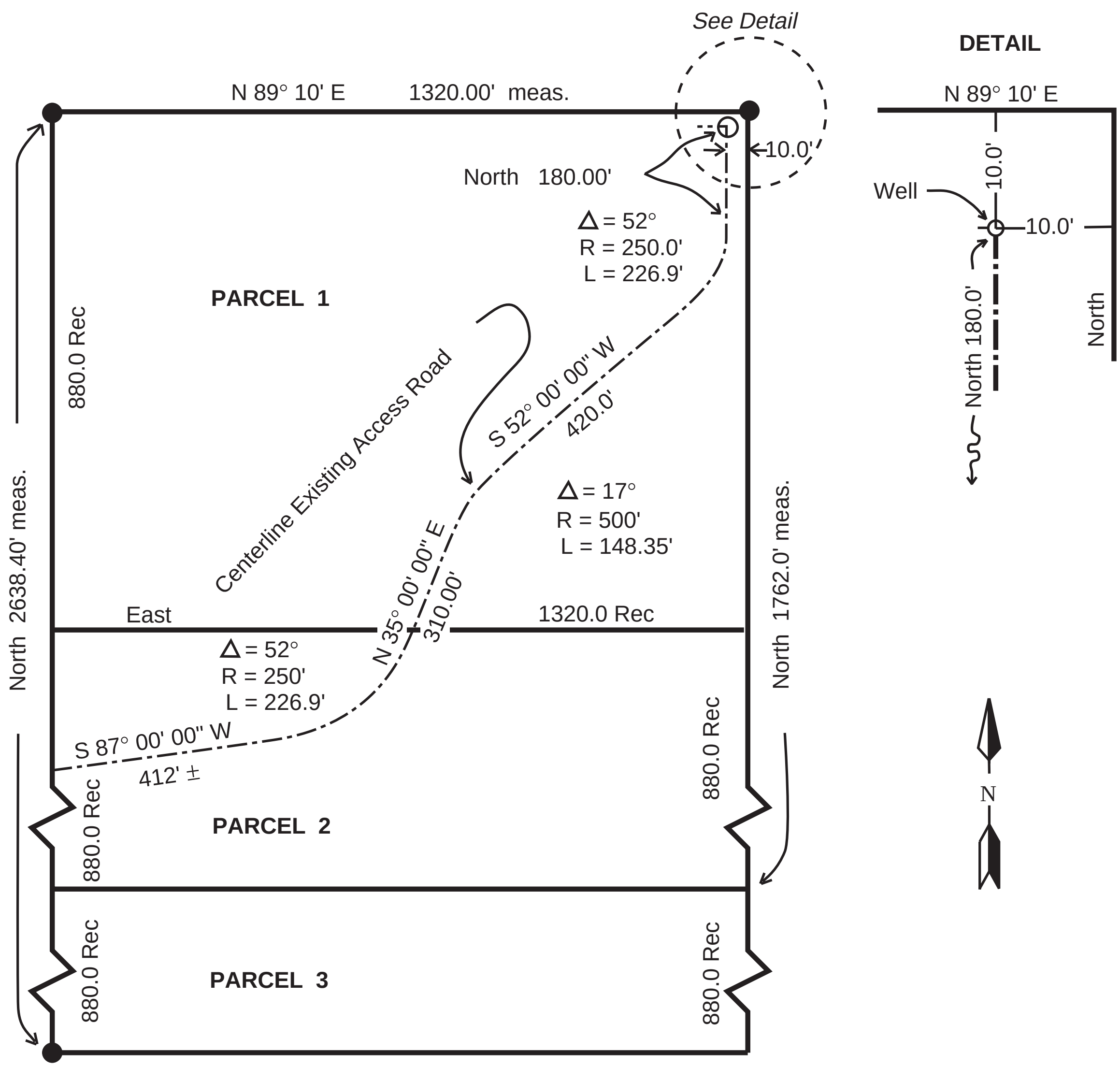
The well easement is to be 25 ft by 25 ft at the northeast corner of Parcel 1. The pipes have not been installed. Provide a 10-ft easement along the easterly line for them. The access easement is to follow the existing access road to the well and should be 20 ft wide.

The owner is not interested in doing a boundary survey of any of the parcels.

The sketch on the following page shows the information gathered from a field survey for the easements. The road and well information shown below were not on the filed parcel map.

Required:

Write complete descriptions for the requested easements.

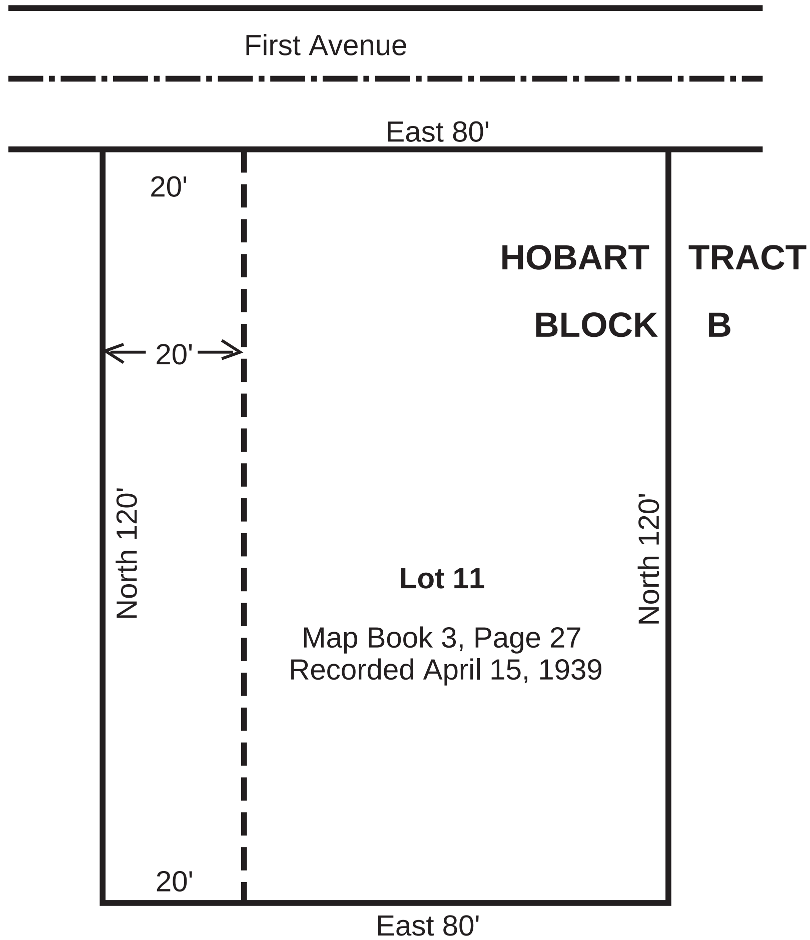


NOTE: All road and well dimensions are your measurements.

Legend

Rec = Record Dimensions per 14/PM/35
meas. = Your Measured Dimensions
● = Found Monument as Shown on 14/PM/35

4. A. Write a description of the west 20 ft of Lot 11 so that it would include the underlying fee in First Avenue.



B. Write a metes and bounds description of the west 20 ft of Lot 11 as measured along the south line of First Avenue such that it will include the underlying fee in First Avenue. Disregard the preamble.

Answer Key

1. The key to planning these descriptions is the statement that surveyors must honor the client's intent and write them in such a way that the intent will be honored even if a later survey altered the bearings and distances on the lot lines. For instance, in Lot 1 the description could be tied to the northwest corner of the lot, but if that were done and a later survey showed the total distance along the North line to be 99 feet, the 75-foot figure in the description would hold, but the buyer would get only 24 feet along the North side.

   Our seven "Planning The Task" questions will be useful on the last two problems so they will be used there. Here are some suggested answers for the descriptions for portions of the five lots asked for in Problem A5 of the 1992 exam:

   Lot 1: Beginning at the southeast corner of Lot 1; thence westerly along the south line of said lot, 80 feet; thence northeasterly to a point on the north line of said lot, distant westerly, 25 feet along said north line from the northeast corner of said lot; thence easterly, 25 feet to the northeast corner of said lot; thence southerly, along said east line, 75 feet to the Point of Beginning.
   or
   That portion of Lot 1 lying easterly of a line connecting a point on the south line of said lot distant along said south line westerly, 80 feet from the southeast corner of said lot with a point on the north line of said lot distant westerly, 25 feet from the northeast corner of said lot.

   Lot 2: Lot 2, EXCEPT the east 40 feet of said lot; said 40 feet measured along the north and south lines of said lot.

   (This one does not depict the client's intent. If a later survey showed the north line to be 99 feet long instead of 100, the client would get only 59 feet along the north side instead of 60.)

   The following descriptions better express the client's intent:

   Beginning at the northwest corner of Lot 2; thence easterly along the north line of said lot, 60 feet; thence southerly to a point on the southerly line of said lot, distant easterly along said south line 35 feet from the westerly terminus of that course with a bearing of East and a distance of 75 feet; thence westerly along said south line, 35 feet to a curve in the lot line; thence, continuing along said lot line, along a tangent curve to the right, having a radius of 25 feet, through a central angle of 90°, an arc distance of 35.36 feet; thence, continuing along the west line of said lot, North, 75 feet to the Point of Beginning.

or

Beginning at the northwest corner of Lot 2; thence southerly along the west line of said lot, south, 75 feet; thence, continuing along the lot line, along a tangent curve to the left, having a radius of 25 feet, through a central angle of 90°, an arc distance of 35.36 feet; thence, continuing along the south line of said lot, East, 35 feet; thence northerly to a point on the north line of said lot that bears easterly along said north line 60 feet from the Point of Beginning; thence, westerly along said north line, 60 feet to the Point of Beginning.

or

That portion of Lot 2 lying westerly of a line connecting a point on the north line of said lot, said point distant easterly along said north line 60 feet from the northwest corner of said lot with a point on the south line of said lot, said point distant easterly along said south line 35 feet from the westerly terminus of that course with a bearing of East and a length of 75 feet.

Lot 3: Beginning at a point on the north line of Lot 3, said point distant westerly along said north line, 30 feet from the northeast corner of said lot; thence southerly to a point on the southerly line of said lot, last said point distant westerly along said south line, 30 feet from the southeast corner of said lot; thence westerly along said south line, 20 feet; thence northerly to a point on the north line of said lot, last said point distant westerly along said north line, 20 feet from the Point of Beginning; thence easterly along said north line, 20 feet to the Point of Beginning.

or

The west 20 feet of the east 50 feet of Lot 3, said distances to be measured at right angles to the east line of said lot.

or

The west 20 feet of the east 50 feet of Lot 3, said distances to be measured along the north and south lines of the lot respectively.

NOTE: These last two will result in different size parcels if a later survey results in different dimensions and directions for the lot lines. One is based on the assumption that the 20- and 30-foot dimensions are intended to be measured at right angles and the other is based on the assumption that they are to be measured along the lot lines. The two would be the same if the opposite lot lines are parallel and all interior angles in the lot are 90° as shown, but a later survey may change that relationship. Since the dimensions are shown along the lot lines rather than at right angles, we can assume that is the intent, making the last description the better choice.

Lot 4: The north 40 feet of Lot 4, measured at right angles to the north line of said lot.

Lot 5: Beginning at the northwest corner of Lot 5; thence East, along the north line of said lot, 75 feet to the angle point with the easterly line of said lot; thence southwesterly to the east terminus of that course on the boundary of said lot having a bearing of east and a length of 50 feet; thence westerly along said boundary, 50 feet to the most westerly line of said lot; thence northerly, along said west line, 75 feet to the Point of Beginning.

or

That portion of Lot 5 lying westerly of a line connecting the northerly terminus of that course on the boundary of said lot having a bearing of N 21° 48' 05" W and a length of 134.63 feet with the easterly terminus of that course on the boundary of said lot having a bearing of East and a length of 50 feet.

NOTE: When calling to the terminus of a certain course, it is important to be clear which one is meant. The northwesterly terminus of that course with a length of 134.63 feet could not have been described as the easterly terminus of that course with a bearing of East and a length of 75 feet because there are two of them on the lot. Care was taken to avoid calling that point the northeast corner of the lot because some would understand it to be that and others would not. In other words, it would be interpreted in more than one way.

2. The seven questions will be helpful in planning the answer to this problem. First, answer the questions and then give a suggested solution.

   A. What kind of description is best for this parcel?

      Answer: The problem did not specify a strip description, but that is the obvious choice.

   B. What kind of description would you use for the adjoining parcels to make sure they are compatible?

      Answer: The problem does not ask us to write the descriptions for the adjoining parcels, but it is a good idea to get in the habit of thinking along those lines. If pipeline A is to extend into Lot 1, the Point of Beginning (POB) for the description in Lot 1 would have to be the same point as the one reached by the description for pipeline A at the North line of the Tarantino parcel. The same is true for pipeline B.

C. What point is best for the POB?

Answer: We are told to begin the description at the southwest corner of Lot 3 and to place the TRUE POINT OF BEGINNING at the South line of the Tarantino parcel so we should “commence” at the southwest corner of Lot 3 and place the TRUE POINT OF BEGINNING at the intersection of the pipeline and the southerly line of the Tarantino parcel.

D. Where should the basis of bearings be established?

Answer: The obvious choice is along the south line of Lot 3. We must be careful to say “along the south line of said Lot 3” and to give the bearing of that line and not just a general direction such as “easterly.”

E. Clockwise or counterclockwise?

Answer: This question does not apply in this case.

F. What calls should be made?

Answer: The southwest corner of Lot 3 should be called out for the POINT OF COMMENCEMENT, the south line of the lot should be recited to establish the basis of bearings and calls should be made to the south, west, and north lines of the Tarantino parcel.

G. What clauses need to be added?

Answer: A clause or clauses need to be added to make sure that the sidelines of the strips will be prolonged or shortened so as to terminate at the south, the north and the west lines of the Tarantino parcel. Also, an exception should be added to except out that portion of the description for pipeline A which will probably be duplicated in the description for pipeline B. A clause stating that the sidelines for the easement for pipeline B should be shortened so as to commence at the westerly sideline of the easement for pipeline A would accomplish the same thing.

Here is how the description might be written:

EXHIBIT A

Parcel 1: An easement for pipeline purposes in and to the northerly 150.00 feet of the easterly 200.00 feet of Lot 3 of Block 14 as shown on Map 3619 filed in Book 36, page 54 of Maps in the Office of the Recorder in the County of Testing, State of California included within a strip of land 30.00 feet wide, lying 10.00 feet on the left and 20.00 feet on the right, looking in the direction of the traverse, of the following described line:

COMMENCING at the southwest corner of said Lot 3; thence, along the south line of said lot, S 89° 57' 17" E, 171.02 feet; thence

N 16° 08' 27" E, 68.37 feet; thence northeasterly along a tangent curve to the right, having a radius of 100.00 feet, through a central angle of 39° 28' 04", an arc length of 68.88 feet to a point hereinafter referred to as Point A; thence N 55° 36' 31" E, 52.27 feet to the south line of the northerly 150.00 feet of said Lot 3 and the TRUE POINT OF BEGINNING; thence continuing N 55° 36' 31" E, 25.17 feet to a point that bears N 55° 36' 31" E, 77.44 feet from said Point A; thence N 7° 51' 30" E, 60.00 feet to a point hereinafter described as Point B; thence N 7° 51' 30" E, 77.04 feet to the north line of said Lot 3.

The sidelines of said strip shall be prolonged or shortened so as to terminate at the southerly and northerly lines of said northerly 150.00 feet of the easterly 200.00 feet of said Lot 3.

Parcel 2: An easement for pipeline purposes in and to the northerly 150.00 feet of the easterly 200.00 feet of Lot 3 of Block 14 as shown on Map 3619 filed in Book 36, page 54 of Maps in the Office of the Recorder of the County of Testing, State of California included within a strip of land 20.00 feet wide, lying 10.00 feet on each side of the following described line:

Beginning at Point B described in Parcel 1 above; thence N 79° 33' 12" W, 103.95 feet to the westerly line of said easterly 200.00 feet.

The sidelines shall be prolonged or shortened so as to begin at the westerly line of said 30.00-foot easement described in Parcel 1 above and to terminate at the westerly line of said easterly 200.00 feet.

3. NOTE: We will dispense with the questionaire for Parcels 1 and 2, but we will use it for Parcel 3. Here are the questions and answers relative to Parcel 3:

   A. What kind of description is best for this parcel?

      Answer: The obvious answer in this case is a strip description.

   B. This question does not apply here.

   C. What point is best for the POB?

      Answer: It is impossible to calculate a POB on the south line of Parcel 1 or the west line of Parcel 2 without mixing field and record data. It is also obvious that the easement must reach the well which is tied to the northeast corner, so let’s use that as the POB.

   D. Where should the basis of bearings be established?

      Answer: The drawing shows the access road to be parallel with and westerly, 10.00 feet from the east line of the parcel so we can use that.

Calling that course parallel with the east line and giving the bearing of that east line should do it.

E. Clockwise or counterclockwise?

Answer: Using the well as the POB forces us to run southerly.

F. What calls should be made?

Answer: The POB must be tied securely to the northeast corner of the parcel. Calls should also be made parallel with the east line of the parcel and to the south line of the parcel.

G. What clauses need to be added?

Answer: The easement for a well (deed Parcel 1) the easement for a pipeline (deed Parcel 2) and the access easement (deed Parcel 3) are separate rights and it is proper that they overlap.

We do need a clause, however, to require that the sidelines of the access easement terminate at the south line of Parcel 1 of the parcel map.

Here are the descriptions for the three easements:

Parcel 1: An easement for a well in and to the northerly 25.00 feet of the easterly 25.00 feet of Parcel 1 as shown on Parcel Map No. 12,000 recorded in Book 14 of Parcel Maps at page 35 in the Office of the Recorder of the County of Calaveras, State of California.

Parcel 2: An easement for a pipeline in and to the easterly 10.00 feet of said Parcel 1 of said Parcel Map No. 12,000.

Parcel 3: An easement for access in and to said Parcel 1 of said Parcel Map No. 12,000, consisting of a strip of land 20.00 feet wide, lying 10.00 feet on each side of the following described line:

Beginning at a point in Parcel 1 of said Parcel Map No. 12,000 lying 10.00 feet westerly, measured at right angles, from the easterly line of said Parcel 1 of said parcel map and 10.00 feet southerly, measured at right angles, from the northerly line of said Parcel 1 of said parcel map; thence South, parallel with said east line of said parcel map, 180.00 feet; thence southerly and southwesterly along a tangent curve to the right, having a radius of 250.00 feet, through a central angle of 52°, an arc length of 226.90 feet; thence S 52° 00' 00" W, 420.00 feet; thence southwesterly along a tangent curve to the left, having a radius of 500.00 feet, through a central angle of 17°, an arc length of 148.35 feet; thence S 35° 00' 00" W to the south line of said Parcel 1 of said Parcel Map.

The sidelines of said easement shall be prolonged or shortened so as to terminate at the South line of said Parcel 1 of said Parcel Map.

The easements described in Parcels 1, 2, and 3 above are intended to be appurtenant to and for the benefit of Parcels 2 and 3 of said Parcel Map No. 12,000.

4. A. The West 20 feet of Lot 11.

   B. Beginning at the northwest corner of Lot 11, Block B of Hobart Tract as shown on Map recorded April 15, 1939 in Book 3, page 27 of Maps in the Office of the Recorder of _______ County; thence East along the north line of said lot, 20.00 feet; thence southerly, parallel with the west line of said lot, 120.00 feet to the south line of said lot; thence West, along said south line, 20.00 feet to the west line of said lot; thence, North, along said west line to the Point of Beginning.

   NOTE: Calling “along the south line of First Avenue” would not convey the underlying fee in the street.

Appendix 15-A

Miscellaneous Clauses

This list includes some specially written clauses and suggested language to be used in deed writing. These phrases may be used as is or adapted to fit specific needs.

Preambles

MC-1 Portion of an Aliquot Part of a Section.

"That portion of the northeast quarter of the northwest quarter of the northeast quarter of Section 10, T2S, R6E, SBM, in the unincorporated portion of the County of _____, State of California, according to the official plat of said land approved September 15, 1884, described as follows:"

MC-2 Lot and Block.

"That portion of Lot 3 of Tract No. 22886, in the City of ______, County of ______, State of California, as shown on map recorded in Book 604 of Maps, pages 20 and 21 in the Office of the County Recorder of said county, described as follows:"

MC-3 Ranchos.

"That portion of the Rancho ____, in the City of ____, County of ____, State of California, as shown on map recorded in Book ___, page ___ of Patents, in the Office of the County Recorder of said county, described as follows:"

MC-4 Line Descriptions.

"That portion of____ lying northerly of the following described line:"

MC-5 Strip Descriptions.

"That portion of____ included within a strip of land, 10.00 feet wide, lying 5.00 feet on each side of the following described line:"

or

"An easement for ________ in and to that portion of____ included within a strip of land, 10.00 feet wide, lying 5.00 feet on each side of the following described line:"

MC-6 Inclusive Descriptions.

"That portion of _____ included within the following described lines:"

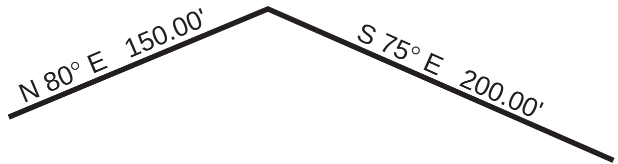
or

"An easement for _______ in and to that portion of ___ included within the following described lines:"

Basic Statements

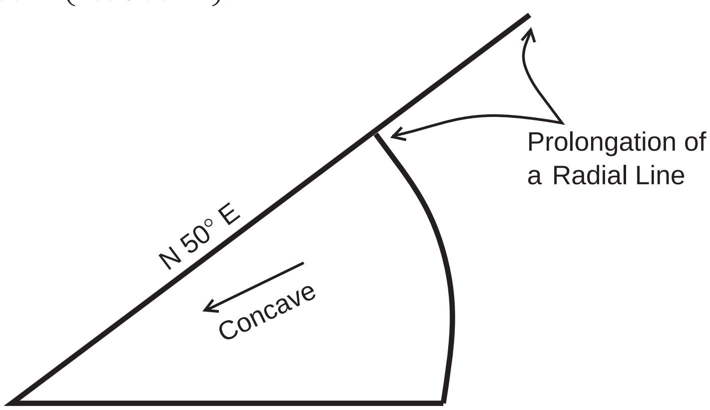
MC-11 Straight Lines.

"...thence N80° E, 150.00 feet; thence S75° E, 200.00 feet..."



MC-12 Radial Lines.

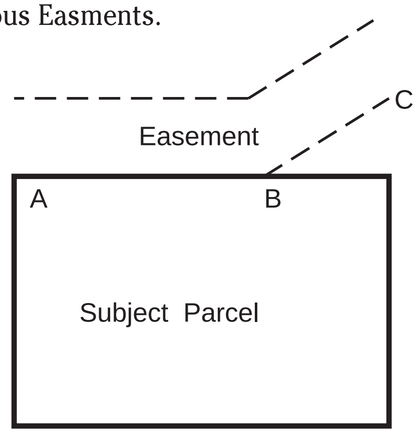
A radial line *radiates* from the center of the circle to a point on the circumference. In this case, the beginning on the radial line would be N 50° E (not S 50° W).



MC-13 Adjacent, Adjoining, and Contiguous Easments.

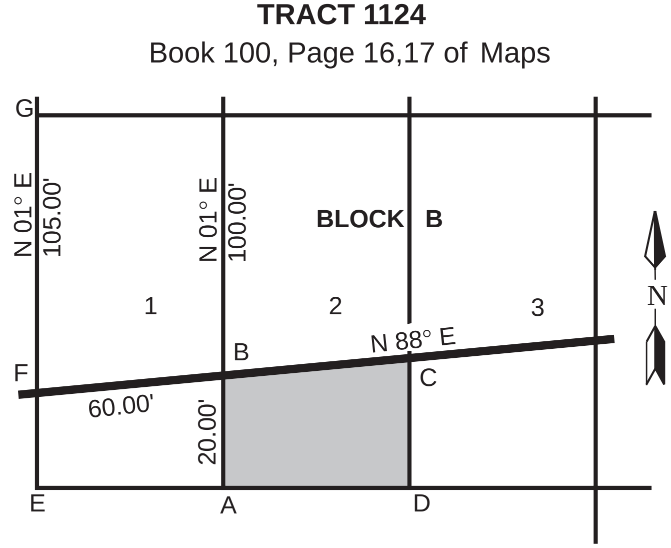
"The entire easement is *adjacent* to the subject parcel; that portion of the easement from A to B *adjoins* the parcel. That portion of the easement from B to C is *adjacent*, but not *adjoining*."

(Definitions of the word *contiguous* are contradictory and the word should be avoided.)



MC-14 Point of Beginning.

Caution: Use only when certain of lot corner's location.



"...BEGINNING at the southwest corner of said Lot 2..."

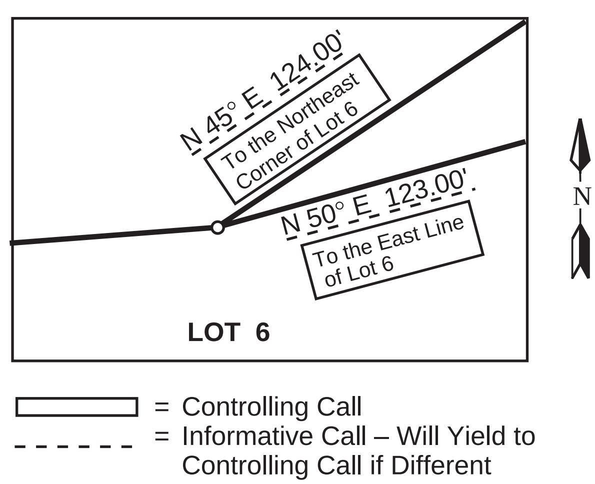
or

"...BEGINNING at a point on the west line of said Lot 2 distant thereon S 1° W, 100.00 feet from the northwest corner of said Lot 2..."

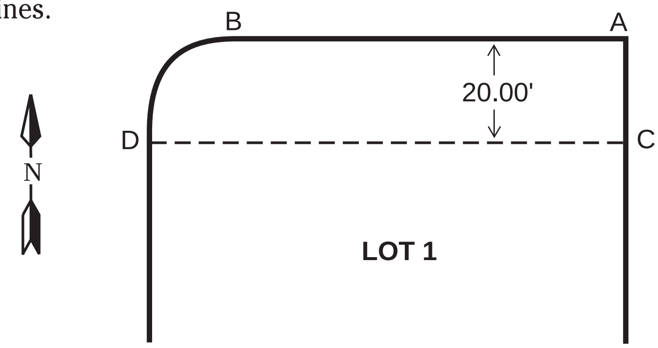
or

"...COMMENCING at the northwest corner of Lot 1 of said Tract 1124, thence along the west line of said Lot 1, S1° W, 105.00 feet; thence N88° E, 66.00 feet to the west line of said Lot 2 and the TRUE POINT OF BEGINNING..." (This last version should be used if the west Line of Lot 2 is uncertain.)

MC-15 Calls.



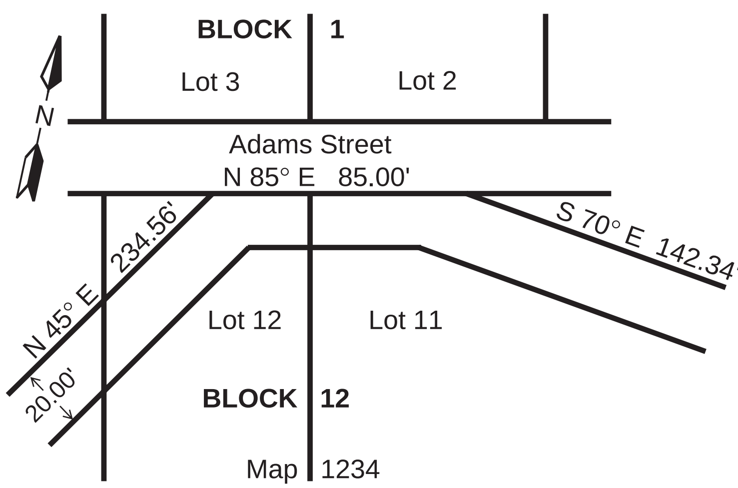
MC-16 Parallel Lines.



All of line C-D is parallel with A-B even though it is longer. It is not necessary to describe C-D as being parallel with A-B "and its westerly prolongation." C-D can safely be described as follows:

"...a line parallel with, and distant southerly 20.00 feet, measured at right angles from the tangent portion of the northerly boundary of said Lot 1..."

MC-17 Street Lines—One Tract.



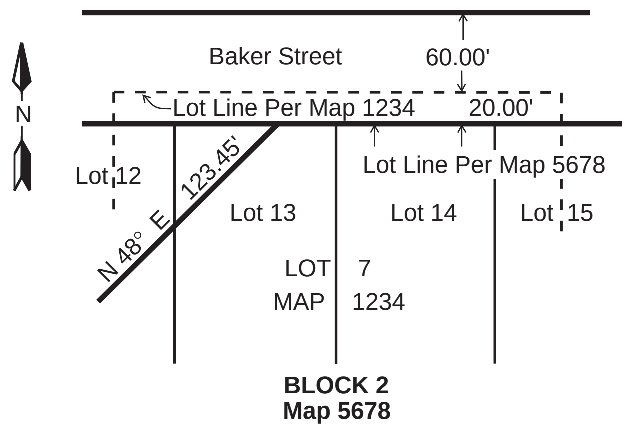
In this case, Adams Street and the lots were created on Map 1234 recorded April 1, 1956 in Official Records of ______ County, CA. When describing the northerly line of the easement:

"...to the south line of Adams Street as shown on Map No. 1234 Recorded April 1, 1956 in the Official Records of ______ County, CA..."

or

"...to the north line of said Lot 12, Block 2 as shown on Map 1234, recorded April 1, 1956 in the Official Records of ______ County, CA ..."

MC-18 Street Lines—Two Tracts.



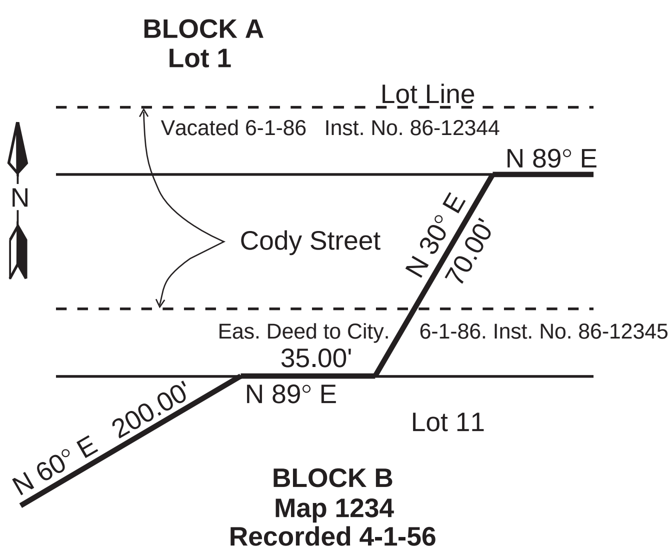
In this case Lot 7 of Map 1234 was further subdivided by Map 5678 which added 20.00 feet to the width of Baker Street.

"...to the north line of Lot 13, Block 2 as shown on Map 5678 recorded July 1, 1971 in Official Records of ______ County, CA..."

or

"...to the south line of Baker Street as shown on Map 5678 recorded July 1, 1971 in Official Records of ______ County, CA..."

MC-19 Street Lines—Easement and Vacation.



Lot lines don't change. Even though a strip had been added to the street on the south and a portion of the street vacated on the north side, the lot lines remain where they were. Calls to the current street sidelines would be made like this:

"...to the southerly line of Cody Street as described in deed to the City of ______, recorded June 1, 1986 as Instrument No. 86-12345 of Official Records of ______ County; thence, along said southerly line, N89° E, 35.00 feet; thence N30° E, 70.00 feet to the northerly line of Cody Street, being the southerly line of that portion of said Cody Street vacated and closed to public use by Instrument No. 12344 recorded June 1, 1986 in said Official Records..."

Curve Descriptions

MC-21 Tangent Curves.

"...thence, northeasterly and easterly along a tangent curve to the right, having a radius of 50.00 feet, through a central angle of 80°, an arc length of 69.81 feet..."

or

"...to a tangent curve concave southerly and having a radius of 50.00 feet; thence northeasterly and easterly along said curve through a central angle of 80°, an arc distance of 69.81 feet..."

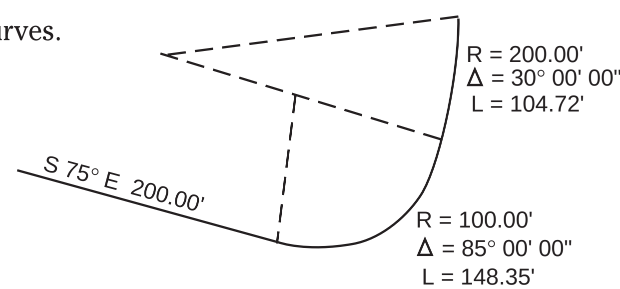
MC-22 Non-Tangent Curves.

"...to a non-tangent curve; thence, easterly from a tangent that bears N 60° E, along a curve to the right, having a radius of 50.00 feet, through a central angle of 80°, an arc length of 69.81 feet..."

or

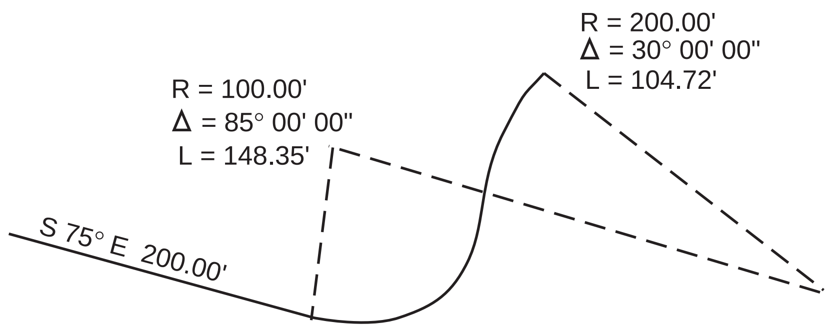
"...to the beginning of a non-tangent curve concave southerly and having a radius of 50.00 feet, to which beginning of curve a radial bears N 30° W; thence easterly along said curve through a central angle of 80°, an arc distance of 69.81 feet..."

MC-23 Compound Curves.



"...thence easterly and northeasterly along a tangent curve to the left, having radius of 100.00 feet, through a central angle of 85° 00' 00", an arc length of 148.35 feet to the beginning of a compound curve concave westerly and having a radius of 200.00 feet; thence northerly along said compound curve through a central angle of 30° 00' 00", an arc length of 104.72 feet..."

MC-24 Reverse Curves.



"...thence easterly and northeasterly along a tangent curve to the left, having a radius of 100.00 feet, through a central angle of 85° 00' 00", an arc length of 148.35 feet to the beginning of a reverse curve concave to the southeast having a radius of 200.00 feet; thence northeasterly along said reverse curve through a central angle of 30° 00' 00", an arc length of 104.72 feet..."

Strip Descriptions

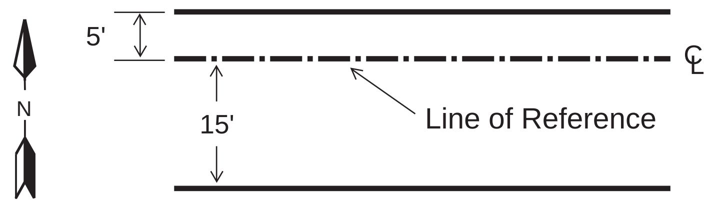
MC-31 Describing the Centerline.

"...A strip of land 20 feet wide, the centerline of which being described as follows..."

or

"...A strip of land 20 feet wide lying 10 feet on each side of the following described line..."

MC-32 Use when the line of reference is inside the strip but is not the centerline and the entire strip runs basically in one direction.



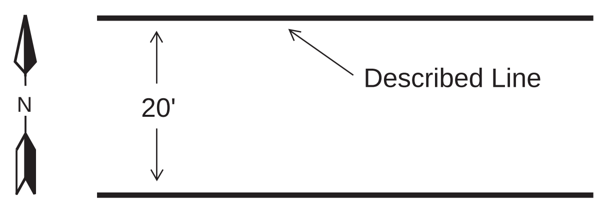
"...A strip of land 20 feet wide lying 5 feet northerly and 15 feet southerly of the following described line..."

or

Use in situations where the strip may change direction.

"...A strip of land 20 feet wide lying 5 feet on the left and 15 feet on the right, looking in the direction of the traverse, of the following described line..."

MC-33 Use when one sideline of the strip is to be the line of reference and the entire strip runs basically in one direction.



"...A strip of land 20 feet wide, the northerly sideline being described as follows..."

or

Use in situations where the strip changes directions.

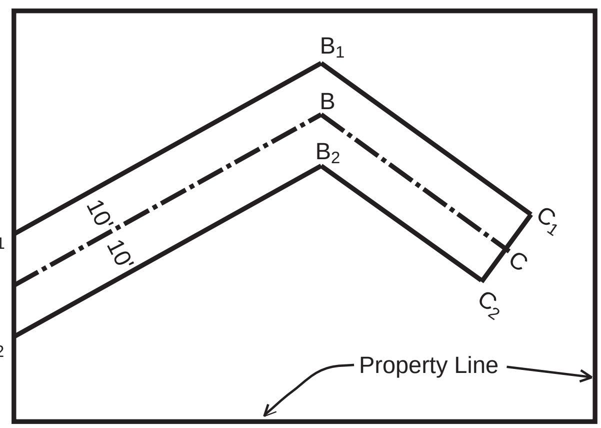
"...A strip of land 20 feet wide, the left sideline of which, looking in the direction of the traverse, being described as follows..."

MC-34 Use when the line of reference must tie to a line parallel with a record line.

"...to a line parallel with and distant southerly, 20 feet, measured at right angles from the north line of said lot, thence along said parallel line..."

MC-35 To make sure that the sidelines terminate at the property line as required, use a clause similar to this:

"The sidelines of said strip should be prolonged or shortened so as to begin at the west line of said property."



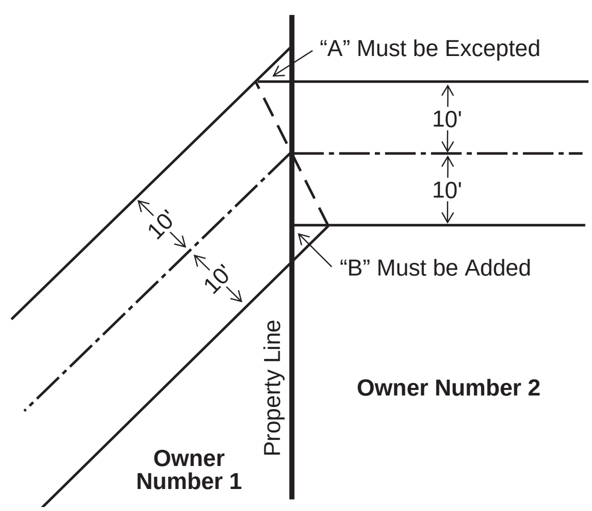
MC-36 If the strip terminates inside the property, the last point on the line of reference should be called the Point of Termination. Then if the strip ends at a course that is at right angles to the line of reference, the following form should be used:

"The sidelines of said strip shall terminate at a line running through said Point of Termination at right angles to last said course."

or

Use this form if the strip ends at a line that is not at right angles to the last course.

"The sidelines of said strip shall terminate at a line that bears ________ running through said Point of Termination."



MC-37 Use this when the strip description from Owner No. 1 terminates at the property line.

"The sidelines of said strip shall be prolonged or shortened so as to terminate at the east line of said property. EXCEPTING therefrom..." (Describe Parcel A.)

(An alternate method of handling this would be to continue the strip description past the property line and then except out that portion east of the line. That way the small triangle would not have to be described.)

or

Use this when the strip description from Owner No. 2 begins at the property line.

"The sidelines of said strip shall be prolonged or shortened so as to begin at the west line of said property. TOGETHER WITH the following described parcel..." (Describe Parcel B.)

(An alternate method of handling this, as in MC-37A would be to begin the description west of the property line and except out that portion west of the line. That way Parcel B would not have to be described.)

Temporary Construction Easements

MC-51 "A temporary easement for a highway detour in and to..."

MC-52 "A temporary easement for contour grading purposes in and to..."

MC-53 "It is understood that said easement is temporary and shall terminate either upon completion of construction of that portion of Interstate (state) Highway Route _______ lying adjacent to the above-described parcel or December 30, 19_____, whichever date occurs first. It is also understood that upon said termination date the State shall have no further obligation or liability in connection with said parcel."

Misc. Equal Area, Coordinate Clause, Underlying Fee

MC-72 May be used on part takes, fee or easement.

"Containing ________ square feet (acres), more or less."

MC-74 May be used on part takes when description includes a portion of a public way and its area is known.

"Containing ______ square feet (acres), more or less, in addition to ______ square feet, more or less, included within the adjoining public way(s)."

MC-76 May be used on part takes when description includes a portion of a public way and its area is unknown.

"Containing _______ square feet (acres), more or less, in addition to that portion included within the adjacent public way."

MC-78 Use on deeds with grid bearings and distances.

"The bearings and distances used in the above description(s) are on the California Coordinate System of 1927 (or 1983, whichever is appropriate), Zone ____. Multiply all distances used in the above description(s) by _____________ to obtain ground level distances."

MC-79 "Together with underlying fee interest, if any, adjacent to the above described property in and to the adjoining public way."

References

_______ , *Black's Law Dictionary,* Fifth Edition, West Publishing Co., 1979.

________ , *California Department of Transportation, Surveys Manual,* Chapter 6.

_______ , *Definitions of Surveying and Associated Terms,* ACSM and ASCE, 1978.

_______ , *Titleman's Handbook,* Ticor Title Insurance Co., 1717 Walnut Grove Ave., Rosemead, CA 91770, Attention Forms Management. (The price is $19.50 including tax, handling, and shipping—highly recommended.)

Brown, Curtis M., *Boundary Control and Legal Principles,* Third Edition, John Wiley and Sons, New York, 1986.

Chelapati, C.V., Editor, *Land Surveyor License Examinations Review Manual,* California State University, Long Beach, 1987. (Highly recommended.)

Walpole, Jane, Ph.D., *The Writer's Grammar Guide,* Simon and Schuster, 1984. (Quick help on grammar).

Wattles, Gurdon H., *Writing Legal Descriptions,* Gurdon H. Wattles Publications, Orange, CA, 1979.

UNIT 16

CALIFORNIA LAW FOR SURVEYORS

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Nowack and Associates, Inc.
Santa Clara, CA

Introduction

There are two specific sections of California statute law that are covered on the Land Surveyor exam. The Professional Land Surveyors Act is the code of law that governs the practice of surveying, outlines the requirements and procedures for licensure, defines surveying, and includes the provisions regulating records of surveys and corner records. The Subdivision Map Act is the series of laws that allows local governments to control and regulate the subdivision of land. The Map Act includes the requirements for the preparation and processing of tentative, parcel, and final maps, along with other subdivision requirements.

Only California law will be covered on the LS exam. Questions normally require citation of sections from the appropriate statutes. Quick retrieval of information is of utmost importance, so surveyors should develop a personalized index system for the latest revision of the statutes.

Performance Expected on the Exams

Learning Objectives

Define surveying practice, what services require licensure, and what titles are reserved for licensure.

Outline the requirements for licensure.

Outline the requirements for a record of survey and corner record.

Explain the various exclusions and exemptions to the Subdivision Map Act.

Outline the purposes and requirements for the three different types of maps defined in the Subdivision Map Act.

Explain the requirements for lot line adjustments and Certificates of Compliance.

Key Terms

Constructive notice

Professional Land Surveyors Act

Board of Registration for Professional Engineers and Surveyors

Improvement

Construction surveying

Topographic surveying

Cadastral surveying

Subdivision surveying

Boundary surveying

Geodetic surveying

Legal descriptions

Photogrammetry

Record of survey (ROS or R/S)

Basis of bearings

Memorandum of oaths

Advisory agency

Appeal board

City engineer/surveyor

Design

Local agency

Local ordinance

Subdivider

Subdivision

Contiguous parcels

Preemption doctrine

Tentative map

Parcel map

Final map

Major subdivision

Minor subdivision

Lot line adjustment

Right of entry	Certificate of compliance
Corner record	Lot merger
Subdivision Map Act	Amended map
Subdivided Lands Act	Certificate of correction

Video Presentation Outline

Professional Land Surveyors Act (PLSA)–Sections 8700 to 8806 of the Business and Professions Code

Article 1: General provisions

Article 2: Administration

Article 2.3: Survey review committee

Article 3: Application of the chapter

Article 4: Issuance of license

Article 5: Surveying practice

Article 5.5: Photogrammetry

Article 6: Disciplinary proceedings

Article 7: Offenses against the chapter

Article 8: Revenues

Article 3: Application of the Chapter

- Definition of survey practice (8726)
- Exceptions to licensure (8730)
- Titles that require licensure (8708, 8775, 8751)

Article 5: Surveying Practice–Record of Surveys

- Mandatory filing requirements (8762)
- Subdivision of land not allowed (8762.5)
- Form (8763)
- Technical requirements (8764)
- Required statements (8764.5)
- Exemptions (8765)
- Examination and fees (8766, 8766.5)
- Corrections (8770.5 – SMA Article 7)

Guide to the Preparation of Records of Survey and Corner Records

See Figure 16-1.

Corner Records (8773–8773.4)

- Requirements and exemptions for filing (8773, 8765d.)
- Form and content (8773.1)

Board Rule 464

See Figure 16-2.

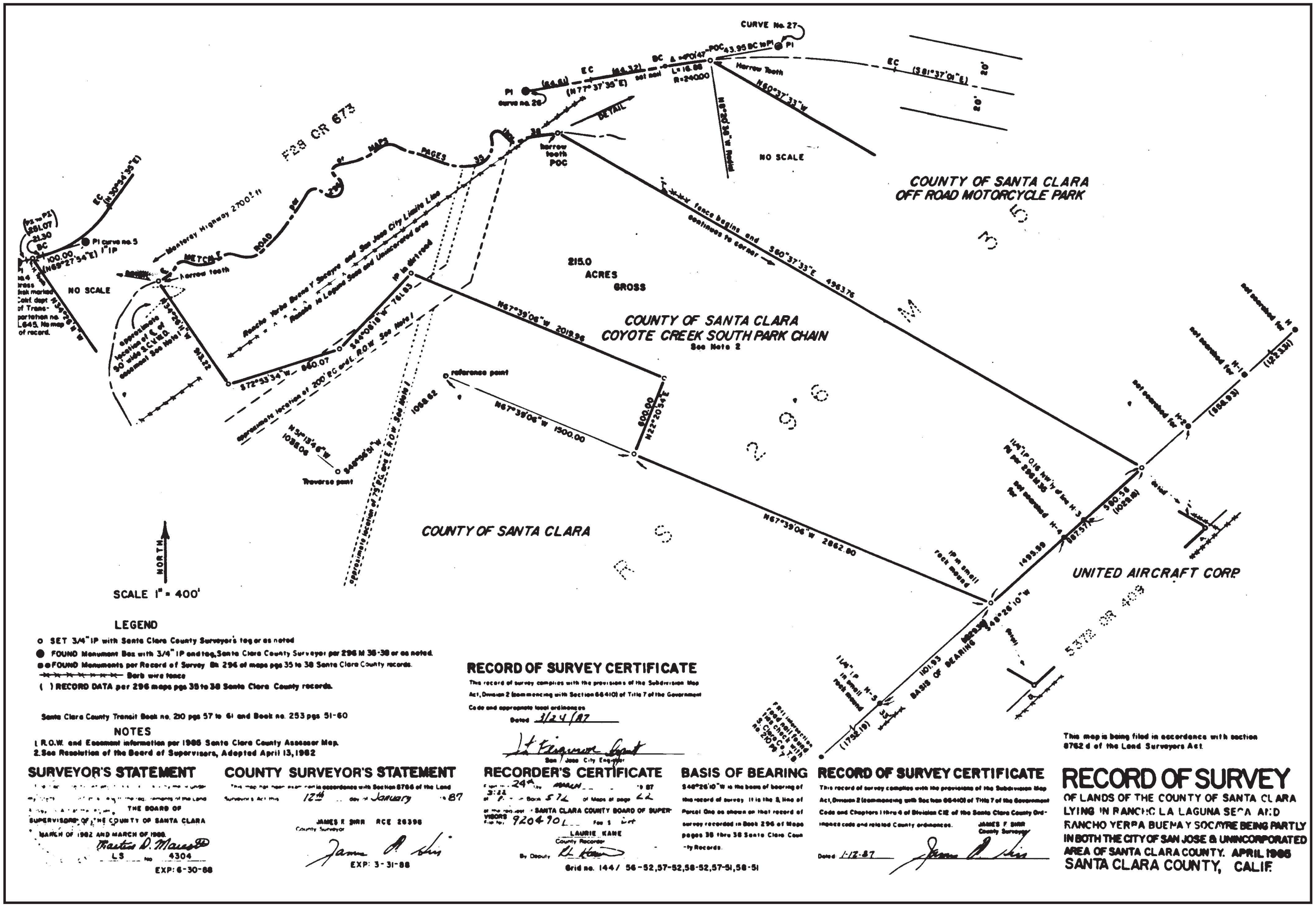


Figure 16-1. Record and survey.

STATE OF CALIFORNIA • DEPARTMENT OF TRANSPORTATION

CORNER RECORD

DPD-3011 (Rev. 9/97) CT# 7541-4548-0

Document Number ____________________

City of ____________________ County of ____________________ , California

Brief Legal Description ____________________

CORNER TYPE

Government Corner ☐ Control ☐
Meander ☐ Property ☐
Rancho ☐ Other ☐

Date of Survey ____________________

COORDINATES (Optional)

N. ____________________
E. ____________________
Zone ____________ NAD27 ☐ NAD83 ☐
NAD83 Epoch ____________________
Elevation ____________________
Vert. Datum: NGVD29 ☐ NAVD88 ☐
Meas. Units: Metric ☐ Imperial ☐

Corner - Left as found ☐ Found and tagged ☐ Established ☐ Reestablished ☐ Rebuilt ☐

Identification and type of corner found: Evidence used to identify or procedure used to establish or reestablish the corner:

____

A description of the physical condition of the monument as found and as set or reset: ____________________

SURVEYOR'S STATEMENT

This Corner Record was prepared by me or under my direction in conformance with the Land Surveyors' Act on ____________ , ________ .

Signed ____________________ P.L.S. or R.C.E. Number ____________
For the State of California
Department of Transportation

SEAL

COUNTY SURVEYOR'S STATEMENT

This Corner Record was received ____________ , ________ and examined and filed ____________ , ________ .

Signed ____________________ P.L.S. or R.C.E. Number ____________

Title ____________________

SEAL

County Surveyor's Comment ____________________

Page 1 of 2

97 37098

Figure 16-2. *Corner record form.*

Subdivision Map Act (SMA)–Sections 66410 through 66499.58 of the Government Code

Chapter 1: General provisions

Chapter 2: Maps

Chapter 3: Procedure

Chapter 4: Requirements

Chapter 4.5: Development rights

Chapter 5: Improvement security

Chapter 6: Reversions and exclusions

Chapter 7: Enforcement and judicial review

Definitions

History of the SMA

Local Subdivision Ordinances

- Preemption
- Limitations on local regulation (66411)

When is a Subdivision Map Required?

- Definition of a subdivision (66424)

  (1) The division of land;
  (2) by any subdivider;
  (3) of any unit or units of improved or unimproved land, or any portion thereof;
  (4) shown on the latest equalized County Assessment Roll;
  (5) contiguous;
  (6) for the purpose of sale, lease, or financing;
  (7) whether immediate or future.

- Contiguous parcels (see Figure 16-3).
- Gifts, inheritances, partitions, etc.
- Condominiums, cooperatives, community apartment projects.
- Government Lots.

Exclusions and Exemptions to the SMA

- Agricultural leases (66424).
- Statutory exclusions (66412).
- Conveyance to public entities (66426.5, 66428).

What Type of Subdivision Map is Required?

- Three types of maps in the Subdivision Map Act.
- Determining which type of map is required (66426).
- Final map exemptions.
- Parcel counting (see Figure 16-4).
- Tentative map content 66424.5, 66474.1 (see Figure 16-5).
- Parcel map content 66444 through 66450 (see Figure 16-6).
- Final map content 66433 through 66443.

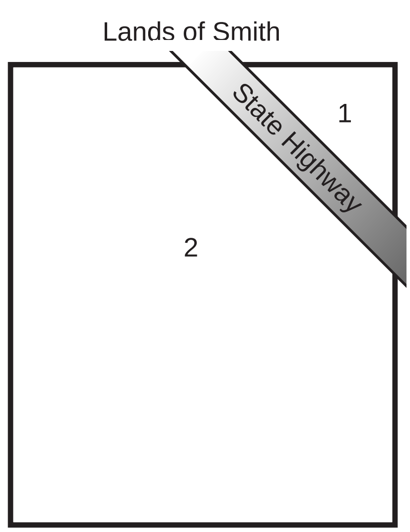


Figure 16-3. *Contiguous parcels.*

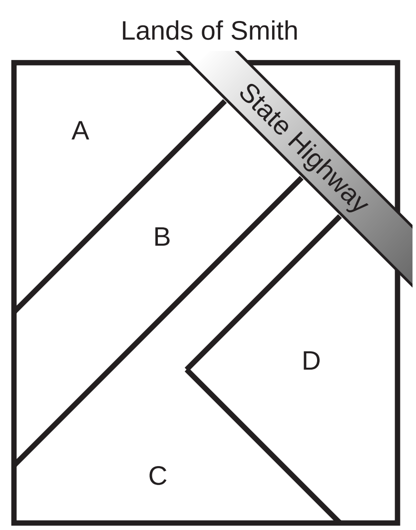


Figure 16-4. *Parcel counting.*

Chapter 3 Procedure

- Time limits (66451.1).
- Local agency fees (66451.2).
- Notice of public hearings (66451.3).
- Tentative maps (66452–66452.10).
- Final maps (66456–66462.5).
- Parcel maps (66463–66463.5).
- Recording of maps (66464–66468.1).
- Amendments to maps (66469) .

Miscellaneous Topics

- Lot line adjustment (66412).
- Mergers (66451.10–66451.21).
- Certificate of compliance (66499.35).

Figure 16-5. Tentative map.

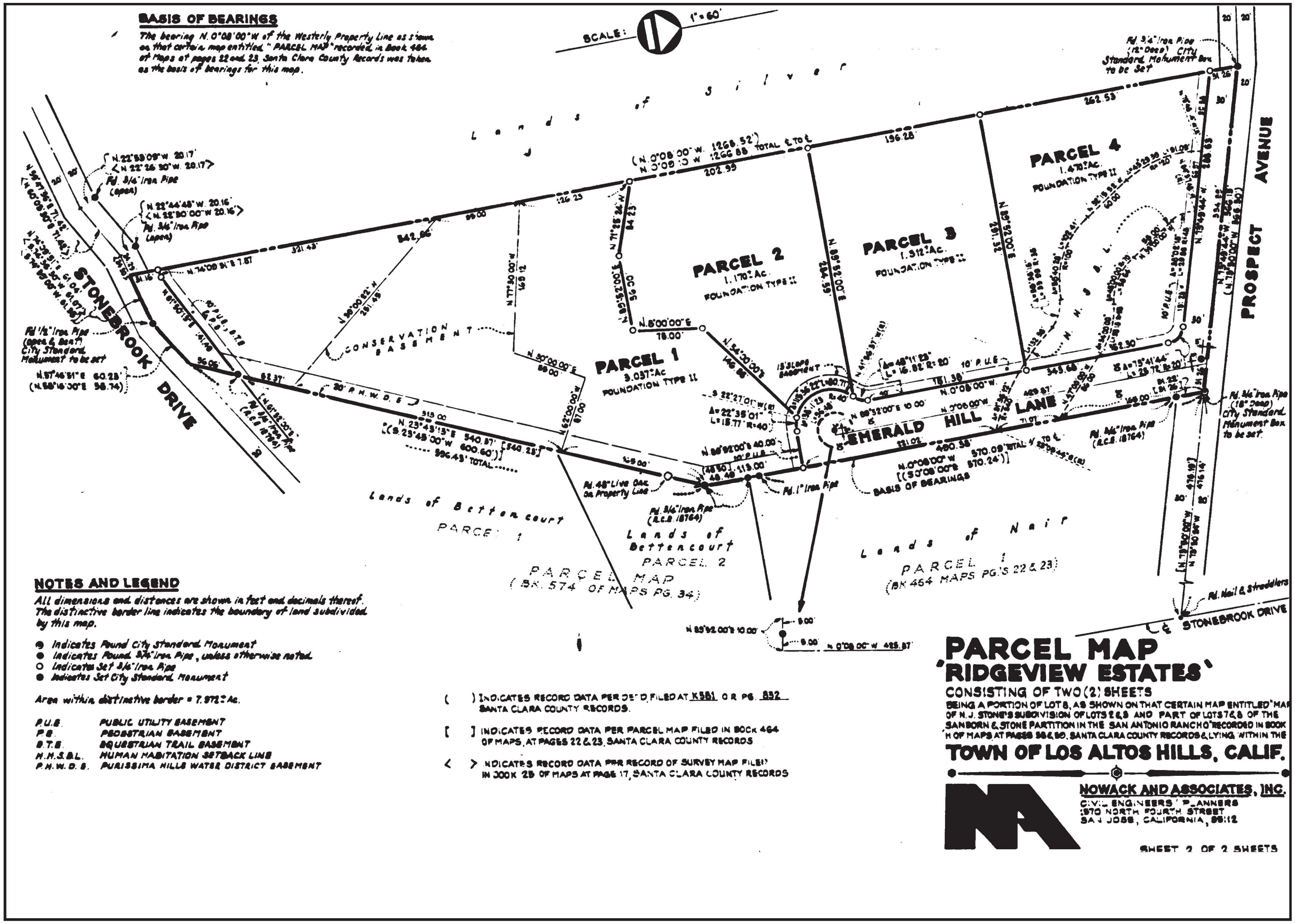


Figure 16-6. Parcel map.

Sample Test Questions

Cite a reference for each answer given.

1. List those titles that are reserved by law to licensed land surveyors.
2. On what items is it mandatory for a licensed land surveyor to place his signature and show his license number or stamp his seal?
3. Define “material discrepancy” with respect to the Professional Land Surveyors Act.
4. Define “responsible charge” with respect to the Professional Land Surveyors Act.
5. Under what conditions may a registered civil engineer practice land surveying?
6. Define “certify” with respect to the Professional Land Surveyors Act.
7. What act or activities performed by a person constitutes the practice of land surveying within the meaning of the Land Surveyors Act?
8. When may a land surveyor administer or certify oaths?
9. On January 29, 1991, you performed a survey and set the corners of the lands of V. Breshears described as follows:

   “The South 75.00 feet of the East 200.00 feet of Lot 2, Tract 500, recorded in Book 8 of Maps, Page 86, Rainbow County Records, State of California.

   The client acquired the property in 1959 from B. Rogers, who still owns the remainder of Lot 1.”

   Is a record of survey required? Explain.
10. What is the “local agency?”
11. “Subdivision” includes a condominium project as defined by what code?
12. Which section of the Subdivision Map Act covers the merger of contiguous parcels of land.
13. In your own words, define “subdivision.”
14. What is the difference between a parcel map, final map, and tentative map?
15. Under what conditions may a parcel map be based upon “record information?”
16. Can the local agency require a tentative map, parcel, map, or final map for a lot line adjustment?
17. Under what circumstances can a record of survey delineate parcels of land not shown on the latest adopted county assessment roll as a unit or contiguous units?

18. Problem D-7 1980 LS

You have been hired to prepare a parcel map from "record data." A recently filed record of survey, prepared by another surveyor, has determined the boundary. In checking for the previously set monuments, you determined that the previous surveyor has used an erroneous section corner to control his survey. This results in a difference in the position of the true corners and the monumented corners of distances from 0.5 ft to 3.2 ft. No improvements have been constructed on the property. Your client is in a rush because interest rate charges are causing problems with his potential sale.

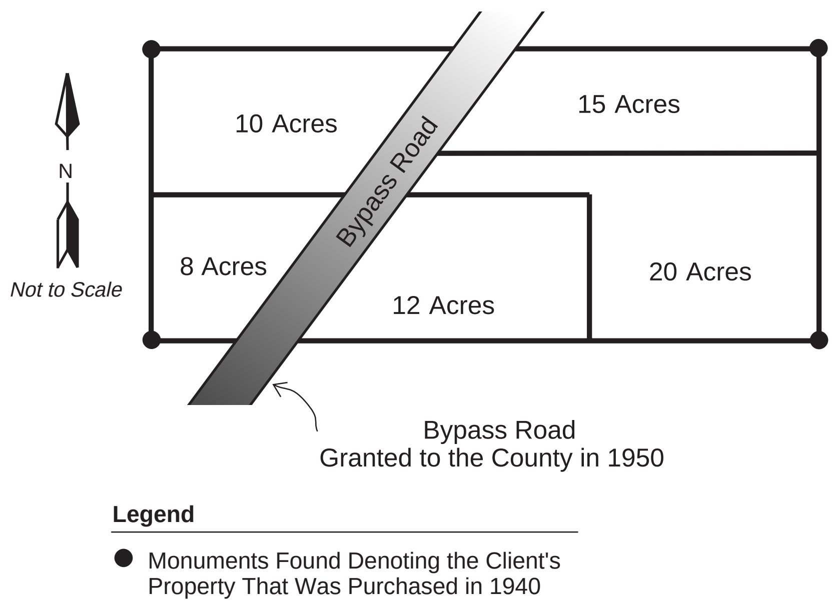
A. Describe your actions with regard to your client.

B. Describe your actions with regard to the other surveyor.

C. Describe your actions with regard to the preparation of the parcel map.

19. Problem A-1, 1989 LS

Given the information denoted in the diagram below, the client has asked you to provide surveying and mapping services necessary to create the parcels as shown.



Required:

The answers to the following questions are based upon current California law. You are to answer each of the questions briefly in your own words, or indicate the appropriate citations.

A. What type of map or maps, if any, are required for this land division?

B. If a map is required, who shall prepare it?

C. Explain whether or not the new parcels are required to be monumented.

D. If monuments are required or requested, when must they be set?

E. If a map or maps are required, under what conditions would holders of beneficial interests not be required to sign?

F. Excluding lawsuits and moratoriums, and if a final map or parcel map is required, what is the maximum time allowed to record the map?

G. Under what conditions may the monuments called for on a parcel map or final map be set by another licensed land surveyor?

H. After the completion of your work on the client's property, a tractor removed the monument that you found at the exterior southeast corner. The contractor asks you to replace the corner. What document, if any, would you prepare?

I. Assuming it is necessary to gain access to the neighboring property to conduct your field survey and the neighbors question your right to be on their property, how would you respond?

J. How can non-title information (i.e., building setback lines, etc.) be filed or recorded with a subdivision map?

Answer Key

1. LS Act 8708: Licensed land surveyor, professional land surveyor, or land surveyor.

   LS Act 8751: Professional engineer in land surveying, land survey engineer, survey engineer, geodetic engineer, and geometronic engineer.

2. LS Act 8761: Maps, plats, reports, descriptions, or other documents.

3. LS Act 8762(b): Difference in position of points, lines, or dimensions between a survey and information found on any subdivision map, official map or record of survey on file with the County Recorder or surveyor or any map or survey record maintained by the Bureau of Land Management.

4. LS 8703: Independent control and direction, by use of initiative, skill, and independent judgment of the observations, measurements, and descriptions involved in land surveying work.

5. LS Act 8731: A civil engineer registered before January 1, 1982 can practice land surveying; otherwise the engineer must obtain a land surveying license.

6. LS Act 8770.6: An expression of professional opinion regarding facts or findings which are the subject of certification. Use of the word certify does not constitute a warranty or guarantee.

7. LS Act 8726.

8. LS Act - 8760: Oaths may be used to take testimony for the identification of obliterated monuments, to perpetuate evidence of monuments found in perishable condition, and when it is desirable to administer an oath to assistants for the faithful performance of their duty.

9. LS Act 8762(d),(e): A record of survey will be required because corners not shown on a record map will be established.

10. SMA 66420: City, county, or city and county.

11. SMA 66424: Section 1350 of the Civil Code.

12. SMA Article 1.5.

13. SMA 66424 - Your definition must include the phrase “for the purpose of sale, lease, or financing.”

14. SBA Chapter 2: Tentative maps are used in the subdivision approval process and are not recorded. Parcel maps are used for land divisions of four or fewer parcels or for divisions listed in 66426 (a),(b),(c), and (d) and are recorded with the County Recorder. Final maps are required for division of five or more parcels and are recorded with the County Recorder.

15. SMA 66448: Parcel maps may be based on record information if local ordinance does not require a field survey and sufficient information exists on filed maps to locate and retrace the exterior lines of the parcel map if the location of at least one boundary line can be established from an existing monumented line.

16. SMA 66412(d): No.

17. LS Act 8762.5 SMA 66499.35: A record of survey can only show new parcels of land if it is accompanied with a Certificate of Compliance.

    SMA 66426.5: A record of survey can show a parcel of land created for conveyance to a public entity for right-of-way purposes.

18. A. I would inform my client that, because of errors on the record of survey, sufficient record data does not exist to file a parcel map based on the record information (SMA 66448).

    B. Inform the other surveyor of the problems and have him correct his survey.

    C. I would inform my client that he could wait until the other surveyor had corrected his survey and then file the parcel map based on record information, or we could go ahead with a parcel map based on a field survey. I would provide my client with estimated costs and time frames for each of these alternatives. The client would have to base his decision on his own budget and time considerations.

19. A. A tentative, final, or parcel map (SMA 66426, 66424).

    B. A licensed land surveyor or civil engineer authorized to practice land surveying (SMA 66434, LSA 8731).

    C. SMA 66495 requires sufficient durable monuments; also see LSA 8771. The surveyor must also set additional monuments as required by local ordinance (SMA 66495). (SMA 66495 does not require parcels to be monumented.)

    D. At least one exterior line shall be monumented prior to the map recording (SMA 66495). The interior monuments may be delayed to no later than a specific date noted in the surveyor’s statement (SMA 66496, 66441).

E. Beneficial interest holders or their trustees are not required to sign parcel maps (SMA 66436, 66445.3E).

F. Two years with a maximum one-year time extension if allowed by local ordinance (SMA 66452.6). (Five years acceptable per 66452.6E.)

G. Death, disability, retirement of original surveyor, or refusal (SMA 66498).

H. Corner record LSA 8773 through 8773.4, board rule 464.

I. Right of Entry (LSA 8774).

J. Supplemental information sheet may be filed concurrently with the final map or recorded as a separate document (SMA 66434.2).

References

Professional Land Surveyors Act, Sections 8700–8806, State of California Business and Professions Code.

Subdivision Map Act, Sections 66410–66499.58, State of California Government Code.

Important: These references are being constantly revised by the State Legislature. The latest version of the statutes can be obtained for $10 from:

California Land Surveyors Association
P.O. Box 9098
Santa Rosa, CA 95405

_______, *Guide to the Preparation of Records of Survey and Corner Records,* County Engineers Association of California, 1989.

This publication can be obtained by sending a check for $3.30 to:

Orange County Surveyor's Office
400 Civic Center Dr., Room 225
Santa Ana, CA 92702